# *Multiplicities in Pb-Pb collisions at the LHC from non-linear QCD evolution*

Javier L. Albacete

THE OHIO STATE UNIVERSITY

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# Outline

@ Energy dependence of nuclear gluon distributions: Balitsky-Kovchegov equation

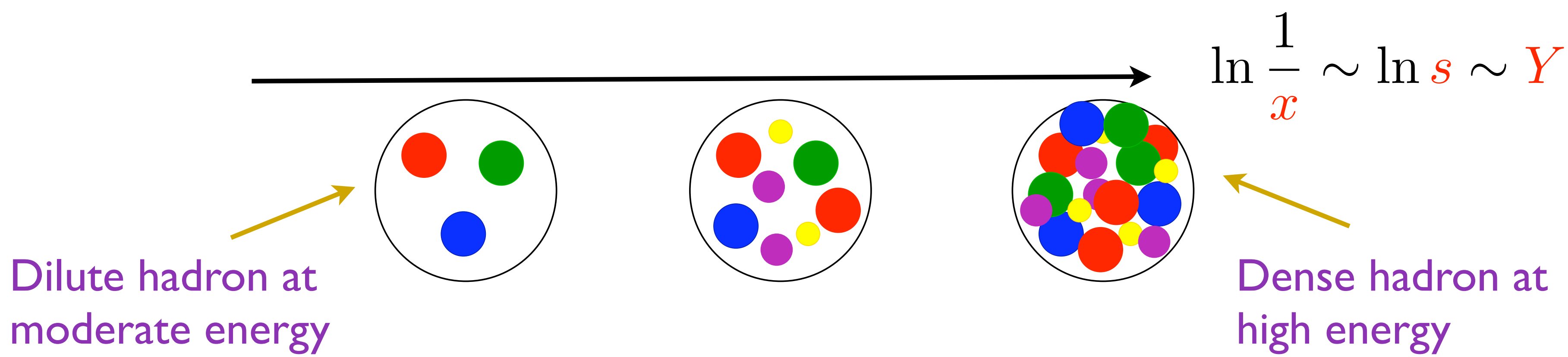
- ⇒ Recent developments: Running coupling corrections
- ⇒ Strong reduction of the speed of evolution

@ Phenomenological consequences:

- ⇒ Energy dependence of multiplicity densities in A-A collisions
- ⇒ Determining initial conditions: RHIC @ $\sqrt{s}$=130 and 200 GeV
- ⇒ Extrapolation to LHC: Pb-Pb collisions @ $\sqrt{s}$=5.5 TeV

# Motivation

- Color Glass Condensate: The high energy limit of QCD is governed by large gluon densities and gluon saturation

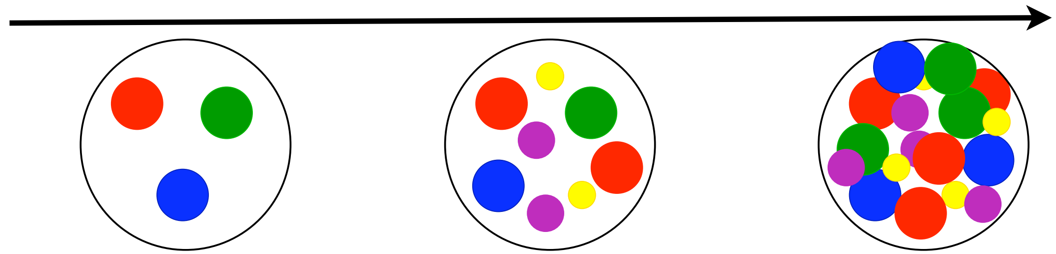



- BK-JIMWLK equations: The evolution of gluon densities to high energy is driven by linear (radiation) and non-linear (recombination) effects

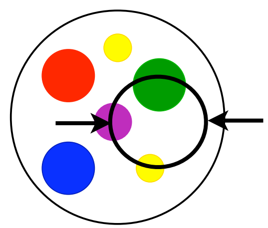
$$\frac{\partial\, \varphi(x,k)}{\partial \ln 1/x} \sim K \otimes \varphi(x,k) - \varphi^2(x,k)$$

# Motivation

⇒ Charge density correlations define the saturation scale:



$$\ln\frac{1}{x} \sim \ln s \sim Y$$



$$\begin{cases} Q_s^2(Y) = Q_0^2\,\exp\lambda Y \\ \lambda = \dfrac{d\ln Q_s^2(Y)}{dY} \end{cases}$$

⇒ LL-theory: $\lambda^{LL} \approx 4.8\alpha_s$ BK-JIMWLK at LL accuracy in $\alpha_s \ln 1/x$

⇒ Data: $\lambda^{data} \approx 0.288$ Fits to DIS and HIC data

Experimental data demand a much slower evolution!
Need of higher order corrections to LL equations

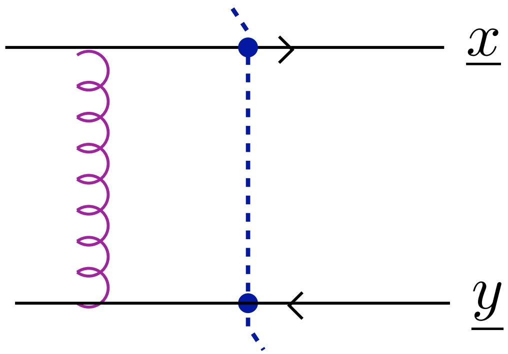
# Running coupling corrections (Kovchegov, Weigert, Balitsky, Gardi, Albacete ...

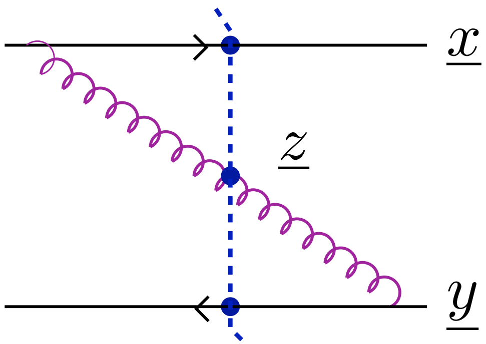
⇒ Strategy: resummation of quark loops to all orders ($\alpha_s N_f$) plus $N_f \longrightarrow -6\pi\beta$

$$\frac{\partial S(\underline{x},\underline{y};Y)}{\partial Y} = \int d^2z\, K^{LO}(\underline{r},\underline{r}_1,\underline{r}_2)\ [S(\underline{x},\underline{z})\,S(\underline{z},\underline{y}) - S(\underline{x},\underline{y})]$$

⇒ Leading log

(fixed coupling)



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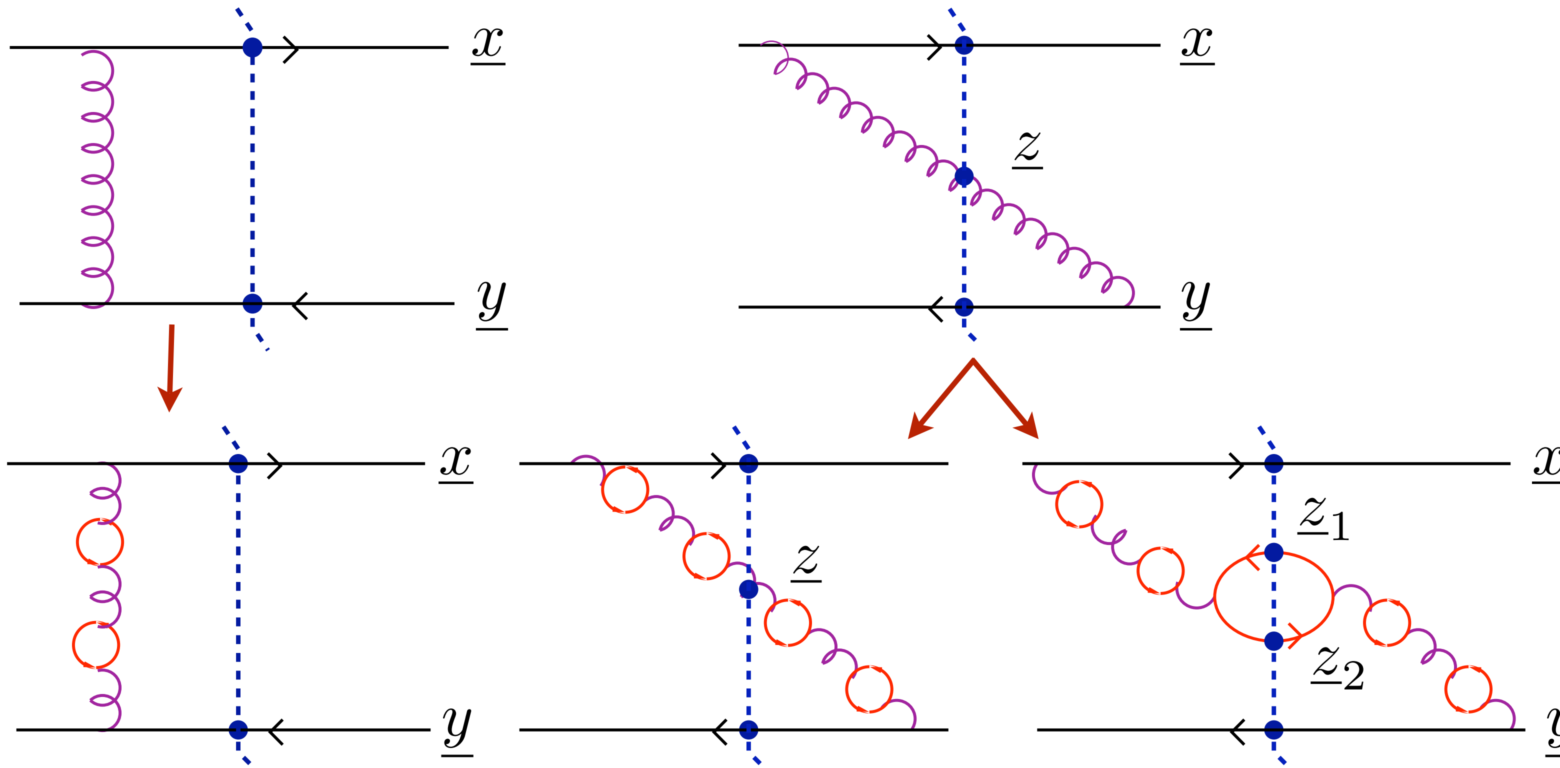
⇒ Leading log

(fixed coupling)

⇒ All orders in $\alpha_s N_f$

$N_f \longrightarrow -6\pi\beta$

(running coupling)




$\frac{\partial S}{\partial Y} \propto$ $S(\underline{x},\underline{y})$ $S(\underline{x},\underline{z})S(\underline{z},\underline{y})$ $S(\underline{x},\underline{z}_1)S(\underline{z}_2,\underline{y})$

"4-point" function

⇕

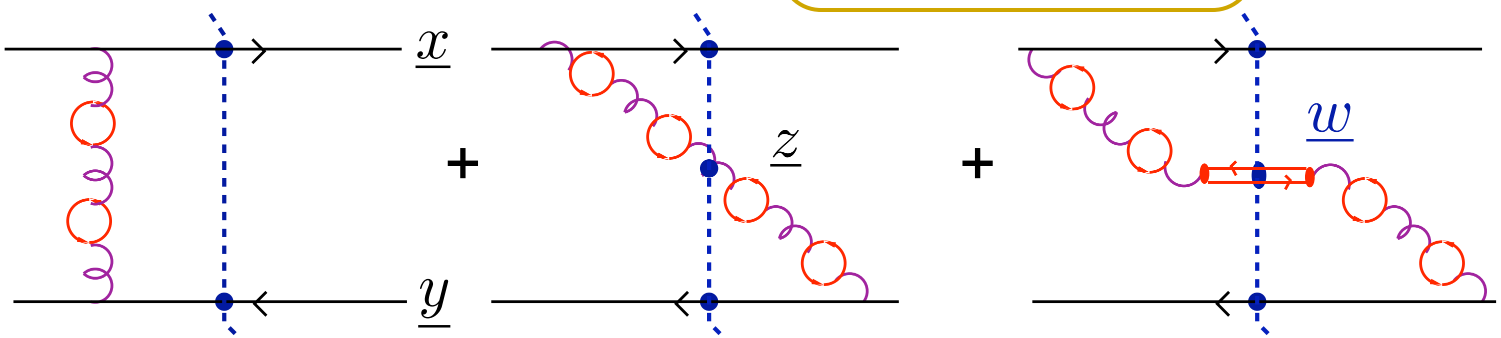
"3-point" function

⇒ The new physical channels modify the interaction structure of the LL equation.

# Complete in $\alpha_s N_f$ Evolution

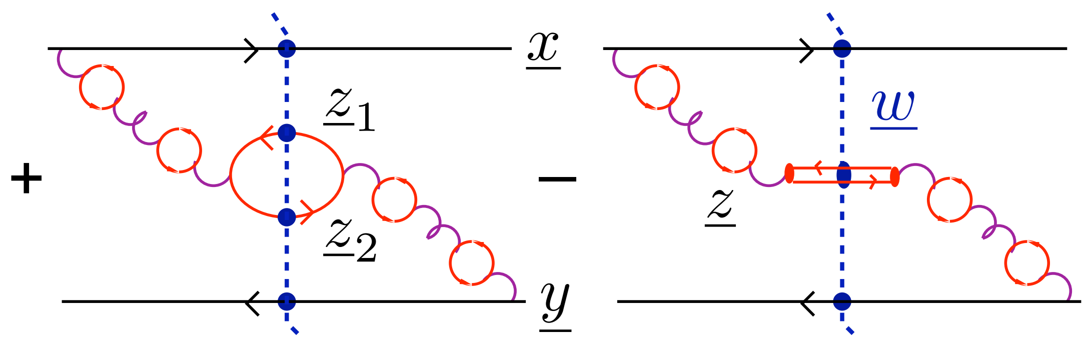
JLA and Y. Kovchegov PRD75 125021

$$\frac{\partial S}{\partial Y} = \mathcal{R}\left[S\right] - \mathcal{S}\left[S\right]$$




$\mathcal{R}[S]$

UV-divergent terms that contribute to the running of the coupling




$\mathcal{S}[S]$

Conformal, non running coupling terms. Neglected in previous calculations

⇒ ***Running*** term: $\mathcal{R}\left[S\right] = \int d^2z\, \tilde{K}(\underline{r}, \underline{r}_1, \underline{r}_2)\left[S(\underline{x}, \underline{z})S(\underline{z}, \underline{y}) - S(\underline{x}, \underline{y})\right]$

⇒ ***Subtraction*** term: $\mathcal{S}\left[S\right] = \int d^2z_1 d^2z_2\, K_{sub}(\underline{x}, \underline{y}, \underline{z}_1, \underline{z}_2)\left[S(\underline{x}, \underline{w})S(\underline{w}, \underline{y}) - S(\underline{x}, \underline{z}_1)S(\underline{z}_2, \underline{y})\right]$

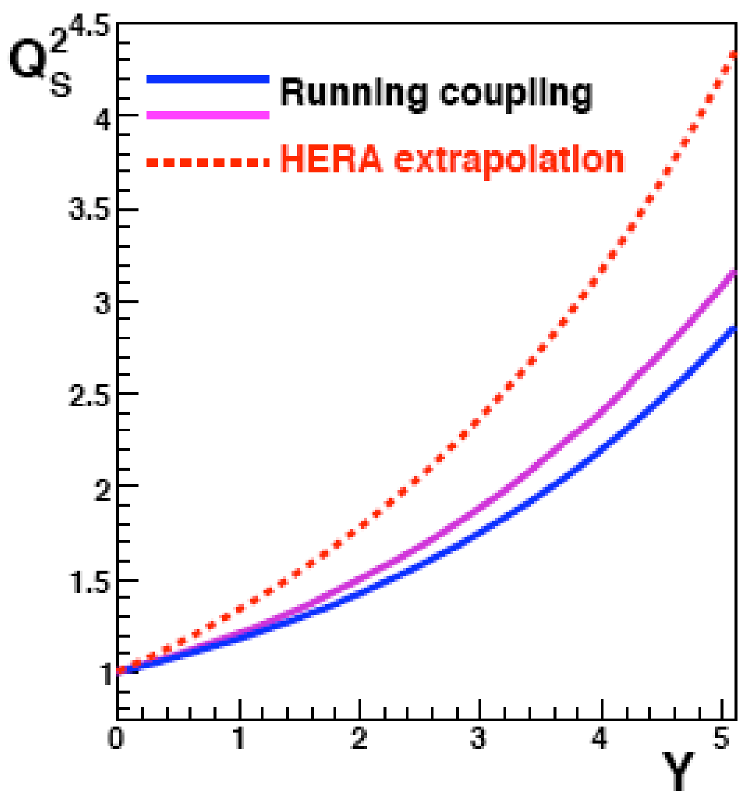
⇒ Running coupling comes in a "triumvirate": $K \sim \frac{\alpha_s(R_1)\,\alpha_s(R_2)}{\alpha_s(R_3)}$

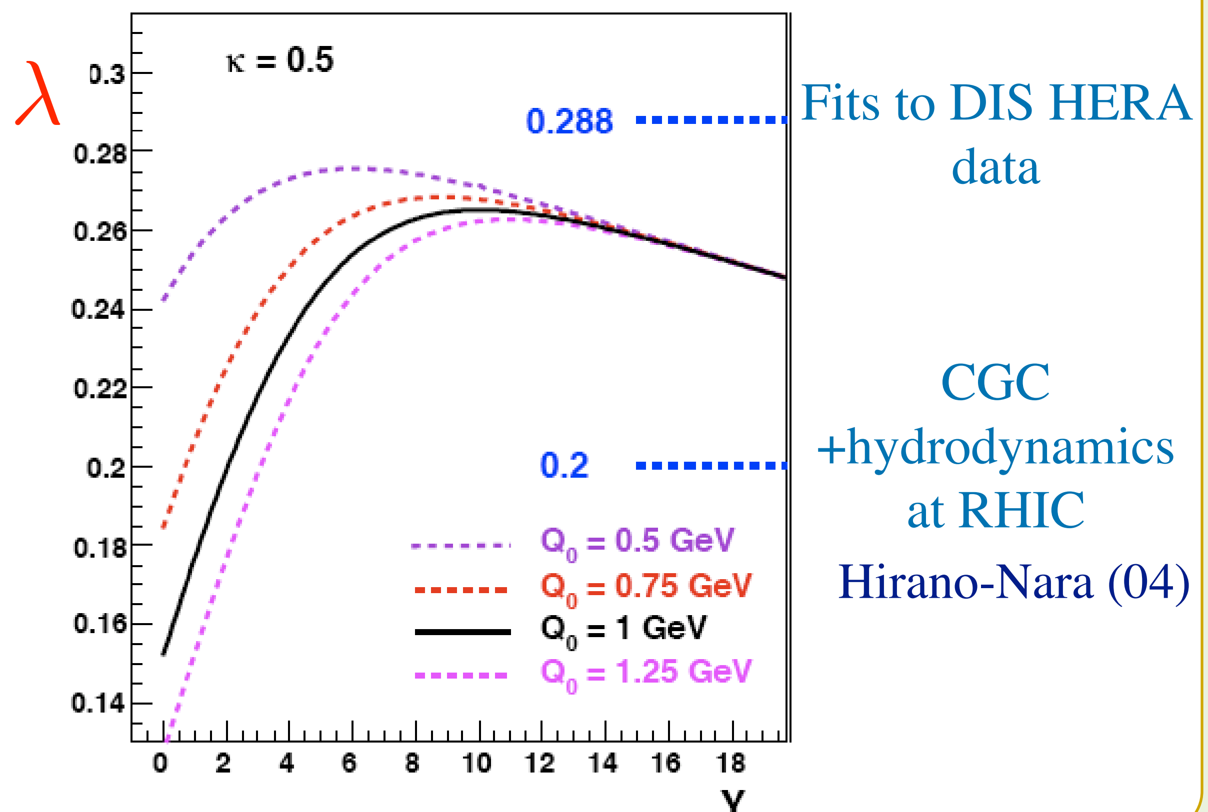
⇒ The running of the coupling reduces the speed of the evolution down to values compatible with experimental data (JLA PRL 99 262301 (07)):

⇒ Energy dependence of multiplicity in saturation models for particle production:

$$\left.\frac{dN_{AA}}{d\eta}\right|_{\eta=0} \sim \sqrt{s}^{\lambda} \qquad \text{where} \qquad Q_s^2(Y) = Q_0^2 \exp[\lambda Y]$$

$$\frac{\partial S}{\partial Y} = \mathcal{R}\,[S] - \mathcal{S}\,[S]$$



# Multiparticle production @ RHIC

$k_t$-factorization + saturation + local parton-hadron duality (Kharzeev-Levin-Nardi)

$$\frac{dN^g_{AB}}{d\eta} = \frac{4\pi N_c}{N_c^2-1}\int\frac{d^2p}{p^2}\int d^2k\,\alpha_s(Q^2)\,\varphi_A(x_1,k)\,\varphi_B(x_2,|p-k|) \quad \text{with} \quad x_{1,2}=\frac{p}{\sqrt{s}}\,e^{\pm\eta}$$

⇒ rapidity ↔ pseudorapidity: average hadron mass

$$y(\eta,p_t,m)=\frac{1}{2}\ln\left[\frac{\sqrt{\frac{m^2+p_t^2}{p_t^2}+\sinh^2\eta}+\sinh\eta}{\sqrt{\frac{m^2+p_t^2}{p_t^2}+\sinh^2\eta}-\sinh\eta}\right]$$

• Running coupling:

$$Q=\max\left\{\frac{|p_t\pm k_t|}{2}\right\}$$

⇒ $$\varphi(x,k)=\int\frac{d^2r}{2\pi^2\,r^2}\exp\left[i\,\underline{k}\cdot\underline{r}\right]\mathcal{N}(Y,r)$$ Solutions of BK equation including all orders in $\alpha_s\beta_2$ corrections $$\times(1-x)^4$$

with $$Y=\ln\left(\frac{0.05}{x}\right)+\Delta Y_{ev}$$

⇒ Initial condition: MV model $$\mathcal{N}(r,Y_0)=1-\exp\left[-r^2Q_0^2\ln\frac{1}{r\lambda}\right]$$

## Free Parameters:

⇒ Average hadron mass, m

⇒ Initial saturation scale $Q_0$

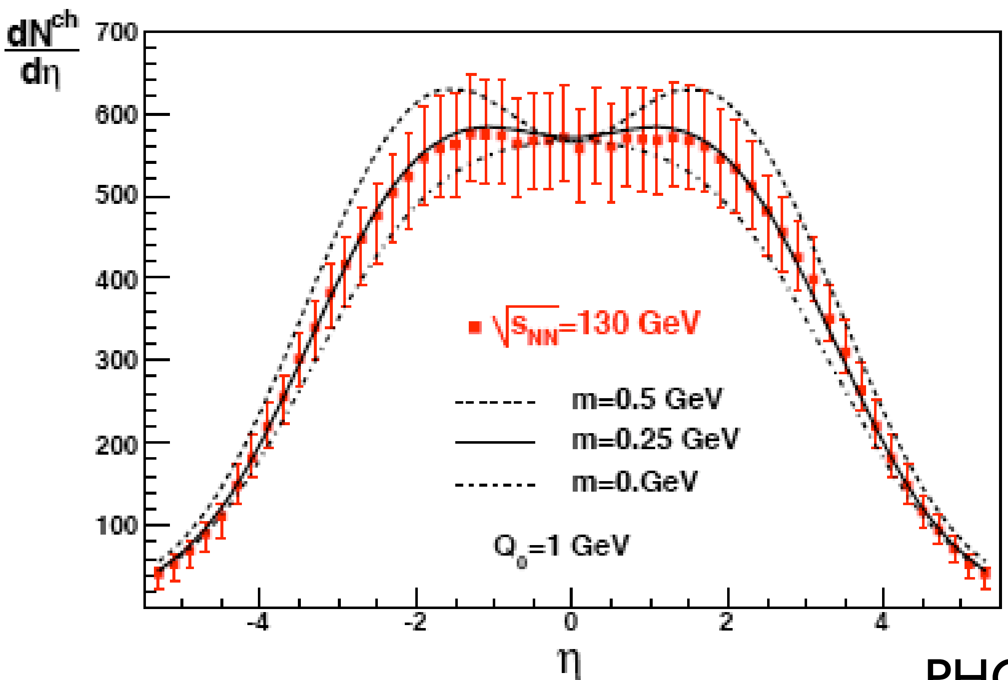
⇒ Is there significant evolution prior to $\sqrt{s}$ =130 ?: $\Delta Y_{ev}$

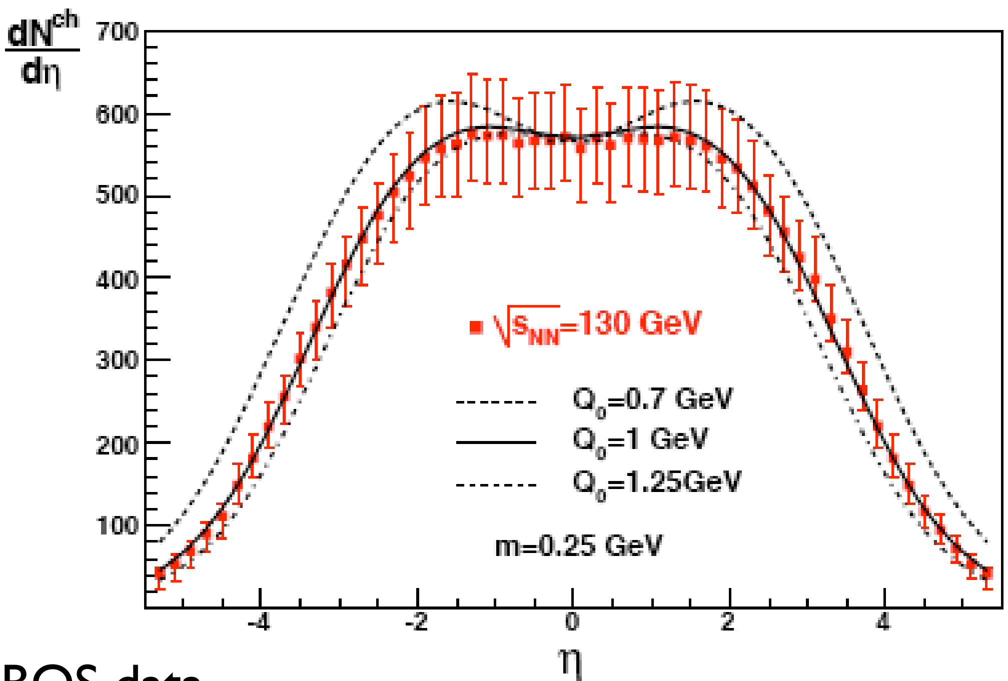
⇒ Energy independent normalization

Initial conditions for evolution:
Au-Au central collisions at RHIC at √s =130

Average hadron mass ~ 0.25 GeV

Initial saturation scale
$Q_0$ ~ 1 GeV



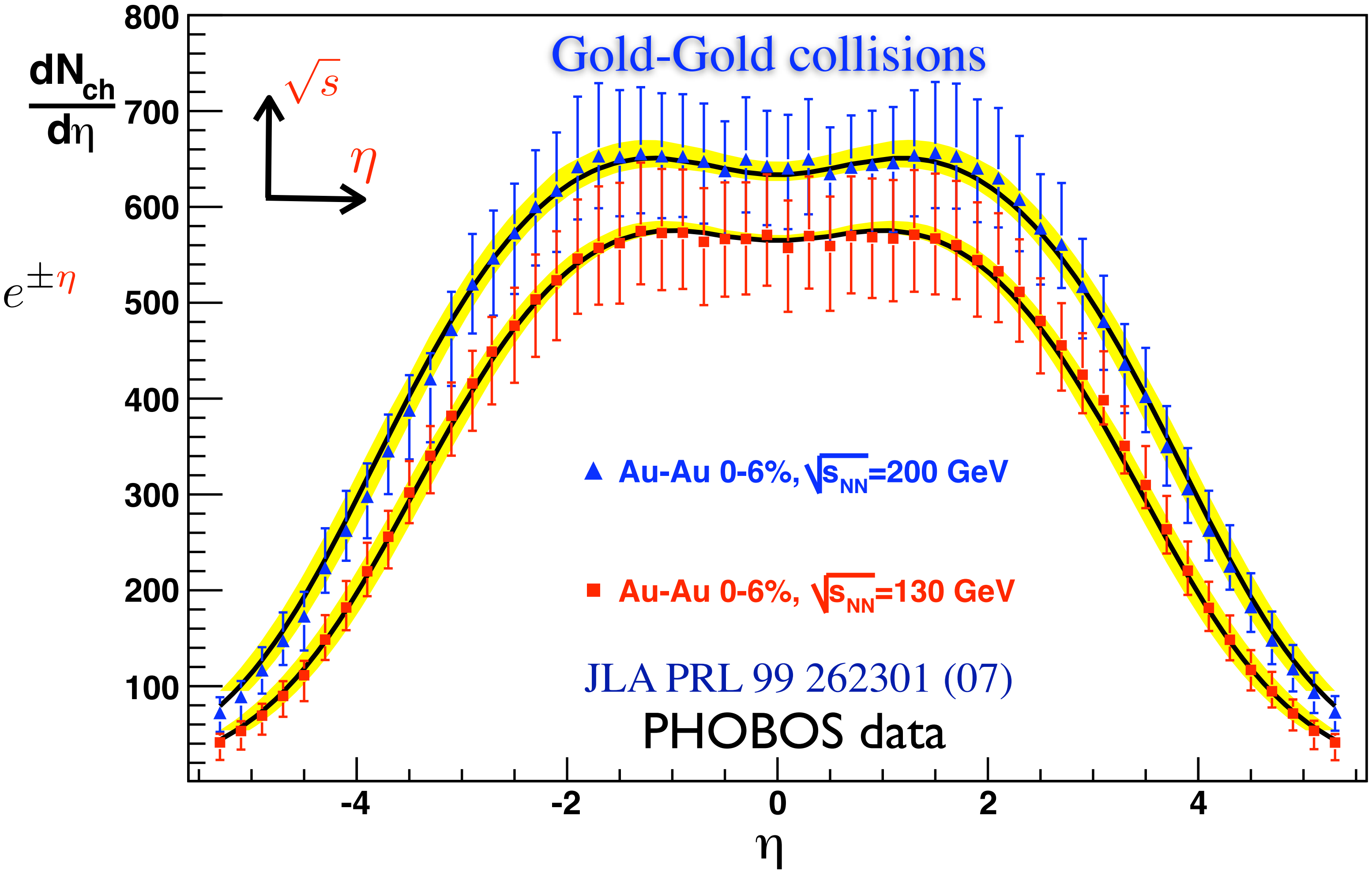

PHOBOS data

# Multiparticle production @ RHIC

Excellent description of both energy and rapidity dependence of RHIC Au+Au multiplicity data using solutions of the BK equation with running coupling

$$m \approx 0.25\,\text{GeV}, \quad 0.75 \le Q_0 \le 1.25\,\text{GeV}, \quad 1 \le \Delta Y_{ev} \le 3$$

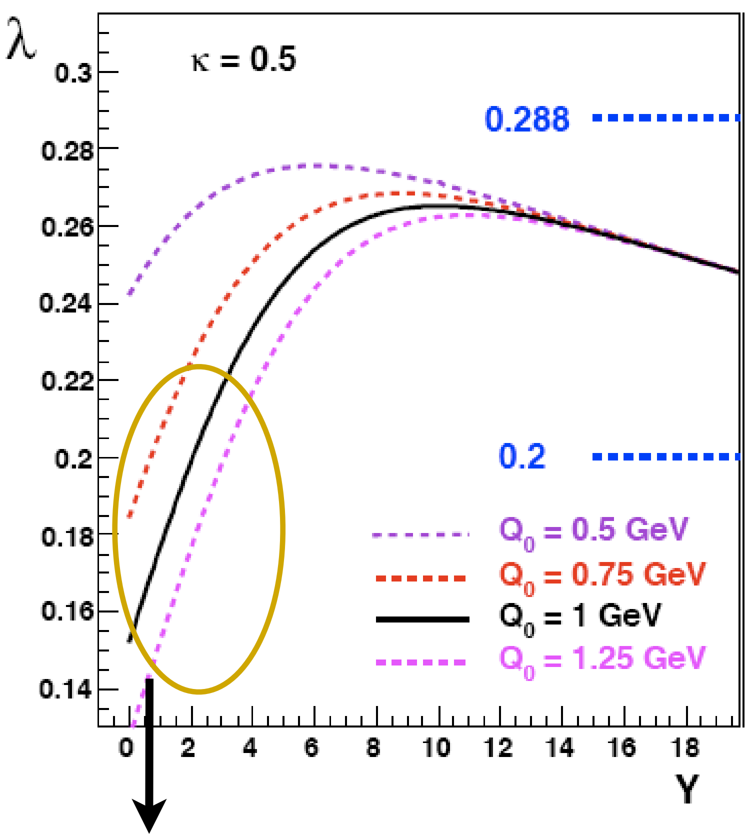
$$x_{1,2} = \frac{p}{\sqrt{s}}\, e^{\pm\eta}$$

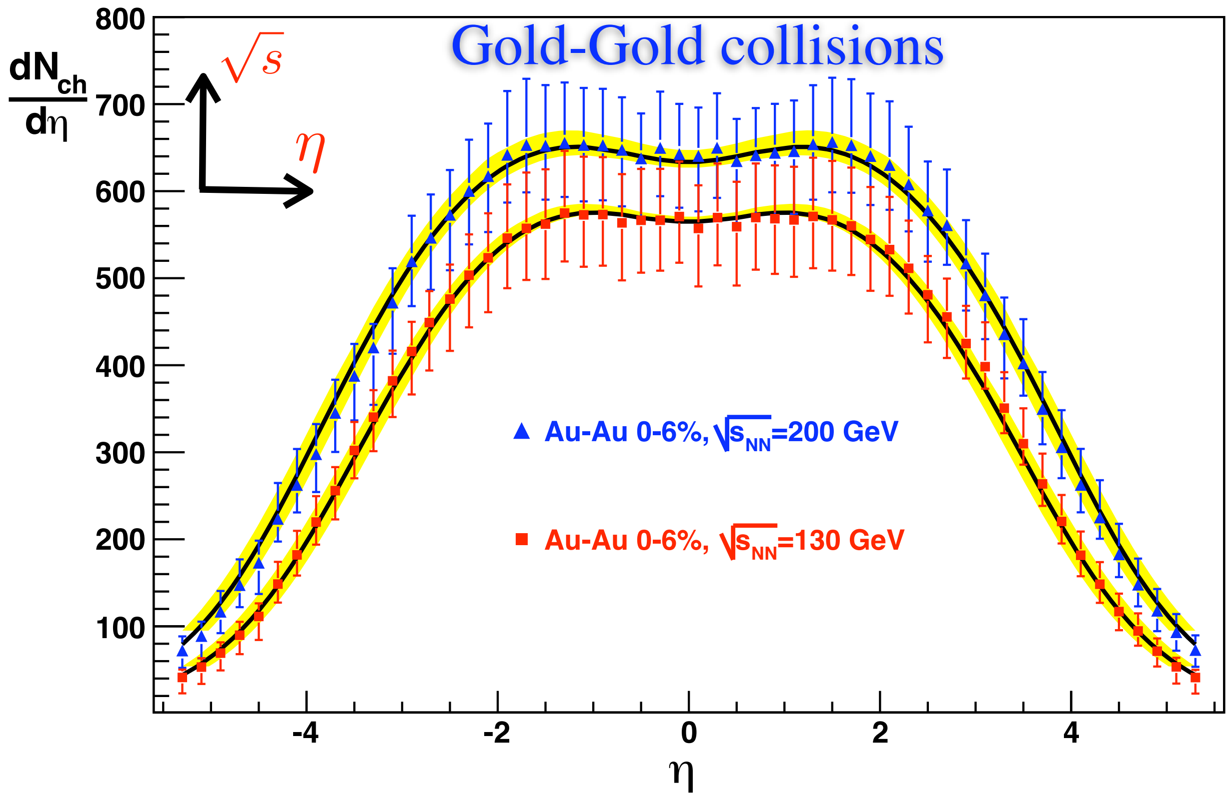


# Multiparticle production @ RHIC

RHIC energies are governed by pre-asymptotics effects (MV model: good i.c.)
Solutions close to the scaling region fail to reproduce RHIC data: No universality

$$m \approx 0.25\,\text{GeV}, \quad 0.75 \leq Q_0 \leq 1.25\,\text{GeV}, \quad 1 \leq \Delta Y_{ev} \leq 3$$



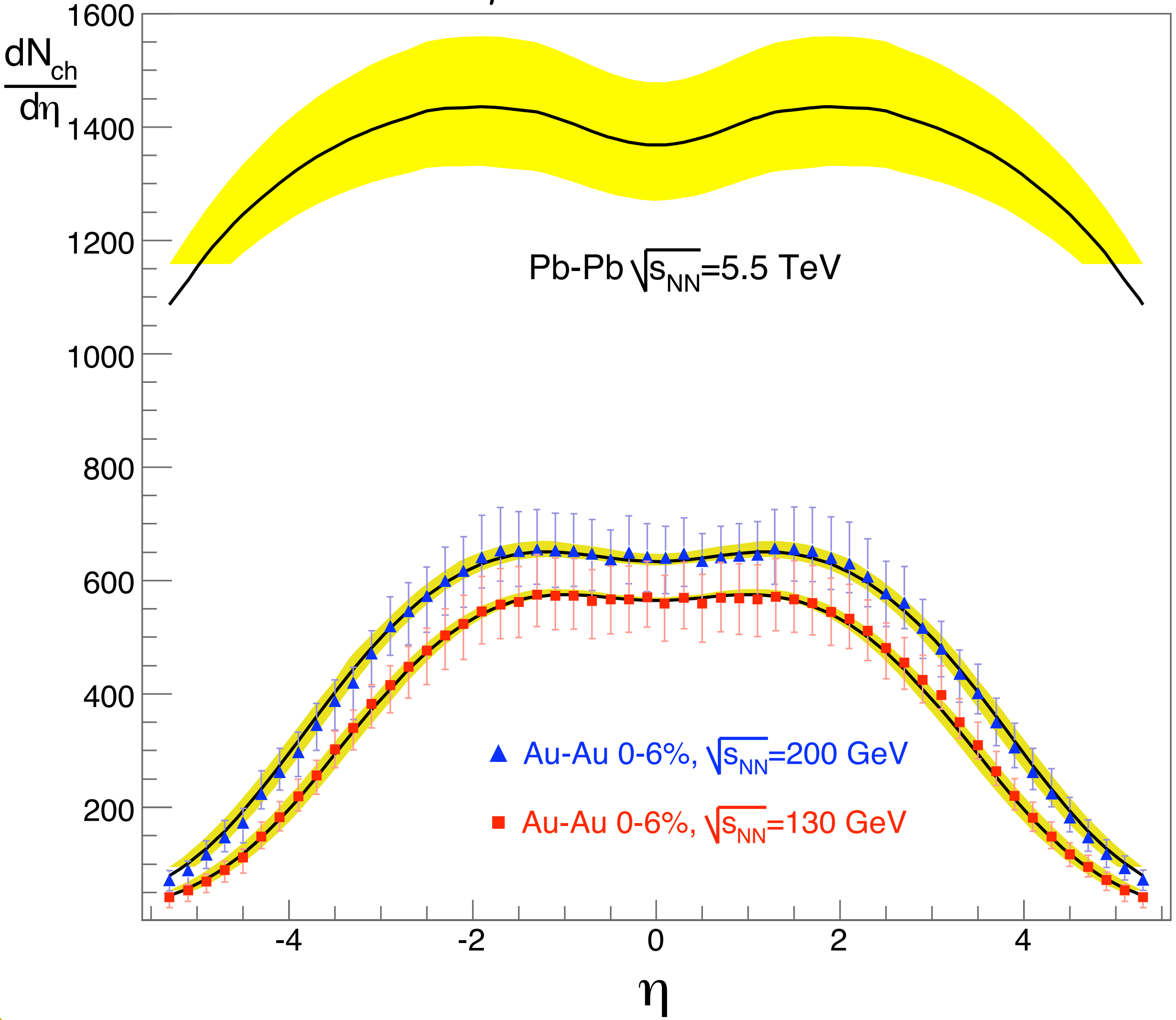




Nuclear gluon distributions probed at RHIC are in the pre-asymptotic regime

# Multiparticle production @ LHC

⇒ The extrapolation to Pb-Pb collisions at the LHC is completely driven by the small-x evolution. Compared to previous calculations, it yields a reduced multiplicity:

$$\frac{dN_{ch}^{Pb-Pb}(\sqrt{s}=5.5\,\mathrm{TeV},\eta=0)}{d\eta} \approx 1290 \div 1480$$




- Other saturation-based calculations:

| | |
|---|---|
| EHNRR arXiv:0705.1770 | $\sim 2500$ |
| KLN[1] hep-ph/0408050 | $\sim 2100 \div 1800$ |
| ASW hep-ph/0407018 | $\sim 1700$ |
| GSV arXiv:0707.1870 | $\sim 1000 \div 1400$ |

- Empiric extrapolation from lower energies data (W Busza nucl-ex/0410035) $\sim 1100$

⇒ The extrapolation to Pb-Pb collisions at the LHC is completely driven by the small-x evolution

Modifications

a) $\varphi(x,k) \to h(x,k) = k^2 \nabla_k^2 \varphi$

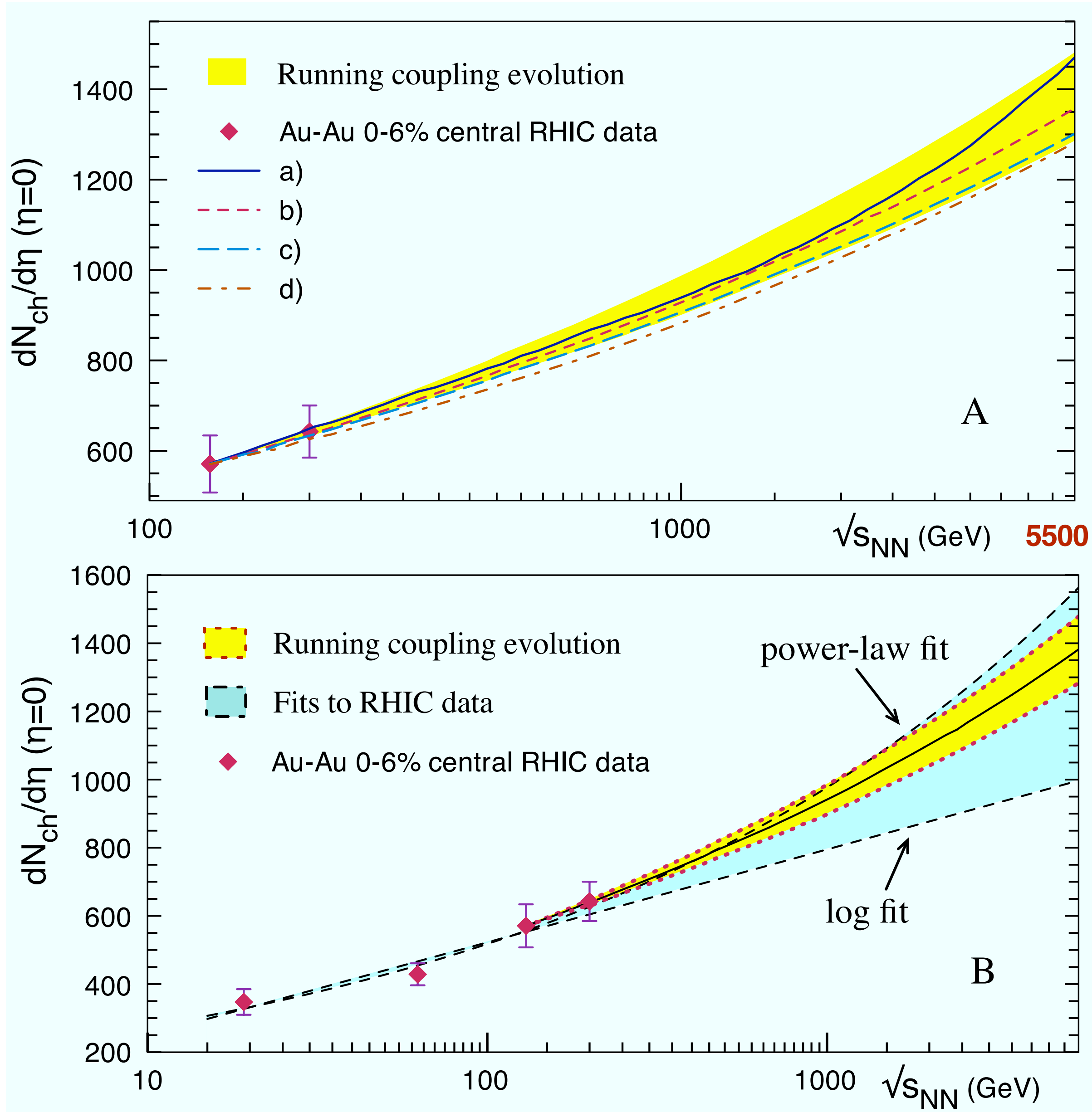
b) $\alpha_{fr} = 0.5$

c) $m = 0$

d) No $(1-x)^4$ corrections

- RHIC data does not discriminate power-law behavior from a logarithmic one

- Logarithmic behaviour seems to be dictated by lower energy data



# CONCLUSIONS

⇒ Running coupling corrections to BK-JIMWLK equations considerably reduce the speed of non-linear evolution

⇒ Multiplicity densities at RHIC can be reproduced using kt-factorization + solutions of the evolution

⇒ hadron mass mass ≈ 0.2 ÷ 0.3 GeV

⇒ $Q_s(\sqrt{s}=130\text{ GeV}, \eta=0) \approx 0.75 \div 1.25$ GeV

⇒ Pre-asymptotic regime: strong scaling violations

⇒ Extrapolation to Pb-Pb central collisions at $\sqrt{s}$=5.5 TeV yields a central value:

$$\frac{dN^{\text{evol}}}{d\eta}(\sqrt{s}=5.5\,\text{TeV})\Big|_{\eta=0} \approx 1400$$

⇒ Smaller than predictions based on HERA information

$$\frac{dN^{\lambda=0.288)}}{d\eta}(\sqrt{s}=5.5\,\text{TeV})\Big|_{\eta=0} \approx 2100 \div 1700$$

⇒ Larger than empiric extrapolations from lower energies data

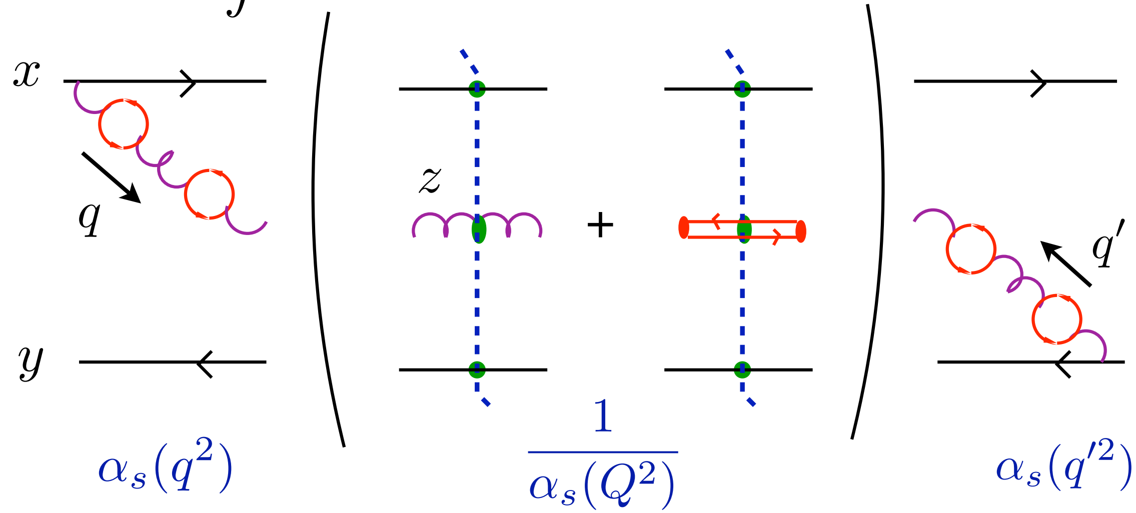
$$\frac{dN^{\text{log ext}}}{d\eta}(\sqrt{s}=5.5\,\text{TeV})\Big|_{\eta=0} \approx 1100$$

# Back up slides

- Complete (all orders in $\alpha_s\beta_2$) evolution equation: $\Rightarrow$ $\frac{\partial S}{\partial Y} = \mathcal{R}[S] - \mathcal{S}[S]$

JLA and Y. Kovchegov PRD 75 125021 (07):

$\Rightarrow$ ***Running*** **term:** $\mathcal{R}[S] = \int d^2z\, \tilde{K}(\underline{r}, \underline{r}_1, \underline{r}_2)\left[S(\underline{x},\underline{z})S(\underline{z},\underline{y}) - S(\underline{x},\underline{y})\right]$




+ virtual terms

$$\tilde{K}_{KW}(\underline{r}, \underline{r}_1, \underline{r}_2) = \frac{N_c}{2\pi^2}\left[\frac{\alpha_s(r_1^2)}{r_1^2} - 2\frac{\alpha_s(r_1^2)\alpha_s(r_2^2)}{\alpha_s(R^2)} + \frac{\alpha_s(r_2^2)}{r_2^2}\right]$$

$$\tilde{K}_{Bal}(\underline{r}, \underline{r}_1, \underline{r}_2) = \frac{N_c\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2\, r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right]$$

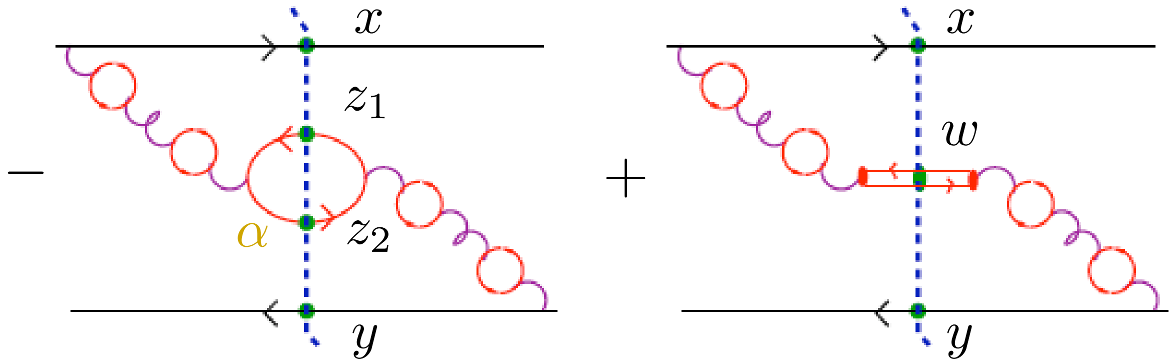
- The qq contribution ensures the renormalizability of the all orders in $\alpha_s\beta_2$ corrections and the right physical behavior of the running term:

$$\mathcal{R}[S] \to 0 \quad \text{for} \begin{cases} S \to 0 & \Rightarrow \text{Probability conservation} \\ S \to 1 & \Rightarrow \text{Unitarity:} \end{cases}$$

$$\frac{\partial S}{\partial Y} = \mathcal{R}\,[S] - \mathcal{S}\,[S]$$

⇒ ***Subtraction*** term:

$$\mathcal{S}\,[S] = \int d^2 z_1 d^2 z_2 \, K_{sub}(\underline{x}, \underline{y}, \underline{z}_1, \underline{z}_2) \left[ S(\underline{x}, \underline{w}) S(\underline{w}, \underline{y}) - S(\underline{x}, \underline{z}_1) S(\underline{z}_2, \underline{y}) \right]$$




$N_f \longrightarrow -6\pi\beta_2$

$$K_{sub}(\underline{x}, \underline{y}, \underline{z}_1, \underline{z}_2) = -\frac{3\beta_2}{2\pi^3} \int_0^1 d\alpha \, \frac{1}{[\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2] \, [\alpha(\underline{z}_1 - \underline{y})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2] \, z_{12}^4}$$

$$\Big\{ \left[ -4\alpha\bar{\alpha} \, \underline{z}_{12} \cdot (\underline{z} - \underline{x}) \, \underline{z}_{12} \cdot (\underline{z} - \underline{y}) + z_{12}^2 (\underline{z} - \underline{x}) \cdot (\underline{z} - \underline{y}) \right] \alpha_s(R_T(\underline{x})) \, \alpha_s(R_T(\underline{y}))$$

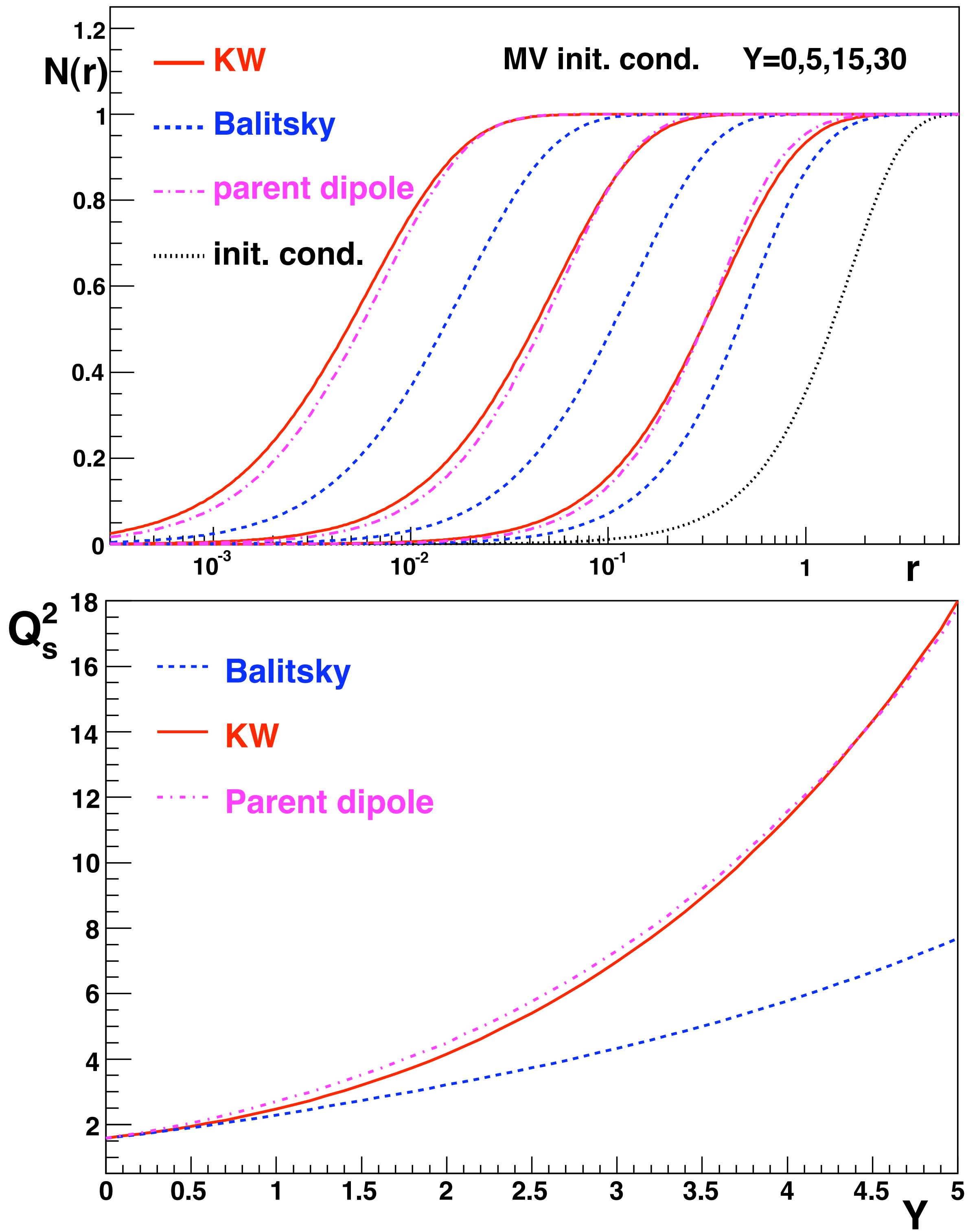
$$2\alpha\bar{\alpha}(\alpha - \bar{\alpha}) z_{12}^2 \left[ \underline{z}_{12} \cdot (\underline{z} - \underline{x}) \, \alpha_s(R_T(\underline{x})) \, \alpha_s(R_L(\underline{y})) + \underline{z}_{12} \cdot (\underline{z} - \underline{y}) \, \alpha_s(R_L(\underline{x})) \, \alpha_s(R_T(\underline{y})) \right]$$

$$4\alpha^2 \bar{\alpha}^2 z_{12}^4 \, \alpha_s(R_L(\underline{x})) \, \alpha_s(R_L(\underline{y})) \Big\}$$

- It receives contributions from transverse (T) and longitudinal (L) gluon's polarization:

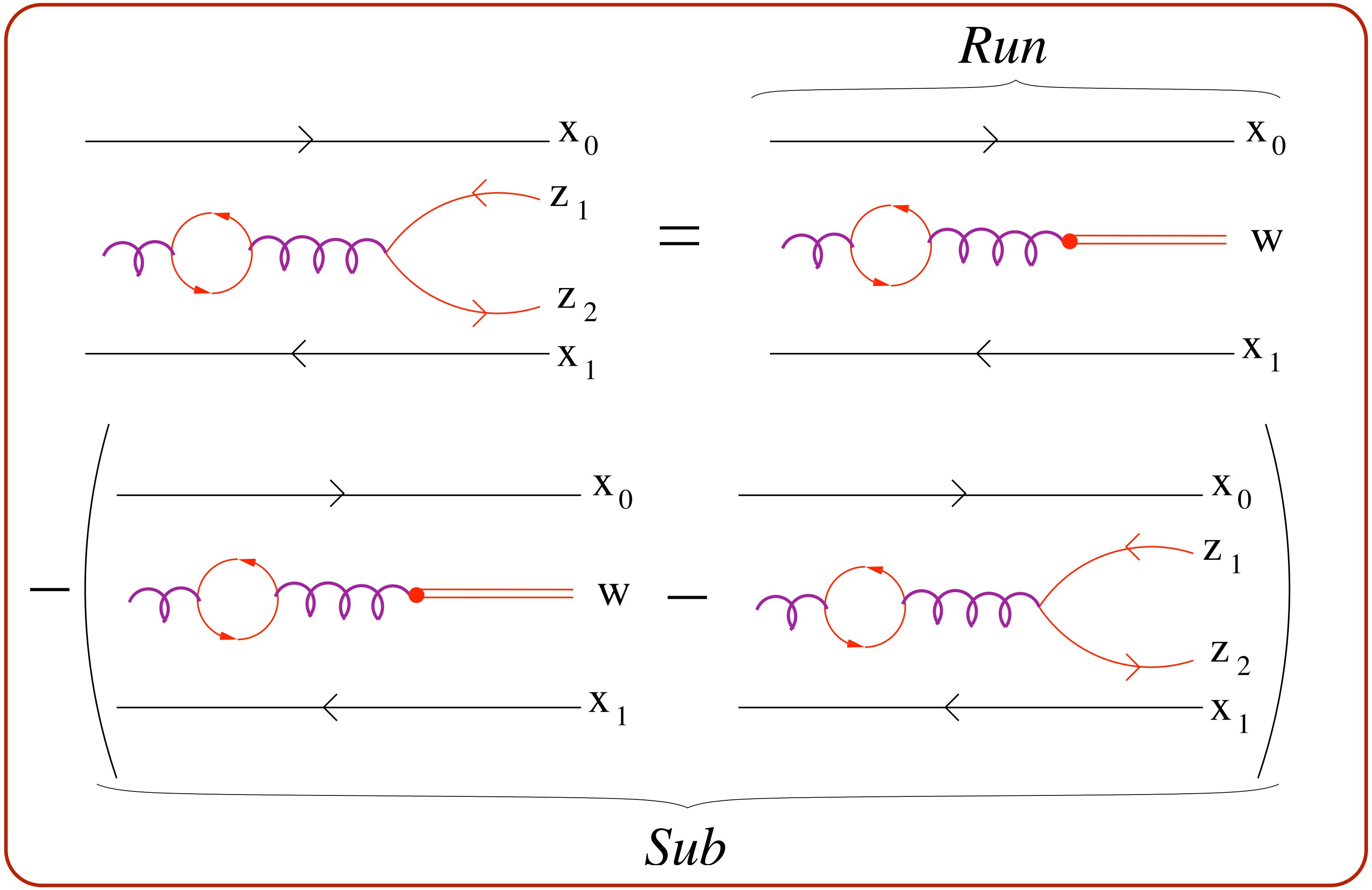
$$\ln\left( \frac{1}{R_T^2(\underline{x}) \mu^2} \right) = \ln\left( \frac{4 e^{-2\gamma - 5/3}}{[\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2] \mu^2} \right) + \frac{\alpha\bar{\alpha} \, z_{12}^2}{(\underline{z} - \underline{x})^2} \ln\left( \frac{\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2}{\alpha\bar{\alpha} z_{12}^2} \right)$$

$$\ln\left( \frac{1}{R_L^2(\underline{x}) \mu^2} \right) = \ln\left( \frac{4 e^{-2\gamma - 5/3} \, \alpha\bar{\alpha} \, z_{12}^2}{[\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2]^2 \mu^2} \right)$$



N(r)
KW
Balitsky
parent dipole
init. cond.
MV init. cond.
Y=0,5,15,30
r
$Q_s^2$
Balitsky
KW
Parent dipole
Y

• The separation procedure is similar in both calculations:




$$\mathcal{S}[S] = \int d^2 z_1 \, d^2 z_2 \, K^{sub} \left[ S(\underline{x}_0, \underline{\mathbf{w}}, Y) \, S(\underline{\mathbf{w}}, \underline{x}_1, Y) - S(\underline{x}_0, \underline{z}_1, Y) \, S(\underline{z}_2, \underline{x}_1, Y) \right]$$

• The differences between the two approaches stem from the choice of the subtraction point, w

- In Balitsky's scheme: $w = z_1$ (or $z_2$), the quark's (anti-q) transverse position :

$$\mathcal{S}^{Bal}[S] \;=\; \int d^2z_1\, d^2z_2\, K^{sub}\left[S(\underline{x}_0,\underline{\mathbf{z_1}},Y)\, S(\underline{\mathbf{z_1}},\underline{x}_1,Y) - S(\underline{x}_0,\underline{z}_1,Y)\, S(\underline{z}_2,\underline{x}_1,Y)\right]$$

An expansion in term of N's result in just non-linear terms ($N^2 << N$ at small-r)

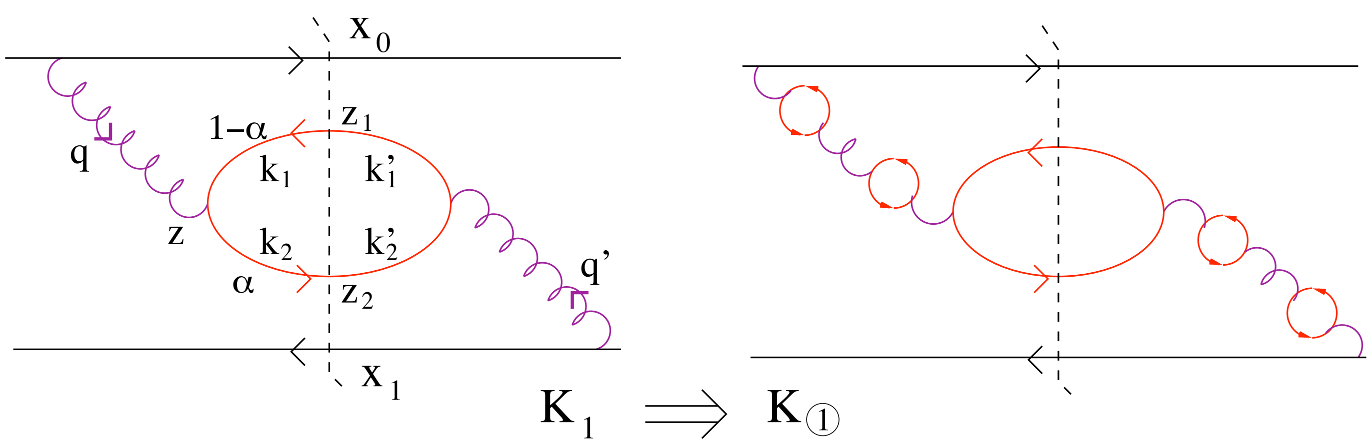
- In KW scheme: w = z =, the gluon's transverse position:

$$\mathcal{S}^{KW}[S] \;=\; \int d^2z_1\, d^2z_2\, K^{sub}\left[S(\underline{x}_0,\underline{\mathbf{z}},Y)\, S(\underline{\mathbf{z}},\underline{x}_1,Y) - S(\underline{x}_0,\underline{z}_1,Y)\, S(\underline{z}_2,\underline{x}_1,Y)\right]$$

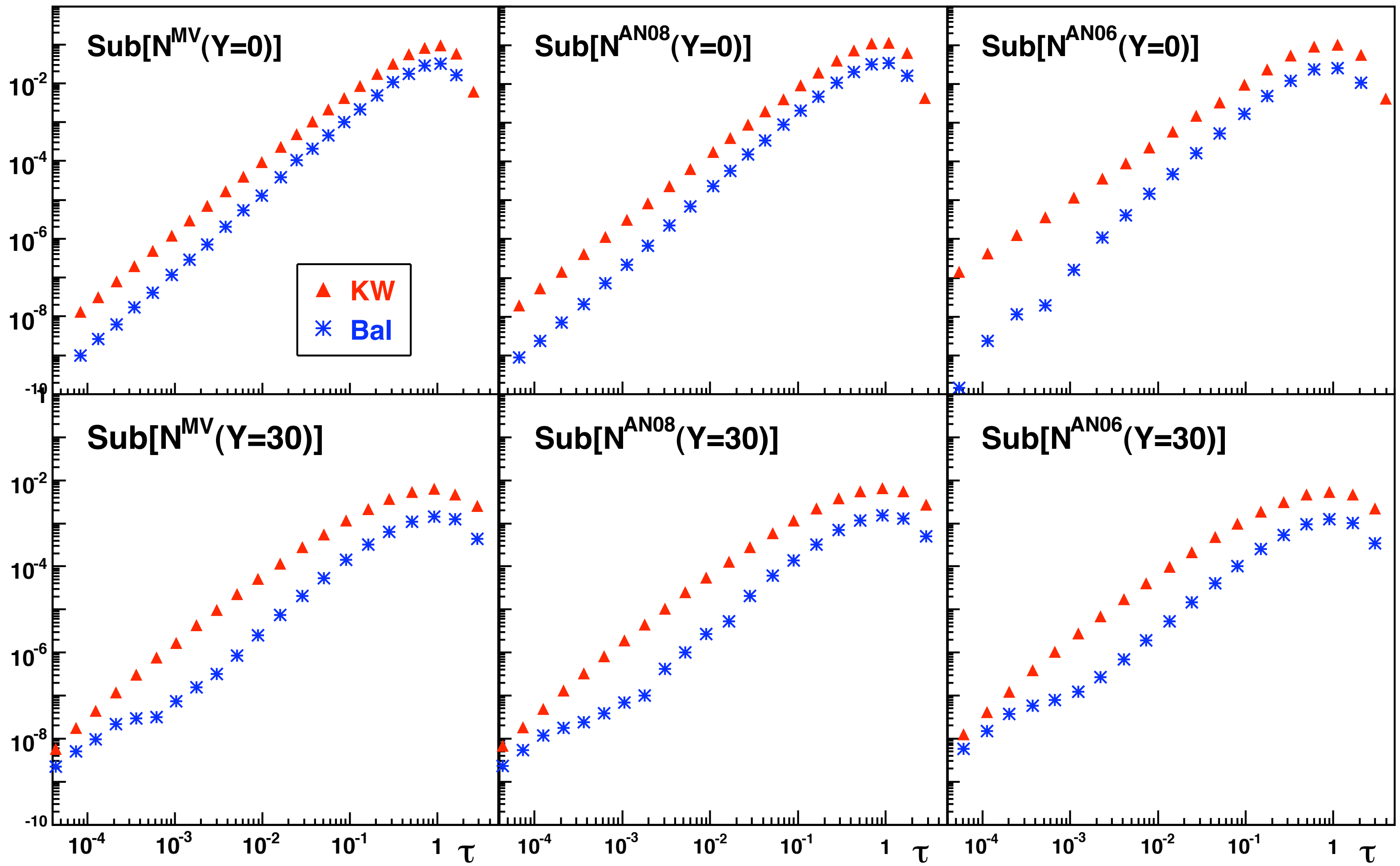
An expansion in term of N's also includes linear terms.

- The kernel of the subtraction contribution is the same in both cases:

Leading-Nf ⇒ All orders Nf (LCPT) ⇒ Fourier transform to coordinate space



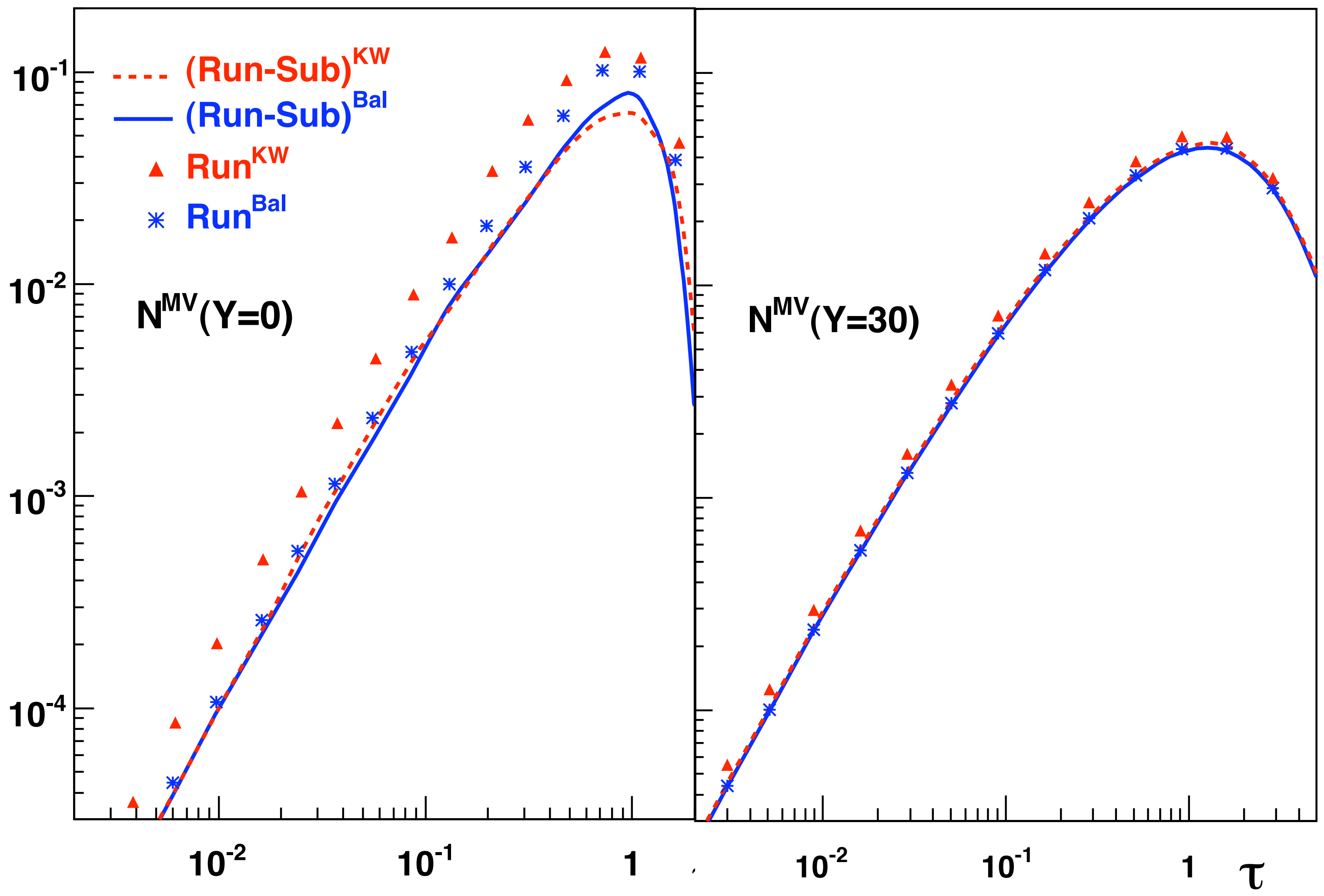
- The subtraction term is larger in KW's scheme than in Balitsky's:




- It has the same sign as the running term: It slows down the evolution

$$\mathcal{F} = \mathcal{R} - \mathcal{S}$$

Once the subtraction term is added back, the two approaches agree:



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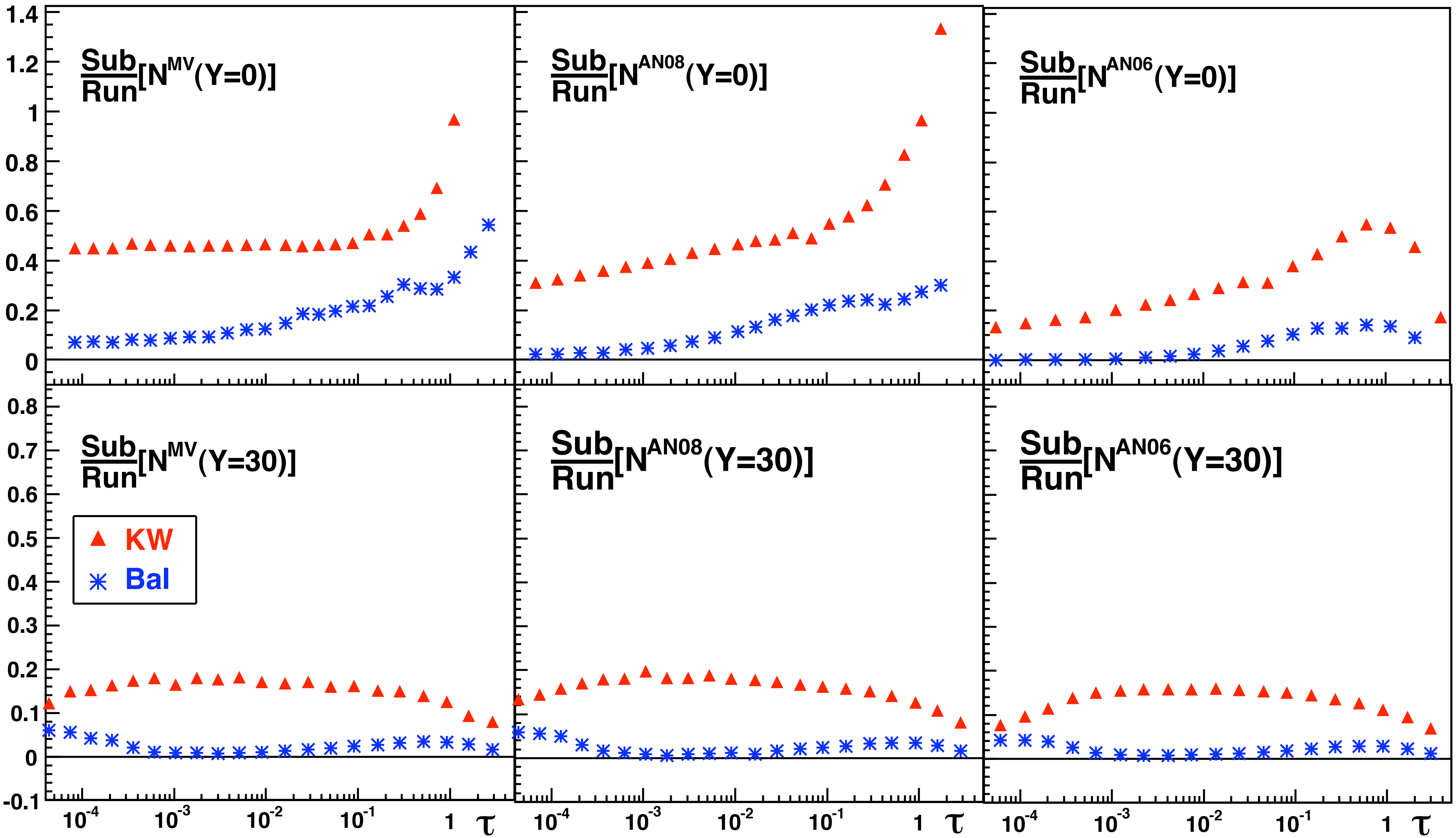


- The relative contribution of the subtraction term to the evolution fades away at large rapidity