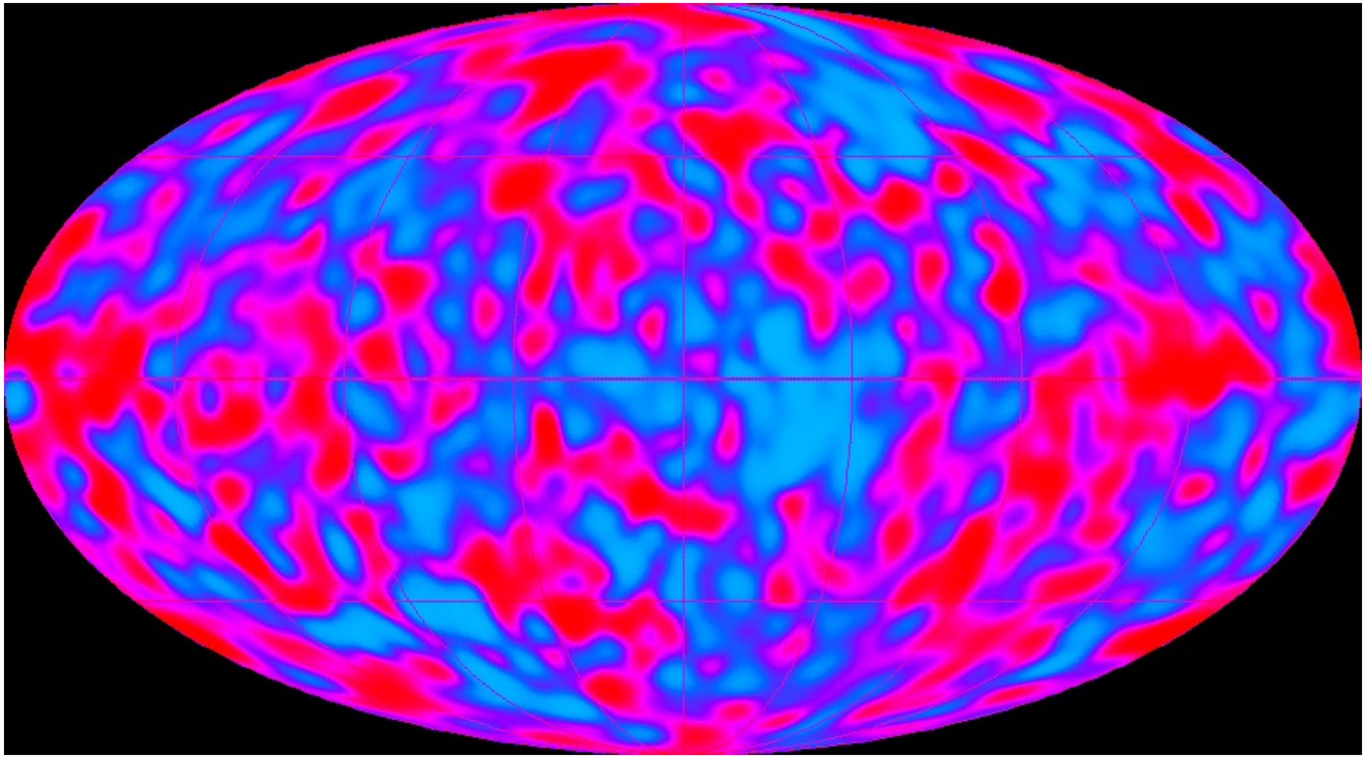
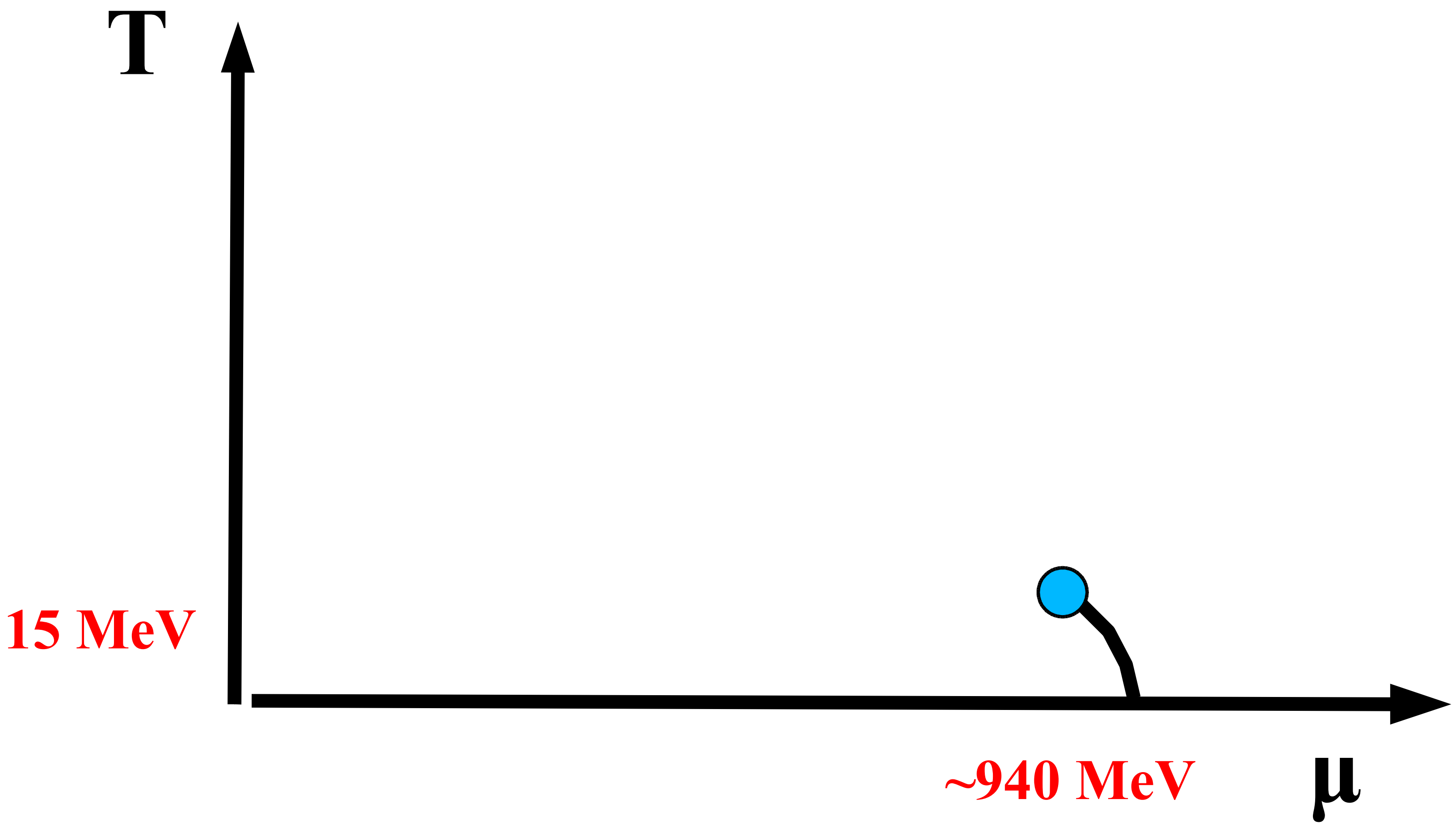
Fluctuations and Correlations

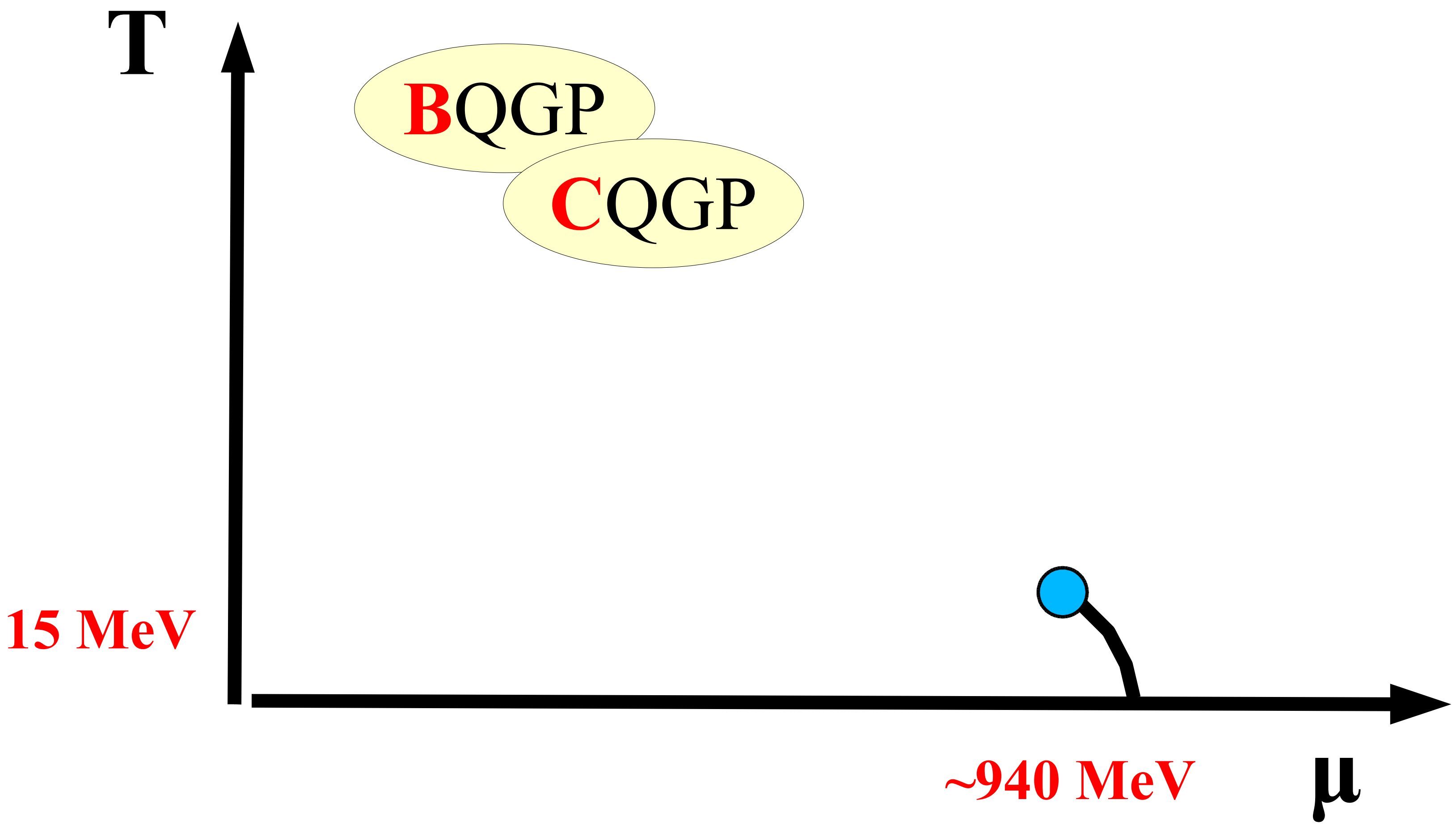
Status and future perspectives



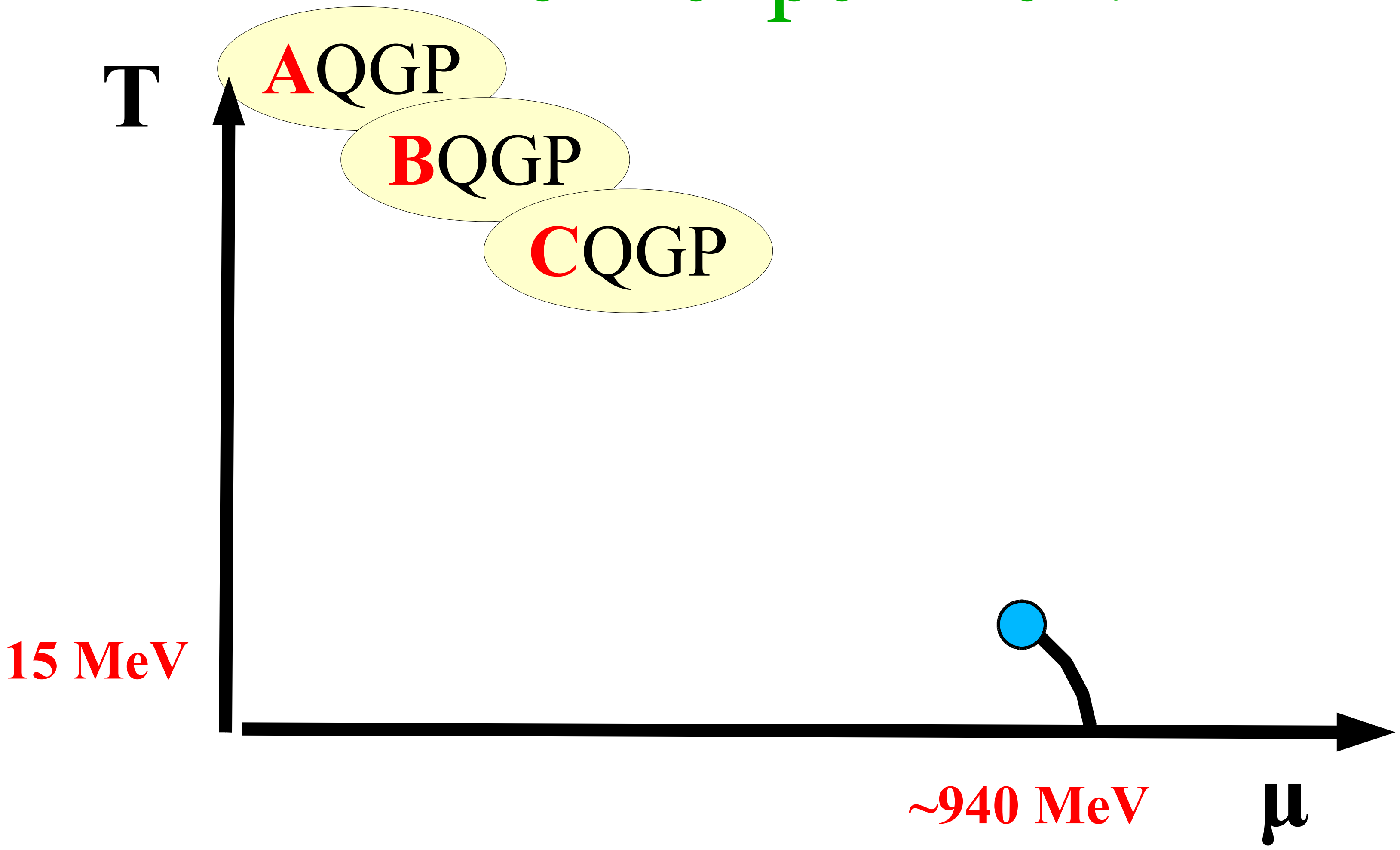
The QCD Phase Diagram from experiment



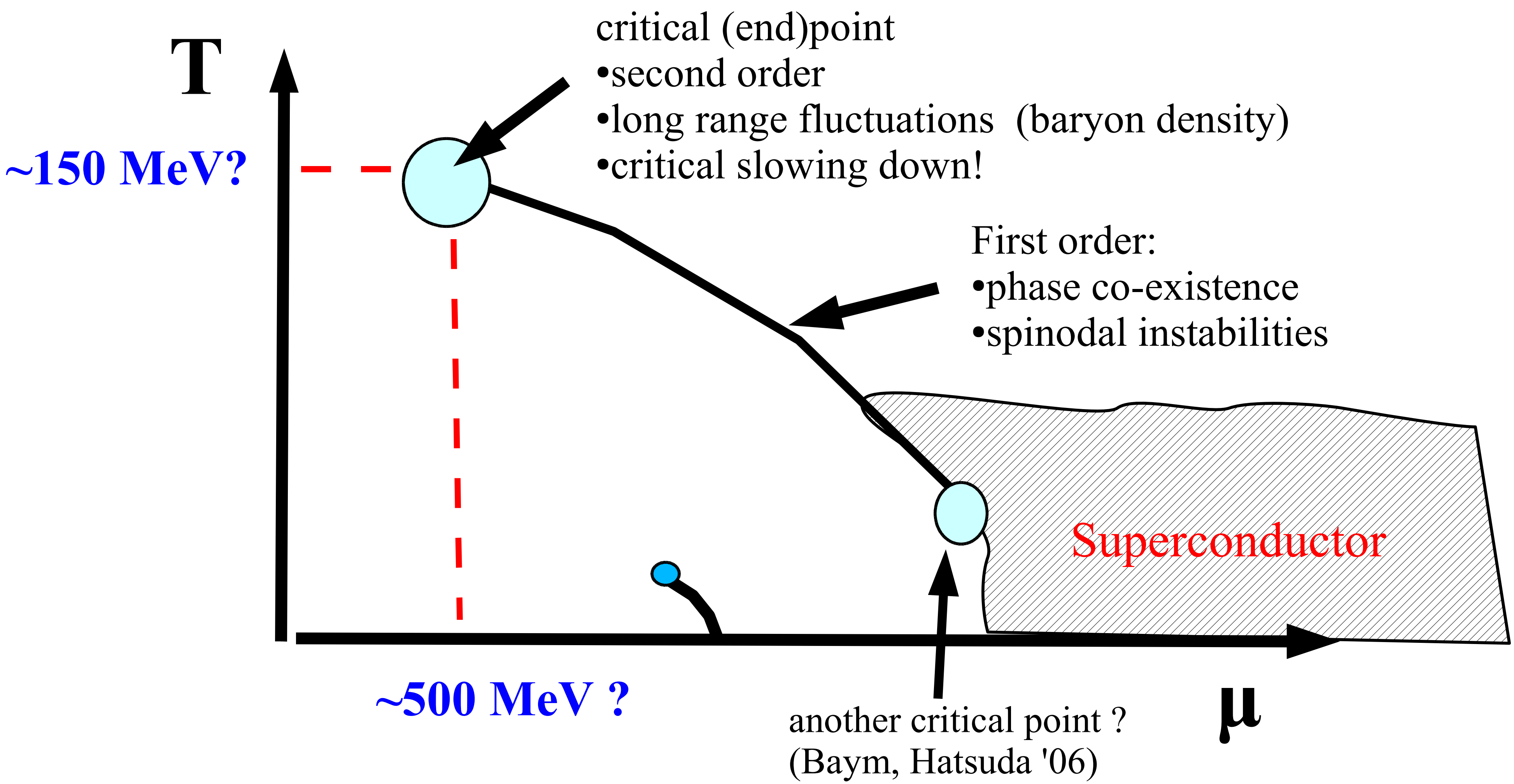
The QCD Phase Diagram from experiment



The QCD Phase Diagram from experiment



The QCD Phase Diagram
(from a theorist's perspective)



N.B.: Critical point of water: T_c=647.096 K, p_c = 22.064 MPa, ρ_c = 322 kg/m3

Fluctuations and Correlations in thermal system

e.g. Lattice QCD

$$Z = Tr\left[\exp\left(-\beta\left(H - \mu_Q Q - \mu_B B - \mu_S S\right)\right)\right]$$

Mean :
$$\langle \alpha \rangle = T \frac{\partial}{\partial \mu_\alpha} \log(Z) = -\frac{\partial}{\partial \mu_\alpha} F$$

$$\alpha, \beta = Q, B, S$$

Variance:
$$\langle (\delta \alpha)^2 \rangle = T^2 \frac{\partial^2}{\partial \mu_\alpha^2} \log(Z) = -T \frac{\partial^2}{\partial \mu_\alpha^2} F$$

Co-Variance:
$$\langle (\delta \alpha)(\delta \beta) \rangle = T^2 \frac{\partial^2}{\partial \mu_\alpha \partial \mu_\beta} \log(Z) = -T \frac{\partial^2}{\partial \mu_\alpha \partial \mu_\beta} F$$

Susceptibility:
$$\chi_{\alpha\beta} = -\frac{1}{V} \frac{\partial^2}{\partial \mu_\alpha \partial \mu_\beta} F = -\frac{1}{V} \frac{\partial}{\partial \mu_\alpha} \langle \beta \rangle$$

Fluctuations and Correlations in thermal system

e.g. Lattice QCD

$$Z = Tr[\exp(-\beta(H - \mu_Q Q - \mu_B B - \mu_S S))]$$

Susceptibility: $\chi_{\alpha\beta} = -\frac{1}{V}\frac{\partial^2}{\partial\mu_\alpha \partial\mu_\beta}F = -\frac{1}{V}\frac{\partial}{\partial\mu_\alpha}\langle\beta\rangle$ $\alpha, \beta = Q, B, S$

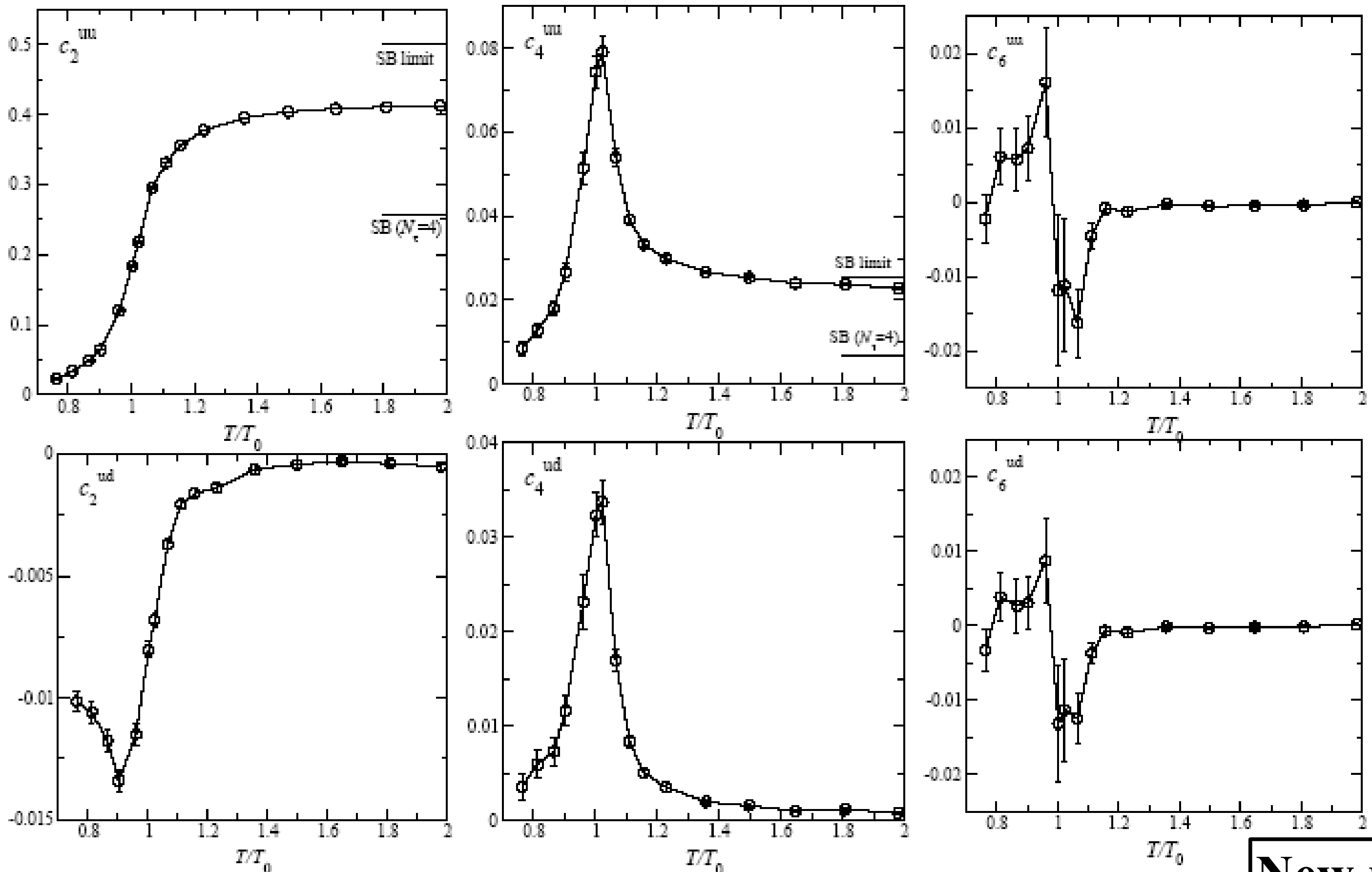
More general: $\chi_{\alpha,\beta} \sim \frac{1}{V}\int d^3x\, d^3y \langle\rho_\alpha(x)\rho_\beta(y)\rangle_{connected} = \int d^3 C_{\alpha,\beta}(r) \sim \xi^2_{\alpha,\beta}$

Correlation Function:

$$C_{\alpha,\beta}(r) = \langle\rho_\alpha(r)\rho_\beta(0)\rangle_{connected} \sim \frac{e^{(-r/\xi_{\alpha,\beta})}}{r} \qquad \xi_{\alpha,\beta} = correlation\ length$$

Lattice-QCD susceptibilities

$$\frac{\chi(T,\mu_q)}{T^2} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4 + \ldots$$



Rule of thumb:

$$c_n \sim \langle X^n \rangle$$
$$X = B, Q, S, \ldots$$

Alton et al, PRD 66 074507 (2002)

New results: C. Schmidt, this afternoon, session X

Susceptibilities and Phasetransitions

$$Z = Tr[\exp(-\beta(H-\mu N))]$$

Susceptibility: $\chi \sim \frac{1}{V}\frac{\partial^2}{\partial\mu^2}\log(Z) = \frac{1}{V}(\langle N^2\rangle - \langle N\rangle^2)$

Poisson: $\chi \sim \frac{\langle N\rangle}{V}$ independent of volume → $\langle(\delta N)^2\rangle = N \sim V$

In general: $\chi \sim \frac{1}{V}\int d^3x\, d^3y \langle\rho(x)\rho(y)\rangle_{connected} = \int d^3r \langle\rho(r)\rho(0)\rangle_{connected} \sim \xi^2$

$$\langle\rho(r)\rho(0)\rangle_{connected} \sim \frac{e^{(-r/\xi)}}{r} \qquad \xi = correlation\ length$$

Cross-over: $\xi = const \rightarrow \chi = const \rightarrow \langle(\delta N)^2\rangle \sim V$

Second Order: $\xi \sim V^{(1/3)} \rightarrow \chi \sim V^{(2/3)} \rightarrow \langle(\delta N)^2\rangle \sim V^{(5/3)}$

First Order: $\langle\rho(r)\rho(0)\rangle = const \rightarrow \chi \sim V \rightarrow \langle(\delta N)^2\rangle \sim V^2$

Susceptibilities and Observables

Susceptibility: $$\chi \sim \frac{1}{V}\frac{\partial^2}{\partial \mu^2}\log(Z) = \frac{1}{V}\left(\langle N^2\rangle - \langle N\rangle^2\right)$$

Fluctuations of some sort!

Cross-over: $$\xi = const \rightarrow \chi = const \rightarrow \langle(\delta N)^2\rangle \sim V$$

Second Order: $$\xi \sim V^{(1/3)} \rightarrow \chi \sim V^{(2/3)} \rightarrow \langle(\delta N)^2\rangle \sim V^{(5/3)}$$

First Order: $$\langle \rho(r)\rho(0)\rangle = const \rightarrow \chi \sim V \rightarrow \langle(\delta N)^2\rangle \sim V^2$$

Since fluctuations diverge at phase transition any sort will do!

System size dependence!

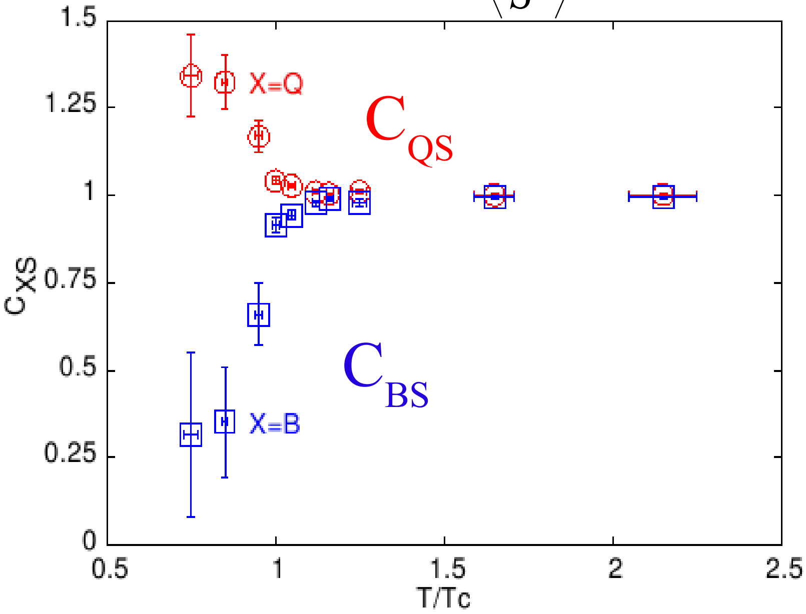
Note of caution: Co-variances also diverge; trigger?

“Charge” (co)-variances
<Q²>, <BS> , <QS>

$$C_{BS} = -3\frac{\langle BS \rangle}{\langle S^2 \rangle}$$

$$C_{QS} = -3\frac{\langle QS \rangle}{\langle S^2 \rangle}$$

- Net Charge fluctuations (QGP)
- Baryon-Strangeness correlations (QGP)
- Baryon number fluctuations (CP)
- Balance Functions
- ....



Fluctuations of conserved charges can survive hadonization!

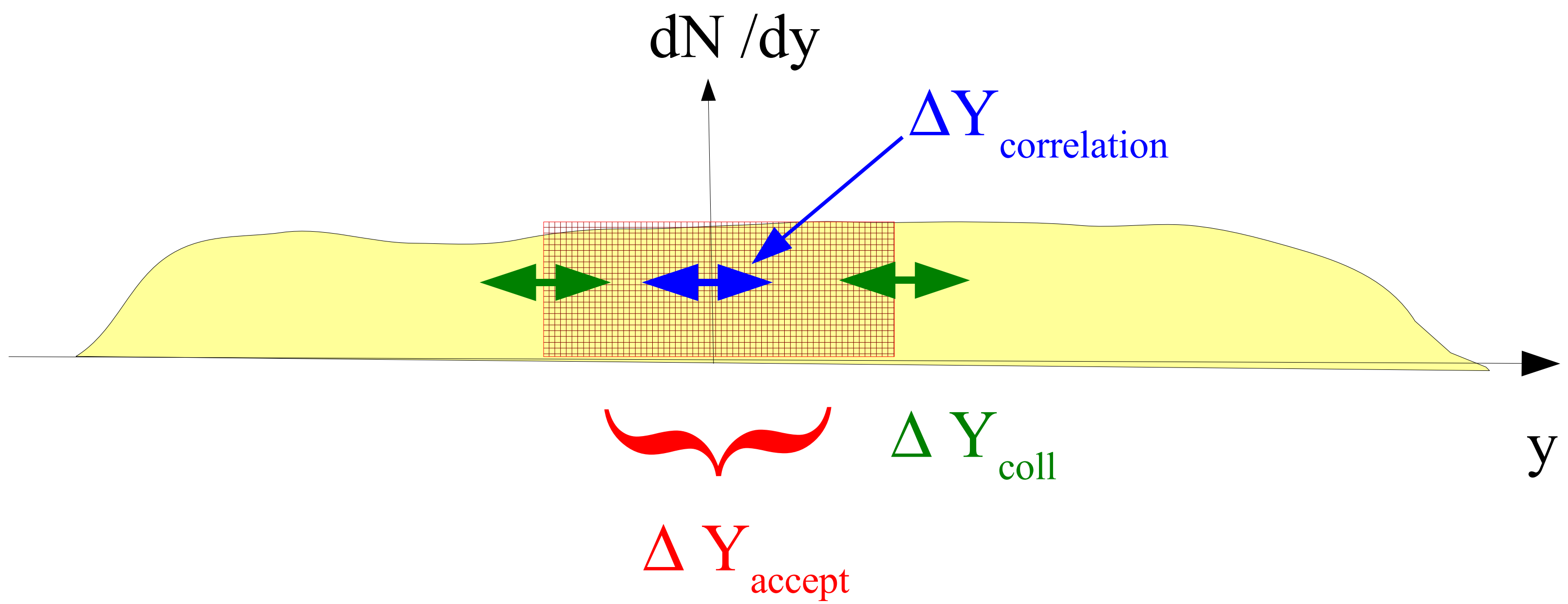
S. Jeon, V.K; Stephanov, Shuryak (2000)

Gavai,Gupta, hep-lat/0510044

V.K, Majumder, Randrup PRL95:182301,2005

Majumder, Mueller Phys.Rev.C74:054901,2006

“Charge” fluctuations

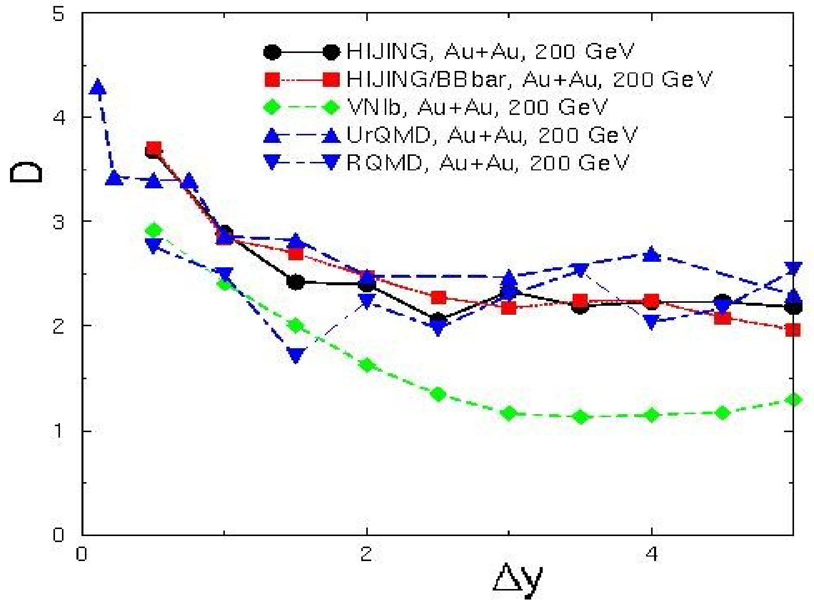


Condition for “charge” fluctuations:

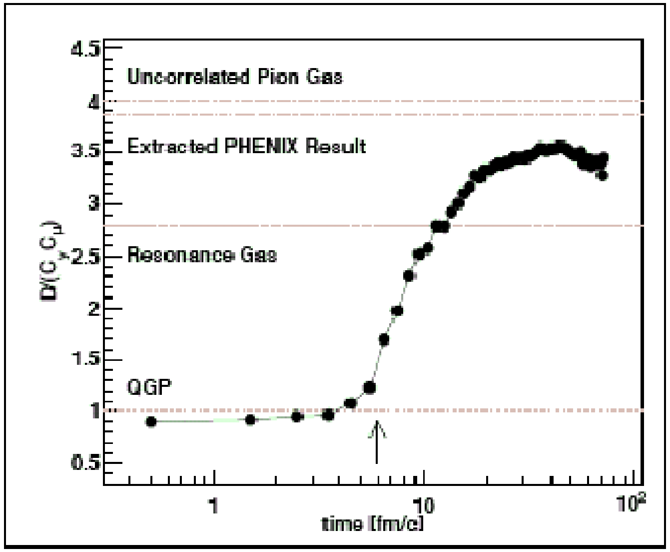
1)$\Delta\ Y_{corrrelation} << \Delta\ Y_{accept}$ **(catch the physics)**

3)$\Delta\ Y_{total} >> \Delta\ Y_{accept} >> \Delta\ Y_{coll}$ **(keep the physics)**

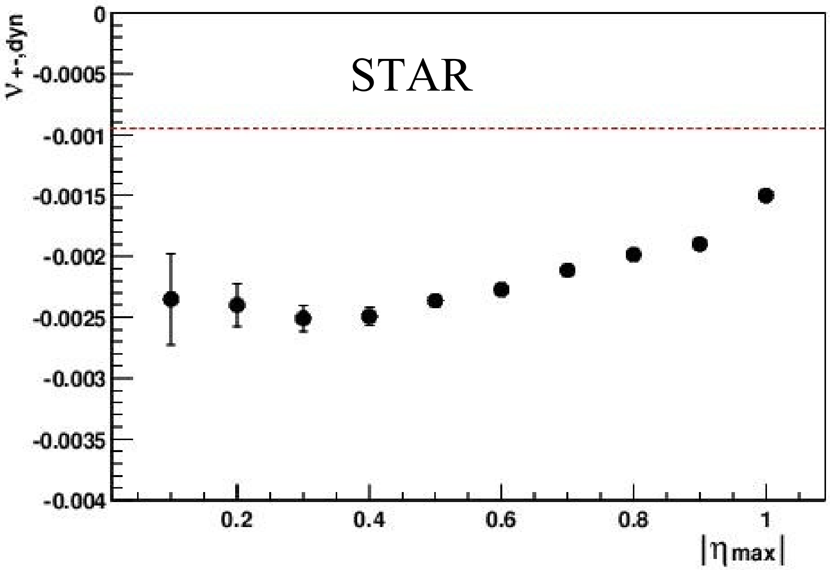
Net-Charge fluctuations

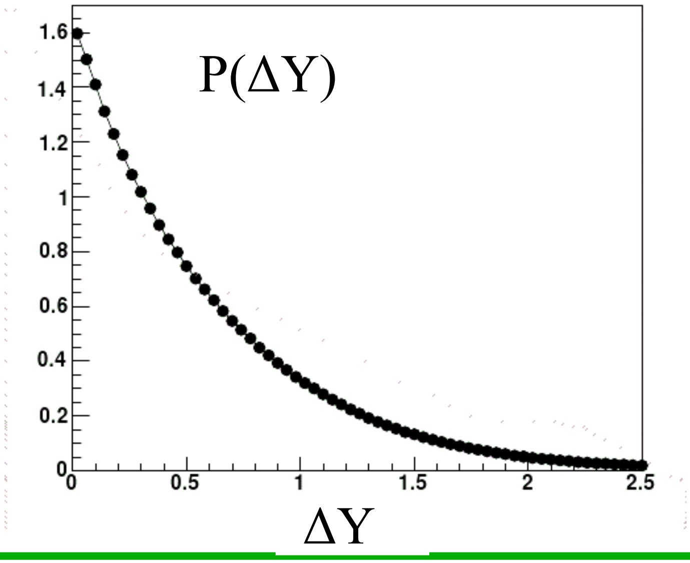


Gale et al.

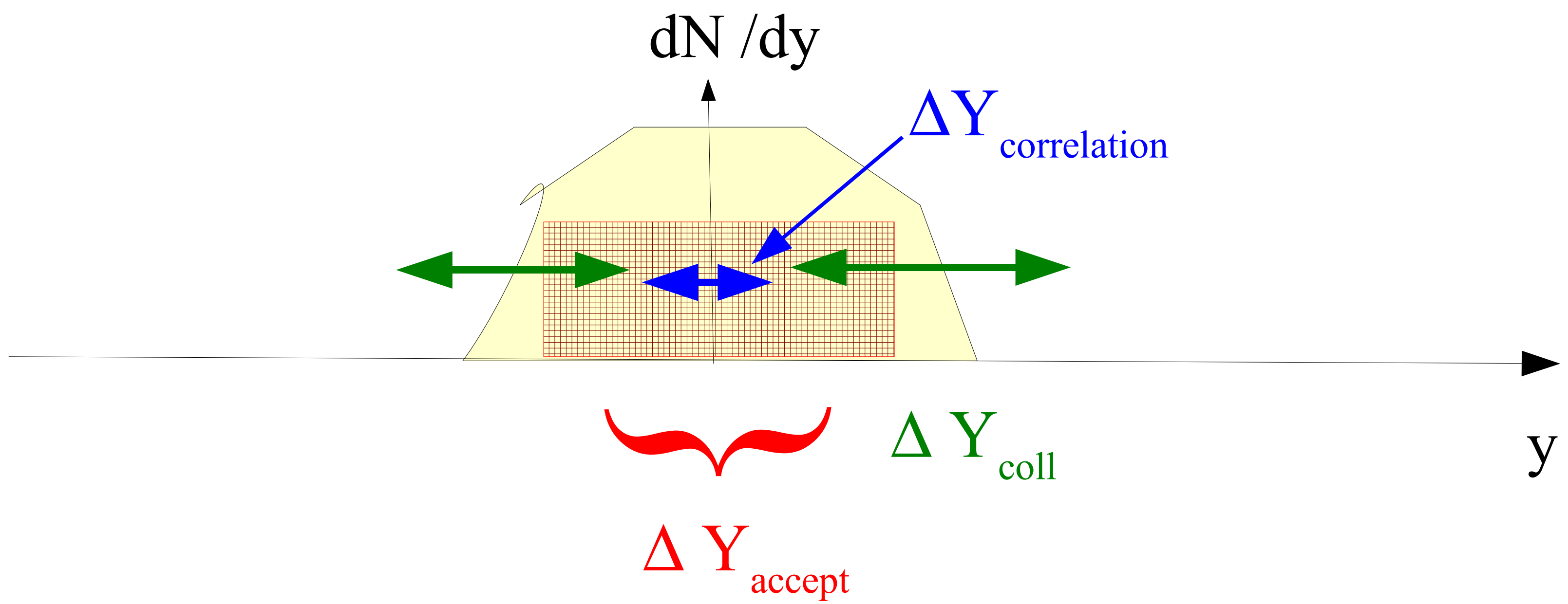


Bleicher et al





“Charge” fluctuations at SPS and below

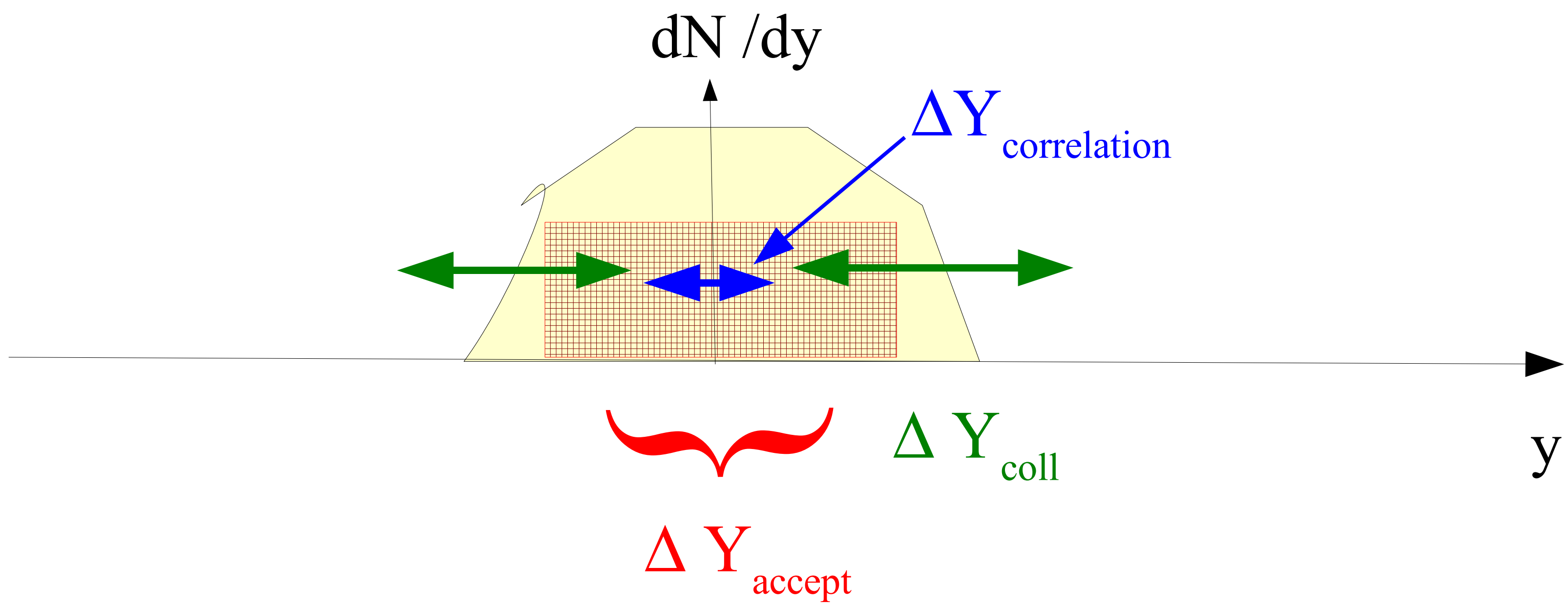


Condition for “charge” fluctuations:

1)$\Delta Y_{corrrelation} << \Delta Y_{accept}$ **(catch the physics)**

3)$\Delta Y_{total} >> \Delta Y_{accept} >> \Delta Y_{coll}$ **(keep the physics)**

"Charge" fluctuations at SPS and below



Condition for "charge" fluctuations:

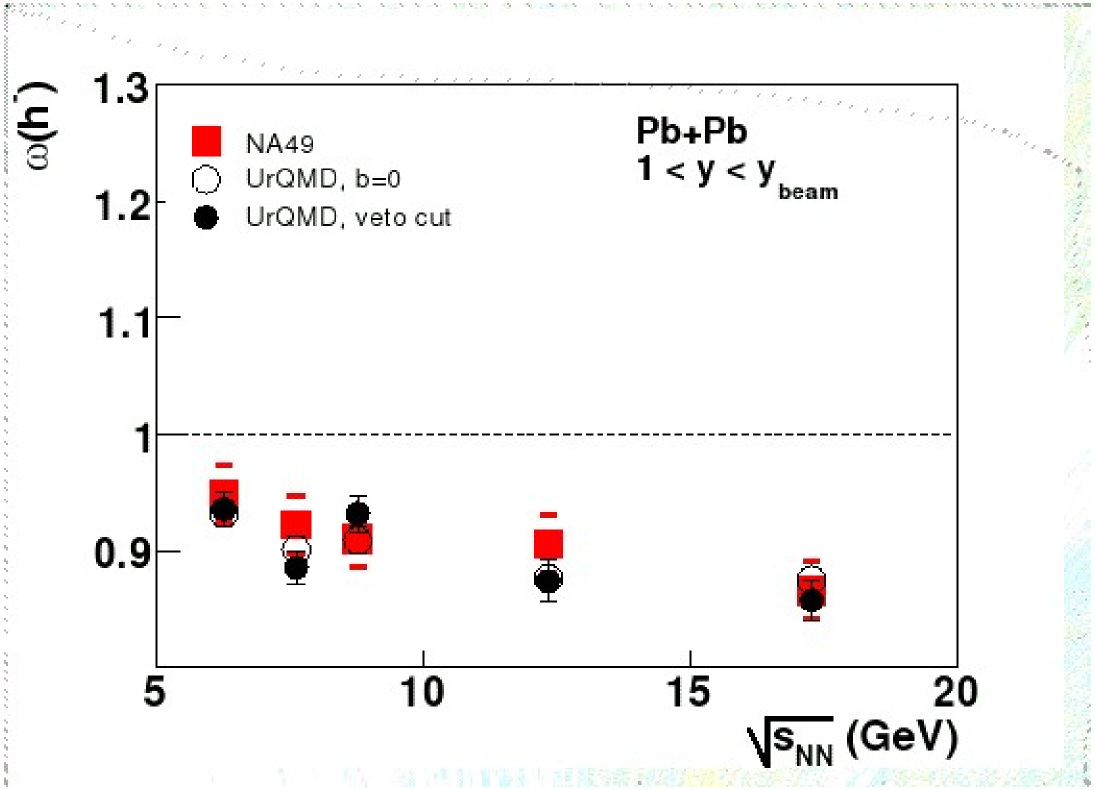
1)$\Delta\ Y_{corrrelation} \ll \Delta\ Y_{accept}$ **(catch the physics)**

3)$\Delta\ Y_{total} \gg \Delta\ Y_{accept} \gg \Delta\ Y_{coll}$ **(keep the physics)**

E-by-E observables

- Net-Charge fluctuations
  - Resonance gas at RHIC
  - no sensitivity at SPS and below
- Multiplicity fluctuations
  - interesting centrality dependence at top SPS energies
  - **No** energy dependence

Multiplicity Fluctuations

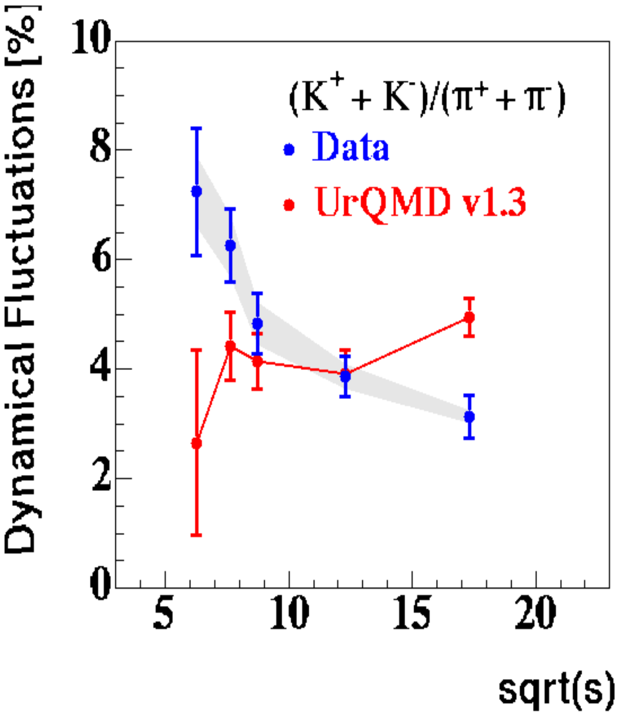


E-by-E observables

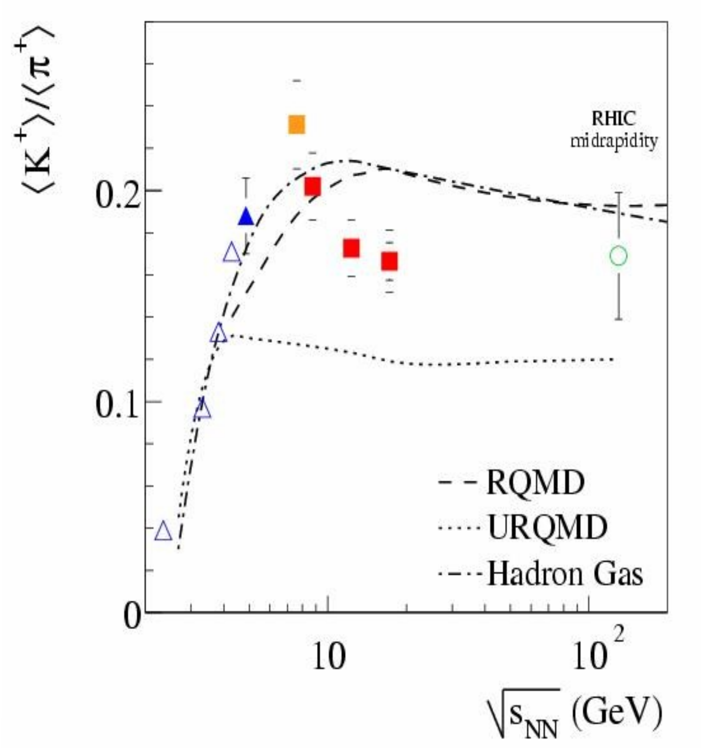
- Net-Charge fluctuations
  - Resonance gas at RHIC
  - no sensitivity at SPS
- Multiplicity fluctuations
  - interesting centrality dependence at top SPS energies
  - **No** energy dependence
- Transverse momentum fluctuations
  - some signal at SPS & RHIC (mostly “jets”)
  - **No** energy dependence
- Ratio (K/π) fluctuations
  - statistical at top SPS, possible signal at low SPS

Strange things...

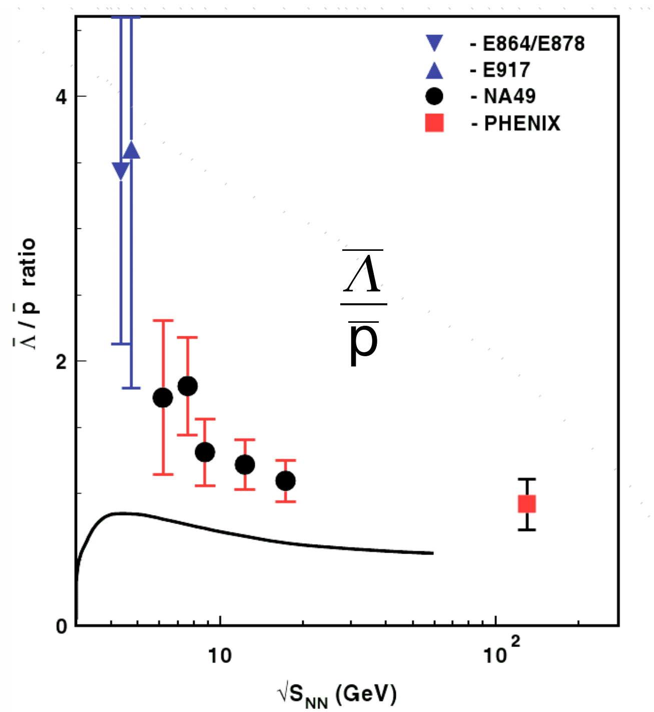
C. Roland, QM04, NA49



NA49



NA49, PRC73, 044910 (2006)

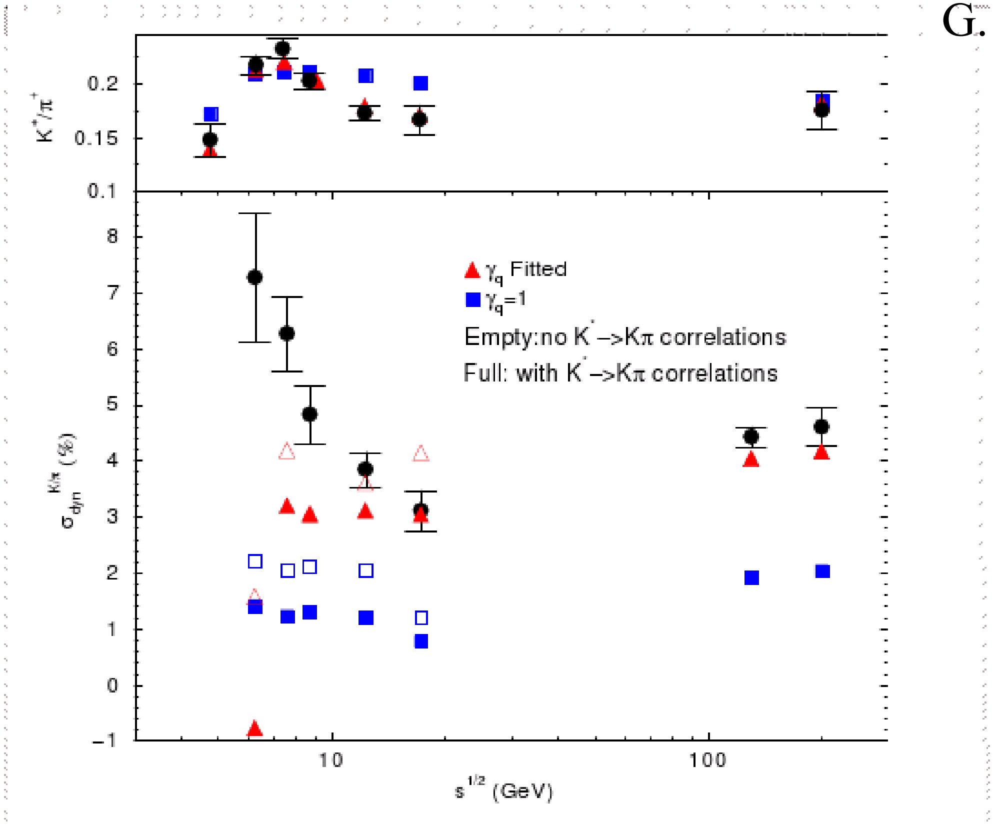


Hadron gas predictions

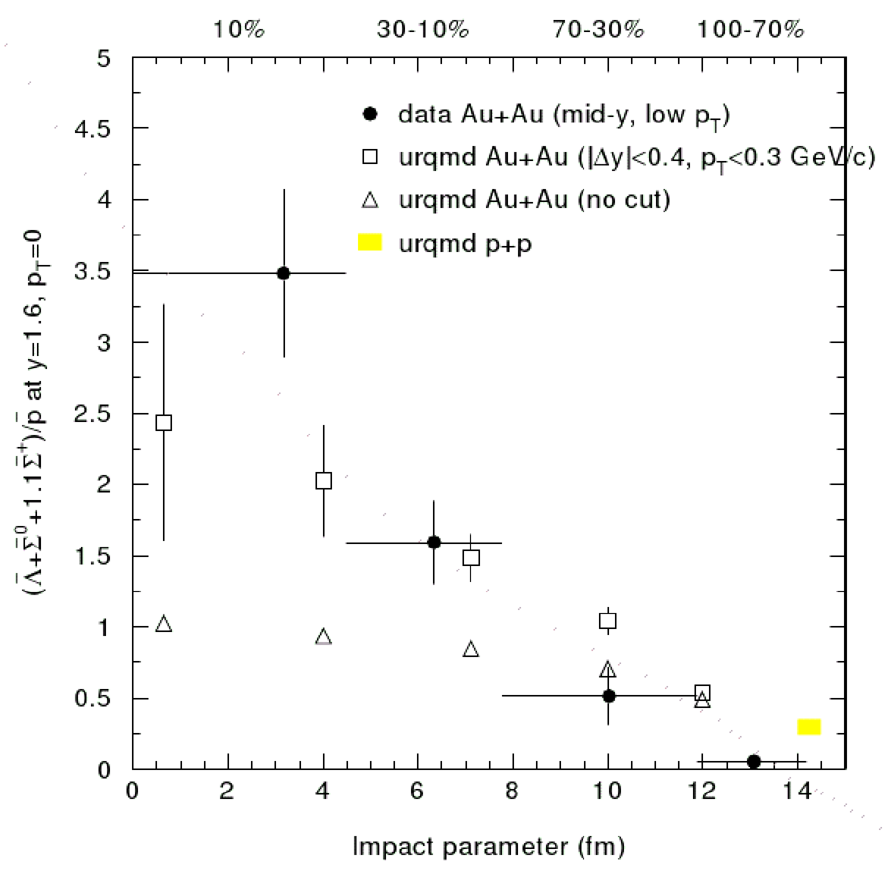
G. Torrieri, QM2006

$\langle \frac{K}{\pi} \rangle$

$\langle \delta (\frac{K}{\pi})^2 \rangle$



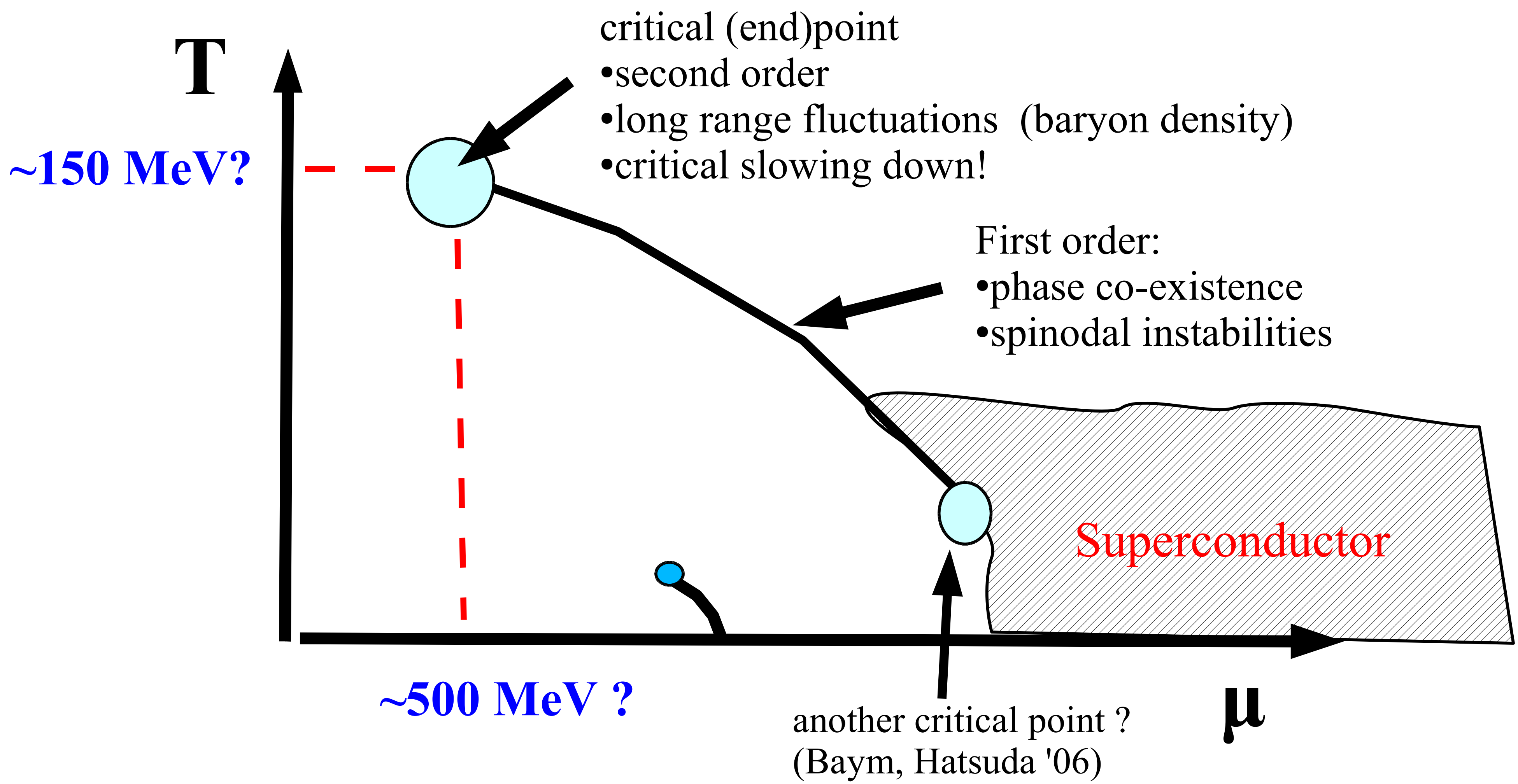
UrQMD and Lambda-bar / p-bar



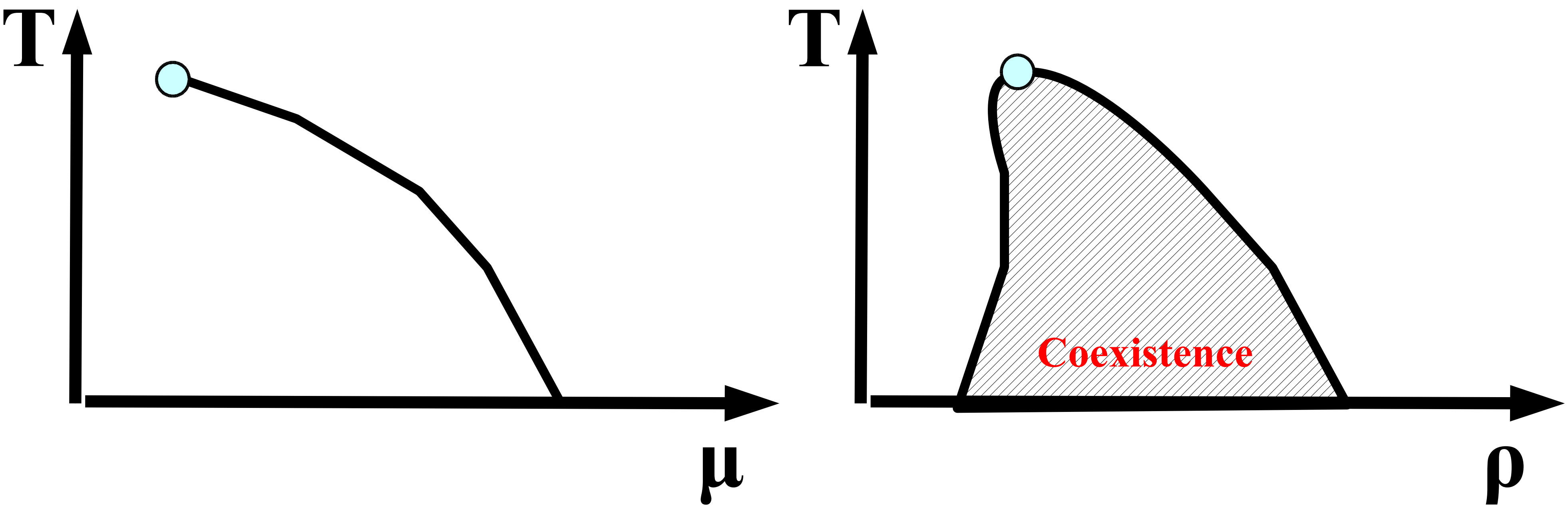
F. Wang
nucl-ex/0010002

Strong enhancement mostly an effect of acceptance cut !?

The QCD Phase Diagram
(from a theorist's perspective)



Order Parameter



Baryon density is a good order parameter
density fluctuations are a good observable (theoretically...)
Baryon Number fluctuations also good in principle, but
same problem as Charge fluctuations at low Energy

Nambu model

(Fuji et al, hep-ph/0401028,0403039)

Sigma remains massive at CP; CP driven by spacelike p-h exitations

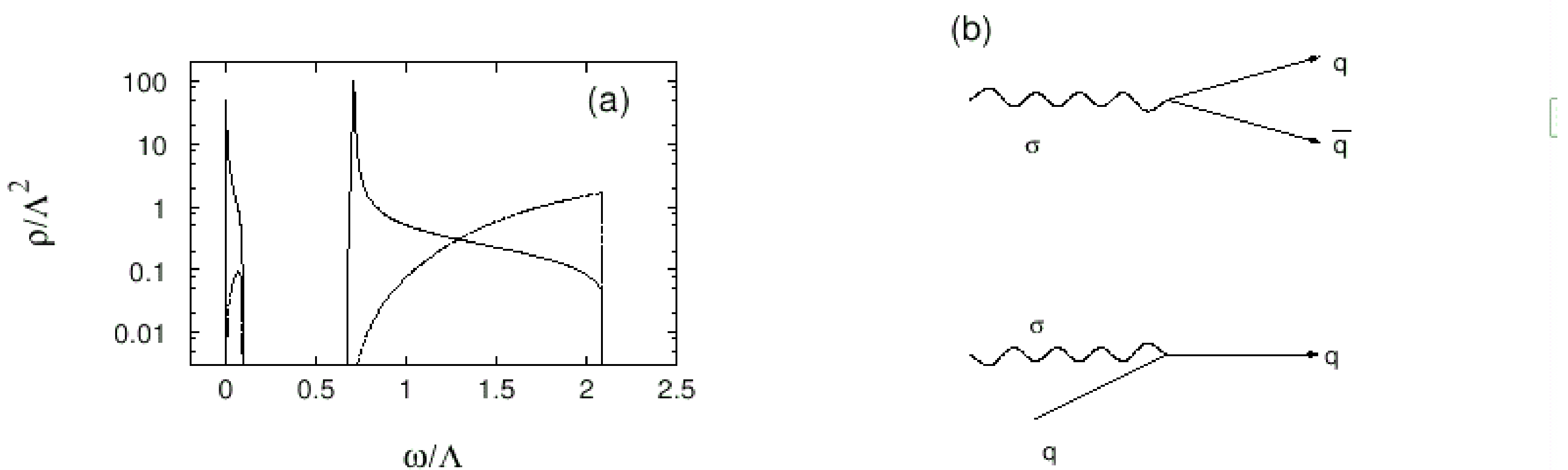


Fig. 2. (a) Spectral function in the scalar channel (solid) with $|\boldsymbol{q}|/\Lambda = 0.1$ at a CEP with $m/\Lambda = 0.01$. The free gas spectrum (dashed) is also shown for reference. (b) Typical processes contributing to the spectrum.

Nambu model
p-h excitations

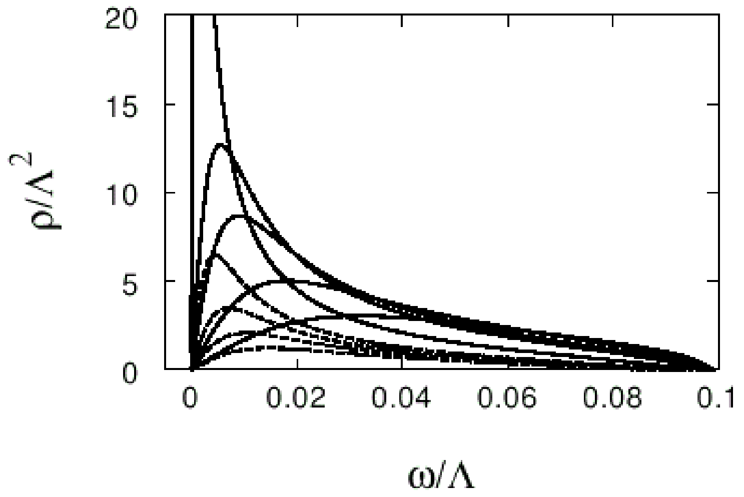
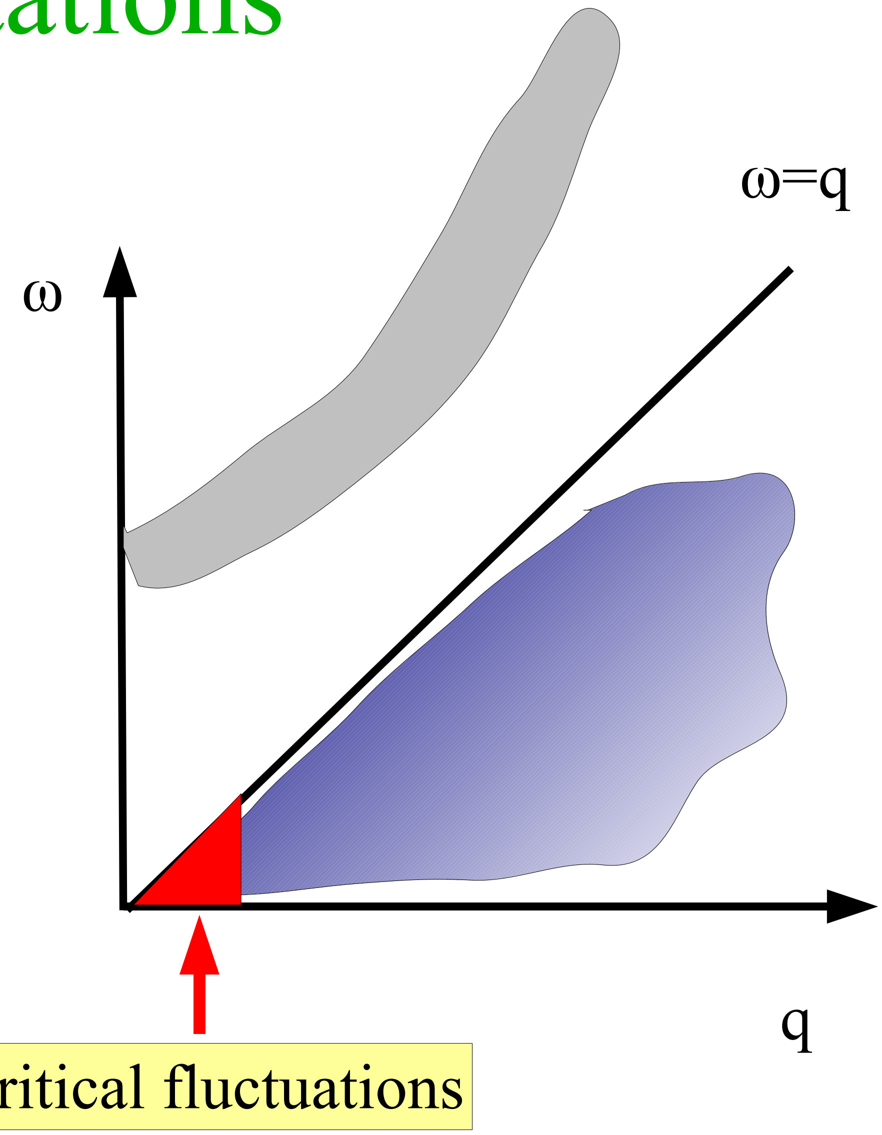


Fig. 3. Spectral function in the spacelike momentum region with $|\boldsymbol{q}|/\Lambda = 0.1$, $T = T_c$ and $m/\Lambda = 0.01$ for several μ (see text).



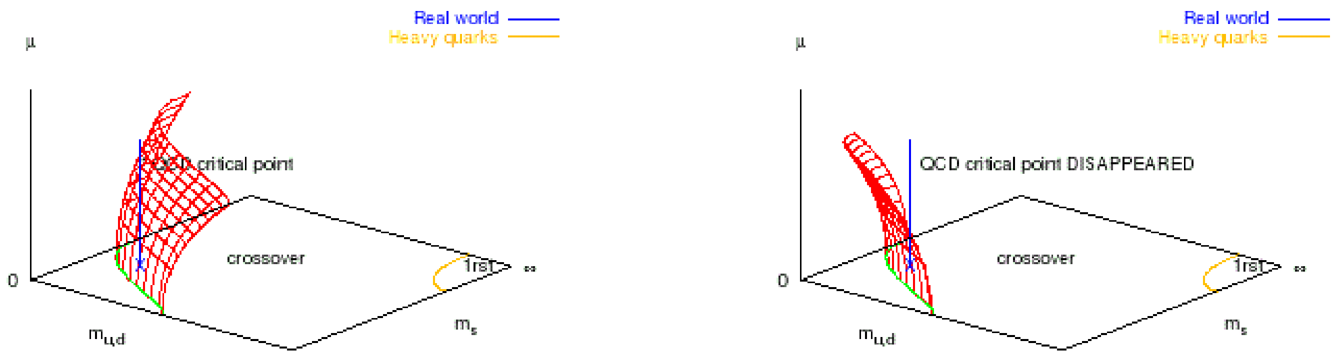
QCD critical point

- Order parameter: baryon density or scalar density
  - Actually it is a superposition
- Both scalar (chiral) an quark number susceptibilities diverge
- **Screening** (“space like”) masses vanish (“omega”, “sigma”)
  - not accessible by (time-like) dileptons
- Is it related to chiral transition at m_q=0 ?

Lattice and the critical point
Forcrand, Philipsen

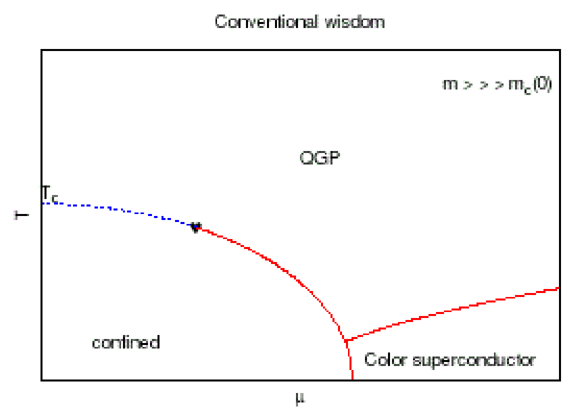
A non-standard scenario: no critical point?

sign of $c_1 = \frac{dm_c(\mu)}{d\mu^2}\Big|_{\mu=0}$



If Phase transition is not chiral but ”liquid gas” and ”anchored at low T high μ the ”conventional scenario is still ok.

Question/Challenge: Can we find a ROBUST phasetransition at high μ e.g. such as in the nuclear liquid gas case?

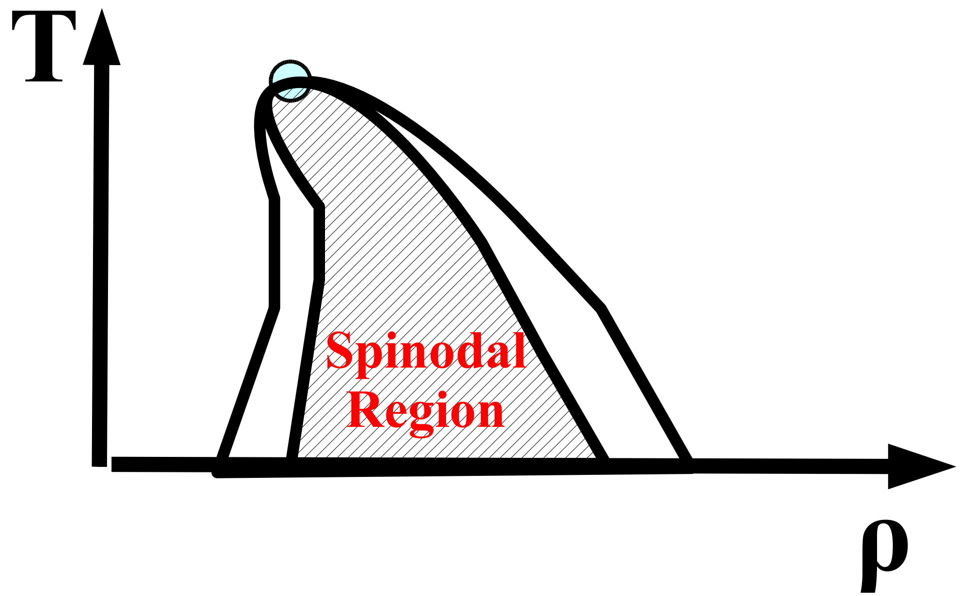


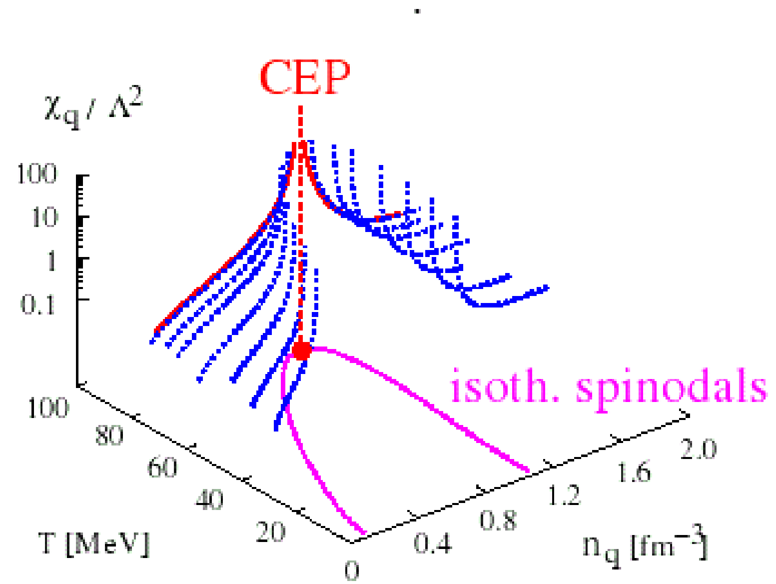
See P. de Forcrand, Parallel Session XI

QCD critical point

- Order parameter: baryon density or scalar density
  - Actually it is a superposition
- Both scalar (chiral) an quark number susceptibilities diverge
- **Screening** (“space like”) masses vanish (“omega”, “sigma”)
  - not accessible by (time-like) dileptons
- Is it related to chiral transition at $m_q=0$?
- The transition is in same universality class as liquid gas! (Son, Stephanov)
  - Fluctuations are driven by density fluctuations; chiral field is just tagging (H. Fuji, parallel Session IV)
- CP “just” the end of of 1^{st} Order transiition
  - Spinodal instabilities

Spinodal region





Inside spinodal region: mechanical instability
formation of “blobs” of typical size;
(may explain K/pi flucts, Majumder et al)

Baryon susceptibility diverges at spinodal boundary
(Sasaki et al, see also Sasaki, parallel session X)

Things to do!

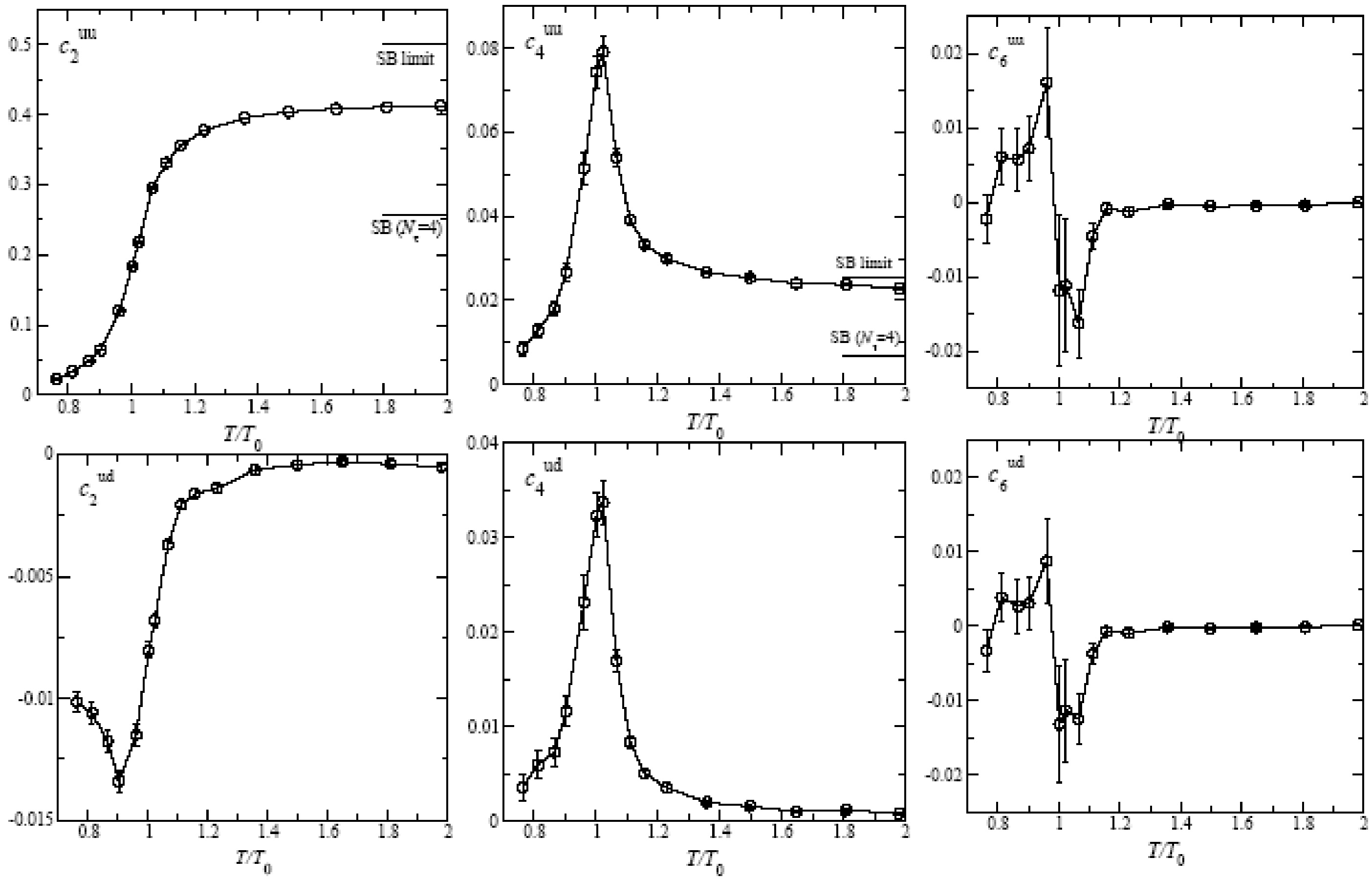
- Characterize the Phases
  - what are useful order parameters
  - what observables couple well to order parameter but survive hadron phase
- Test observables using static and dynamical models
  - Effects are small, comparable with “trivial ones” such as quantum statistics, dynamics etc.
  - Only a well chosen observable / set of observables will prevent us from seeing Poisson
    - e.g. can we live without neutrons?
  - CONSERVATION LAWS
- **Don't forget the first order phase co-existence!**

Observables for CP and co-existence

- Fluctuations (probably not of conserved charges)
- Correlations (spiondal blobs)
- Energy scan
- System size dependence (finite volume scaling)
  - centrality may not do
- Good, HOMOGENIOUS acceptance
- Be prepared to measure everything
  - not clear (yet?) which observable couples strongest to baryon density
  - want to see finite volume scaling in more than one observable
- Get you hands dirty now!

Lattice-QCD susceptibilities

This should be in your present (RHIC) dataset !



Rule of thumb:

$$c_n \sim \langle X^n \rangle$$
$$X = B, Q, S, \ldots$$

Alton et al, PRD 66 074507 (2002)

Critical Point and viscosities

CP is in universality class of liquid gas (Son, Stephanov)

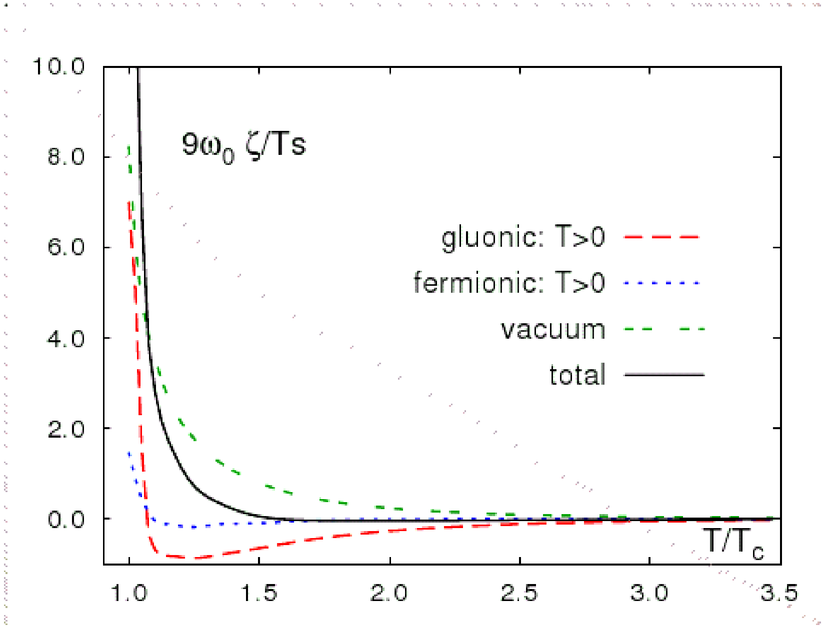
Hohenberg - Halperin Model H (Rev. Mod. Phys 49 (1977)):

$$\eta \sim \xi^{0.065,} \qquad \xi = \text{Correlation Length}$$

Shear viscosity diverges at CP

Bulk viscosity also diverges:
(Kharzeev, Turchin, Karsch arXiv:0711.0914)

Note: even large increase without PT
due to vacuum contribution



INT Summer Program
The QCD Critical Point
July 28 - August 22, 2008

Organizers:G. Roland, M. Stephanov and V.K.

The End

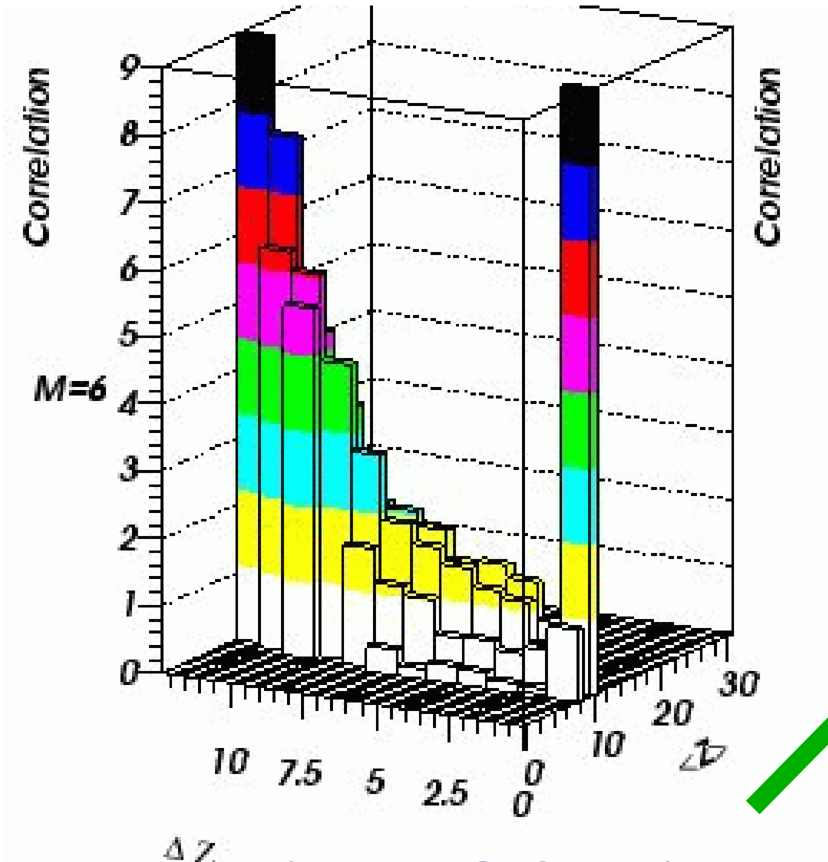
Charge Fluctuations and Balance Functions

$$\frac{\left\langle \delta Q^2 \right\rangle_{\Delta\eta}}{\left\langle N_{\mathrm{ch}} \right\rangle_{\Delta\eta}} \approx 1 - \int d\eta \, B(\eta|\Delta\eta)$$

Spinodal decomposition in nuclear multifragmentation

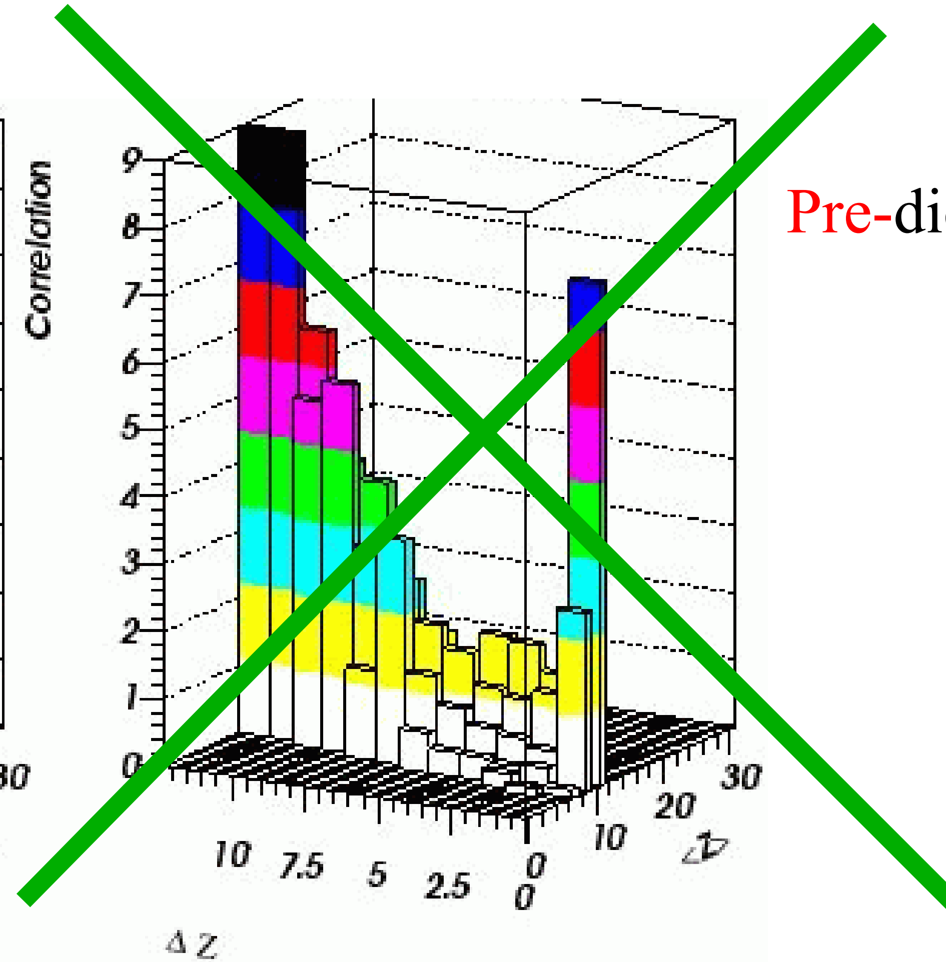
occurs!

Data speak for themselves!



Experiment (*INDRA @ GANIL*)

Borderie *et al*, PRL 86 (2001) 3252



Pre-diction

Theory (*Boltzmann-Langevin*)

Chomaz, Colonna, Randrup, …

Correlation length and correlation functions

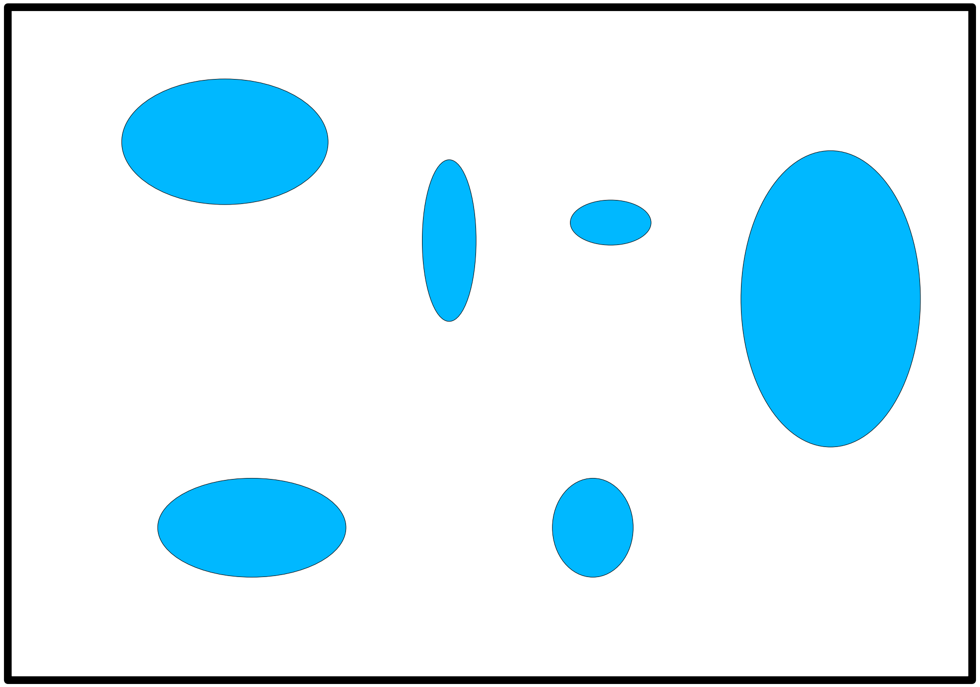
Theory: $\langle \rho(r)\rho(0) \rangle$ Correlation in coordinate space

Experiment: $\frac{dN}{dp_1 dp_2} \sim \langle \tilde{\rho}(p_1)\tilde{\rho}(p_2) \rangle$ Correlation in momentum space

Flow helps maybe....

Better: Calculation of momentum space correlations

Baryon number fluctuations
(the obvious choice?)



Strong spatial fluctuations

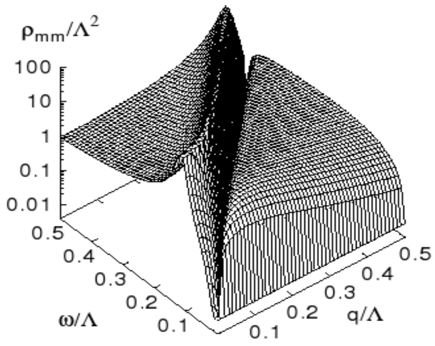
If $V_{domain} << V$, small effect on integrated Baryon Number fluctuations

$$\frac{\langle (\delta N)^2 \rangle}{\langle N \rangle} \approx \left(1 + \frac{(\Delta \rho)^2}{4 \bar{\rho}^2} \right)$$

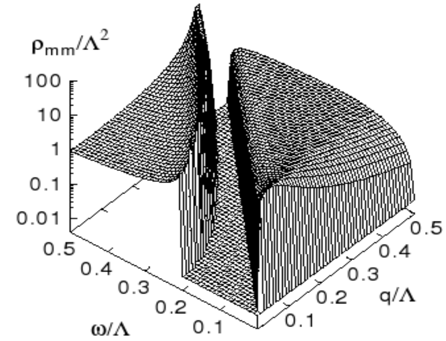
CP and the chiral transition

Chiral transition $m_q=0$

$T>T_c$



(a)

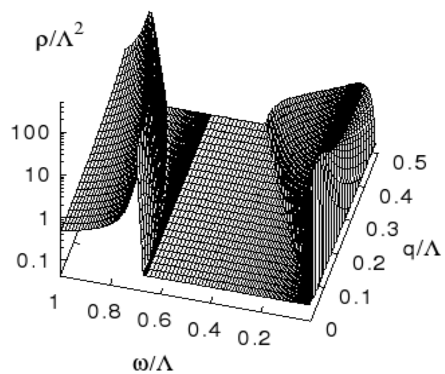


(b)

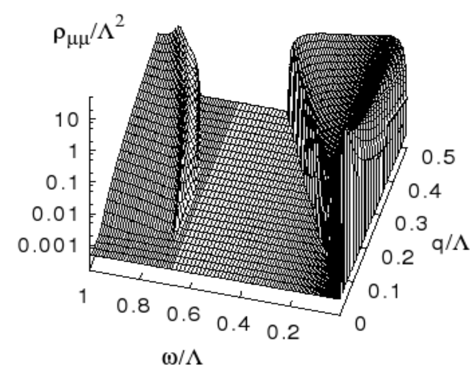
$T<T_c$

Critical point $m_q>0$

Massive Scalar

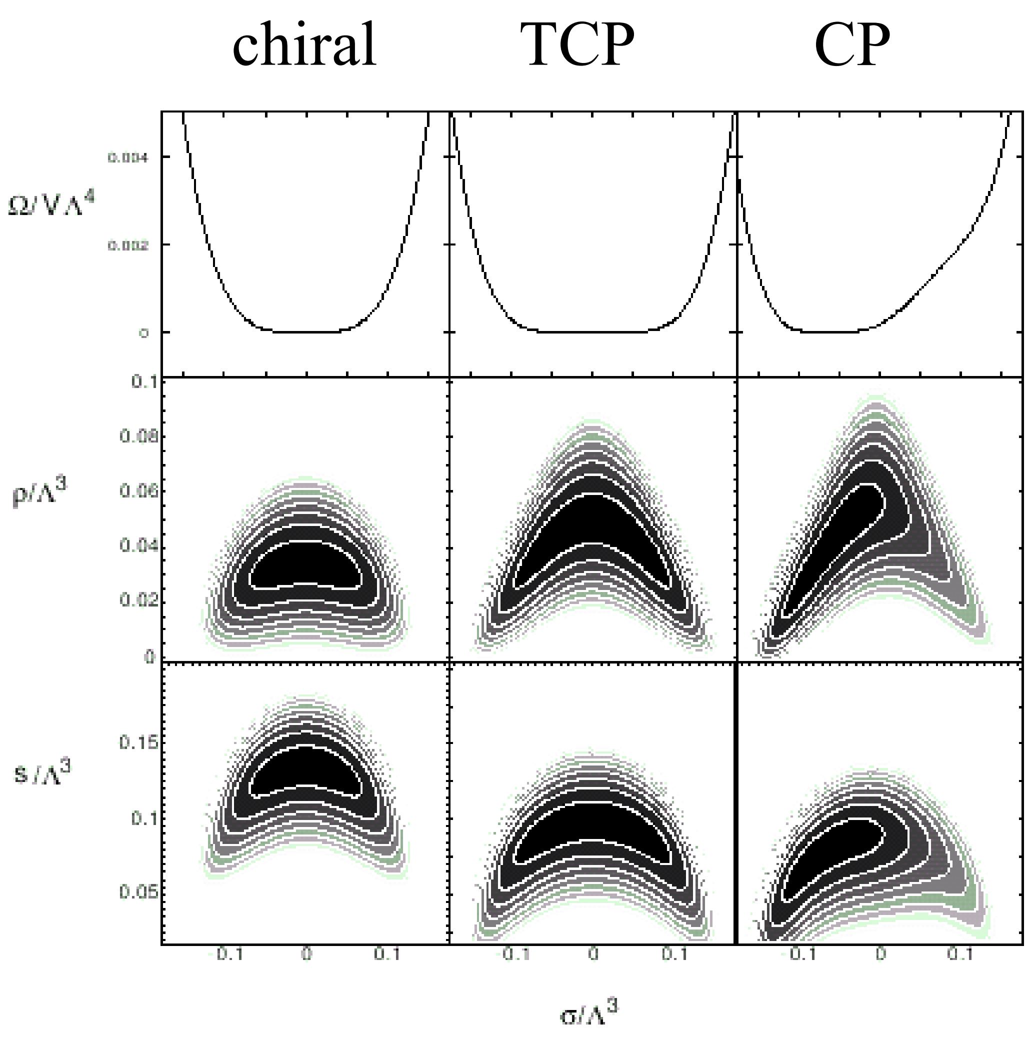


Scalar



Vector (Baryon-density)

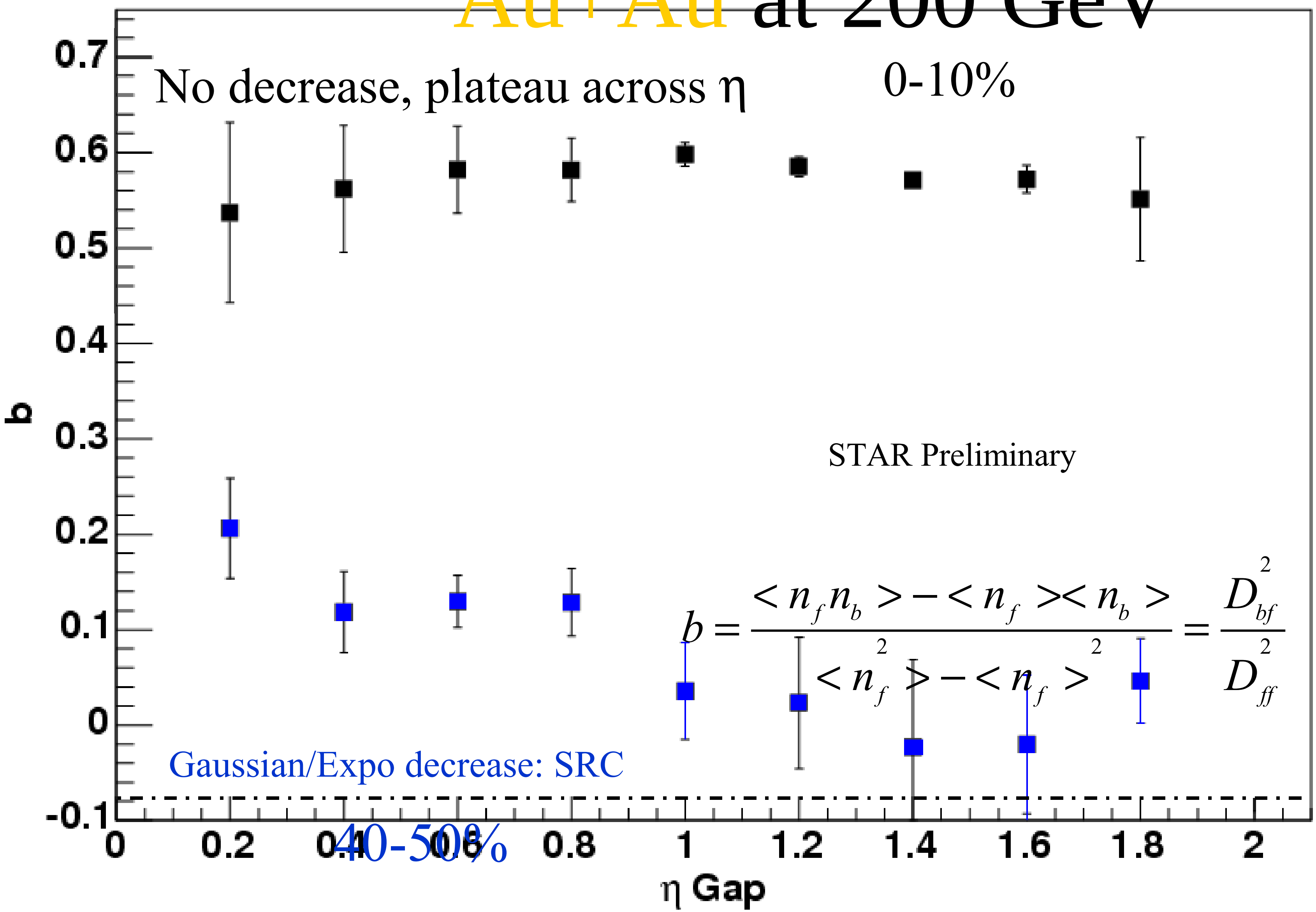
Mixing of scalar and vector



Note: mixing of scalar vector (p-h) only for spacelike excitations close to q=0 (Friman, Soyeur et al)

Note: entropy also mixes in!
Multiplicity fluctuations
Trigger?

Centrality Dependence: Au+Au at 200 GeV

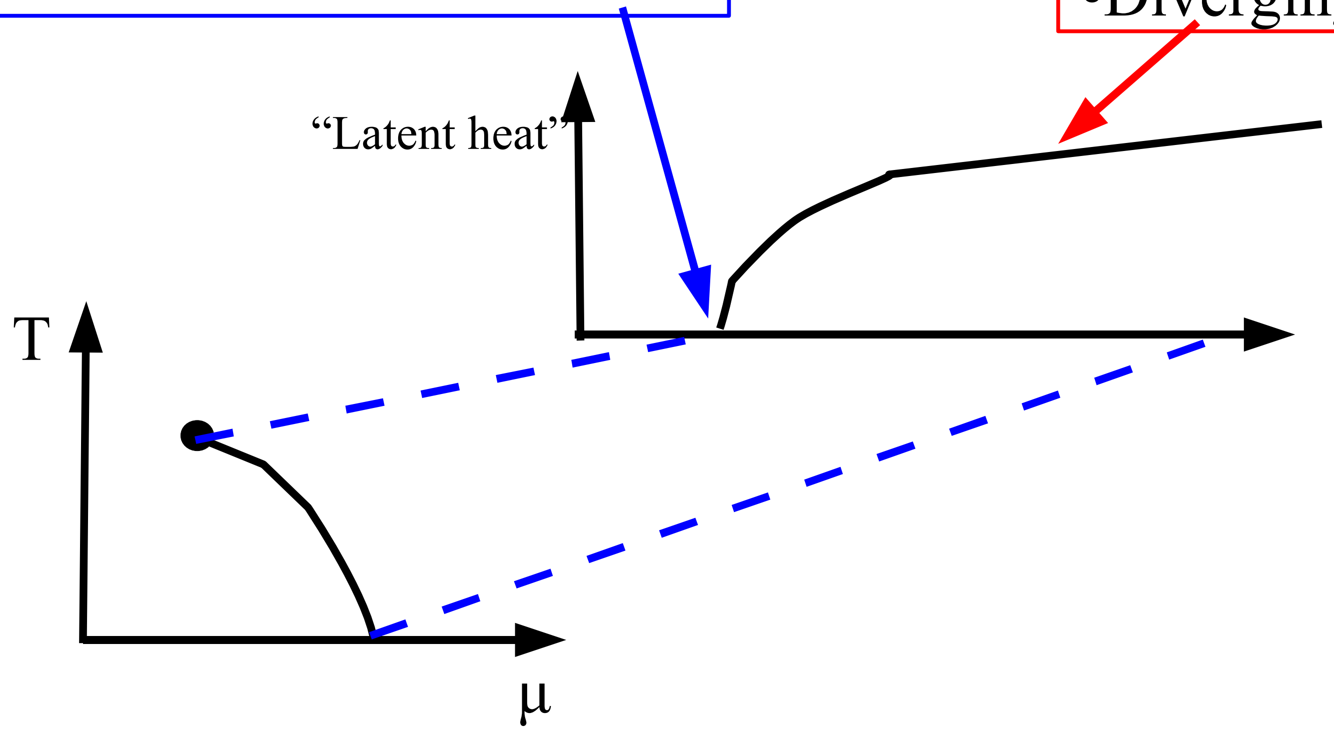


- SRC < Δη 1.0.
- LRC > Δη 1.0.
- Central collisions demonstrate a strong, LRC.
- For mid-peripheral collisions:
  - LRC is absent.

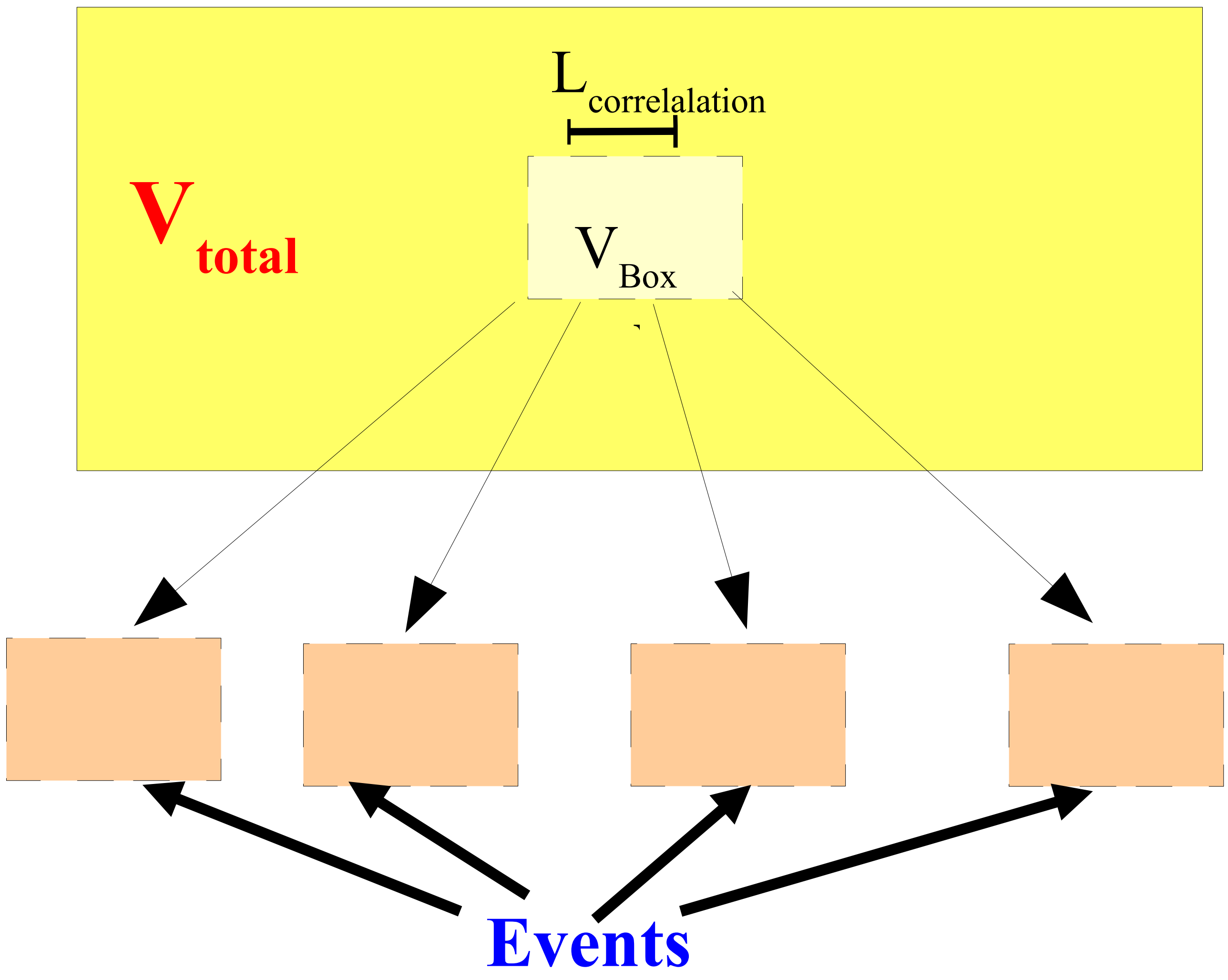
First order or second order?

Second order:
- Critical fluctuations
- Diverging Susceptibilities

First order:
- Phase coexistence, bubbles
- Spinodal instabilities
- Diverging Susceptibilities



“Charge” Fluctuations



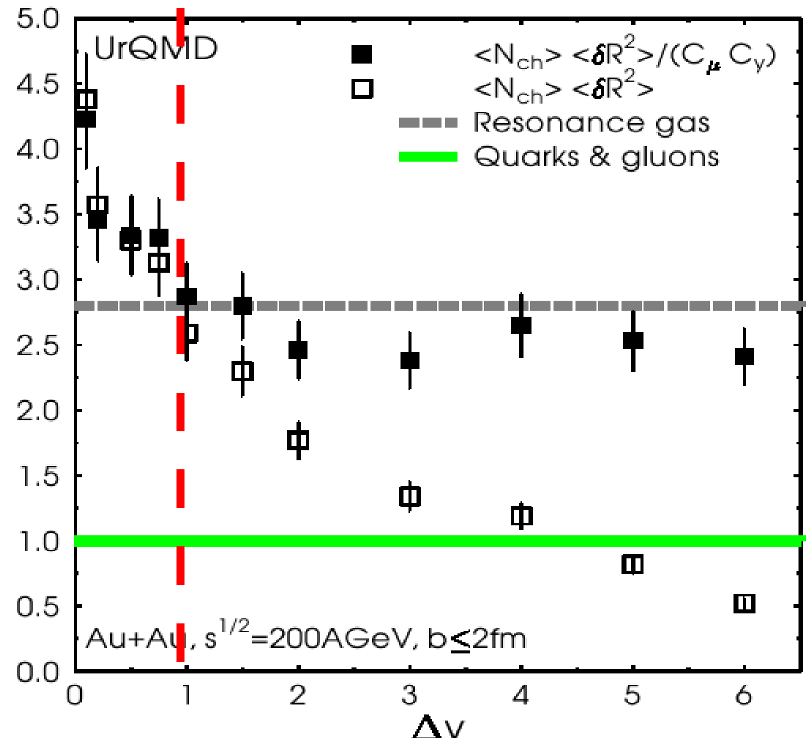
$V_{box} << V_{total}$

$L_{correlation} \sim V_{box}^{1/3}$

Charge conservation

$$C_\mu = \tilde{R}^2_{\Delta y} = \frac{\langle N_+\rangle^2_{\Delta y}}{\langle N_-\rangle^2_{\Delta y}}$$

$$C_y = 1 - P = 1 - \frac{\langle N_{ch}\rangle_{\Delta y}}{\langle N_{ch}\rangle_{total}}$$



Correlations (meson decay)

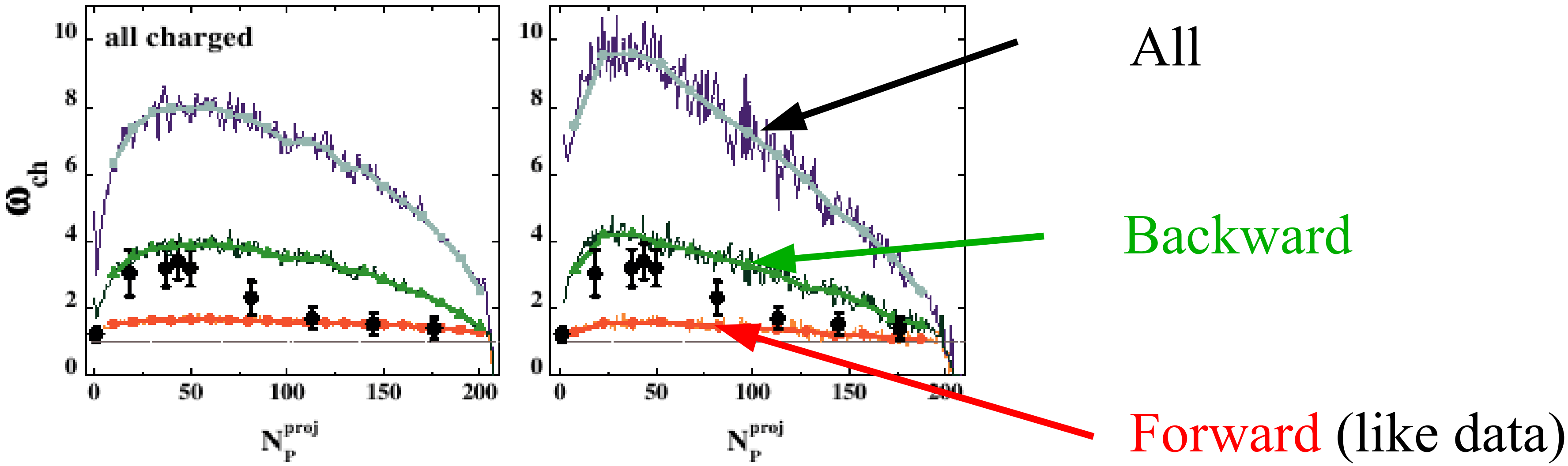
Bleicher et al.
Phys.Rev.C62:061902,2000

QCD first order transition

- Order parameter: baryon density or scalar density
- Both scalar (chiral) and quark number density are discontinuous
- Density fluctuations / blobs due to spinodal instability.
- If latent heat is sizable: Long lived mixed phase
  - two peak structure of omega a la C.M. Ko for phi.
  - assuming the TAPS results hold up in this case you want to measure **low p_t** omegas

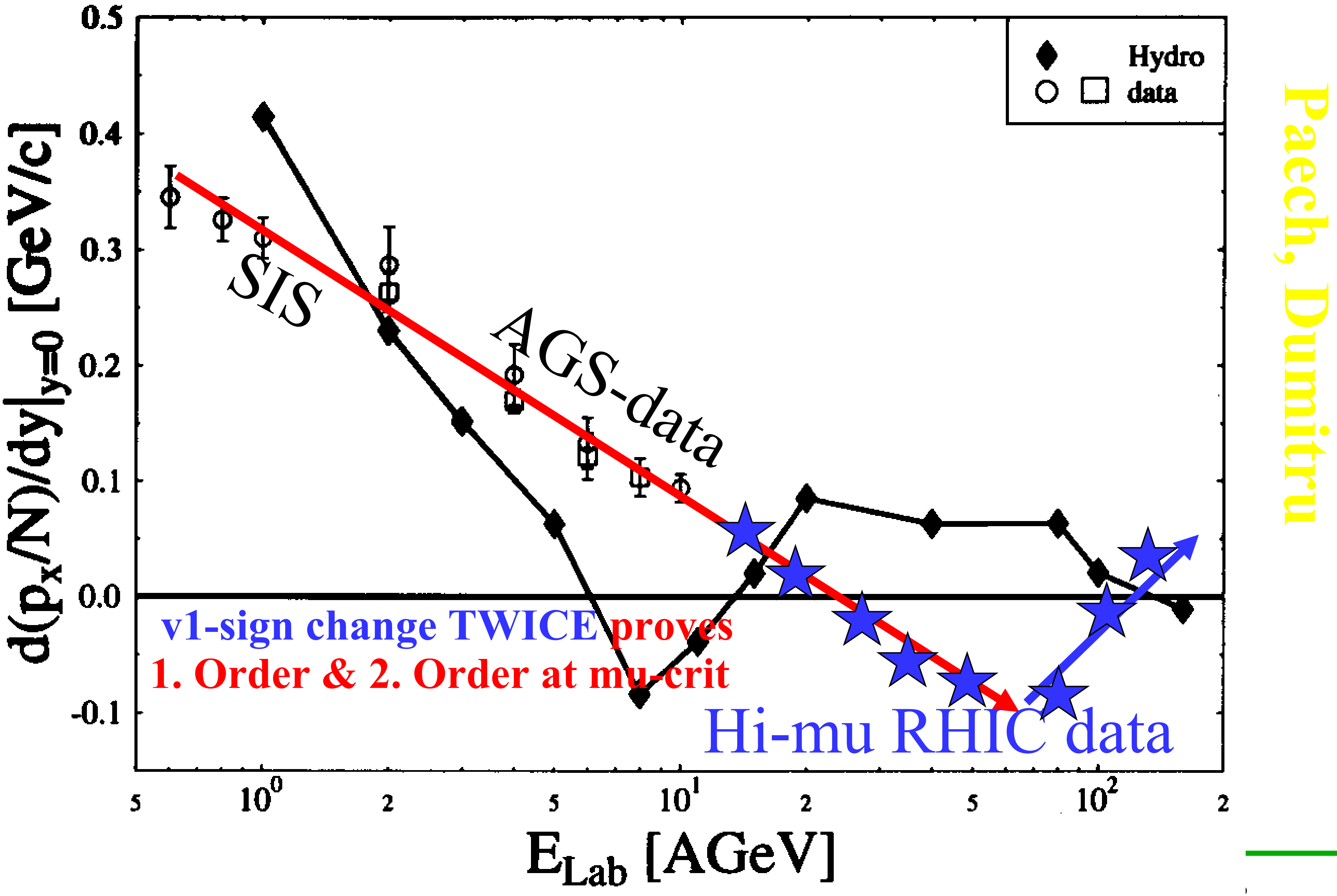
Dynamics, event selection ...
(or why a symmetric detectors are good)

Konchakovski et al, nucl-th/0511083



•Fluctuations are sensitive to dynamics (mixing of projectile and target material?)

•Event selection/trigger affects fluctuations → large Acceptance!



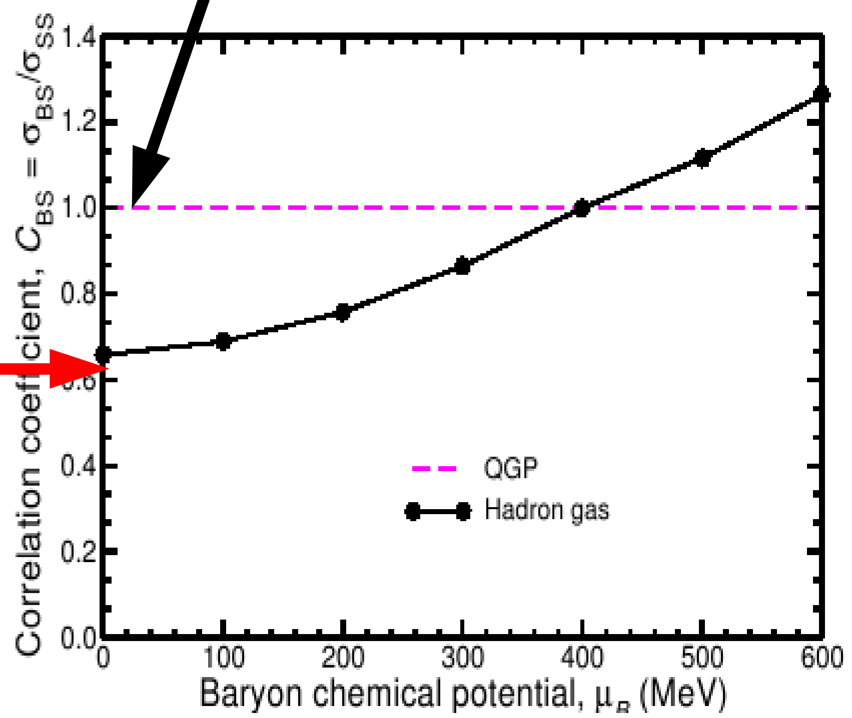
Hydro
data
SIS
AGS-data
v1-sign change TWICE proves
1. Order & 2. Order at mu-crit
Hi-mu RHIC data
$d(p_x/N)/dy|_{y=0}$ [GeV/c]
E_{Lab} [AGeV]
Paech, Dumitru

V

Something else: <BS> , <QS>

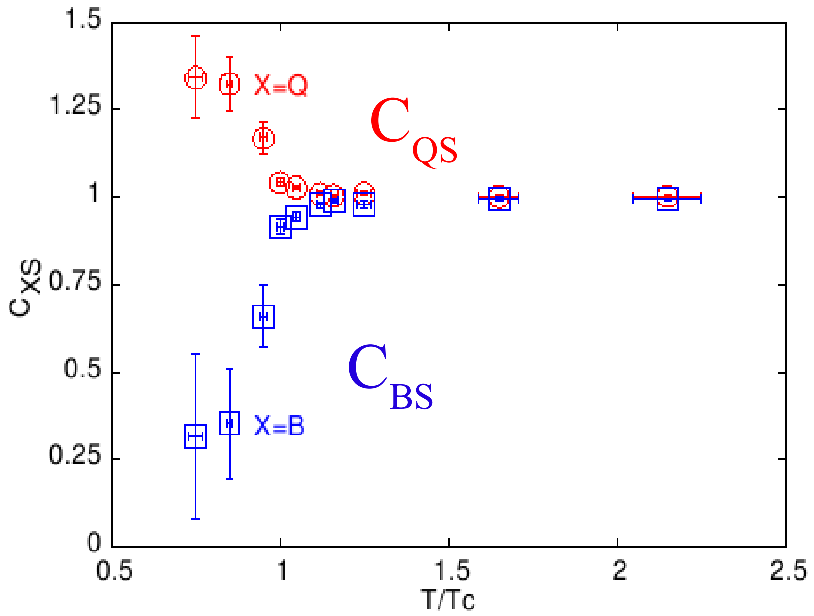
Independet quarks and
LATTICE QCD for T>1.1T_c

Bound state
QGP



V.K, Majumder, Randrup PRL95:182301,2005

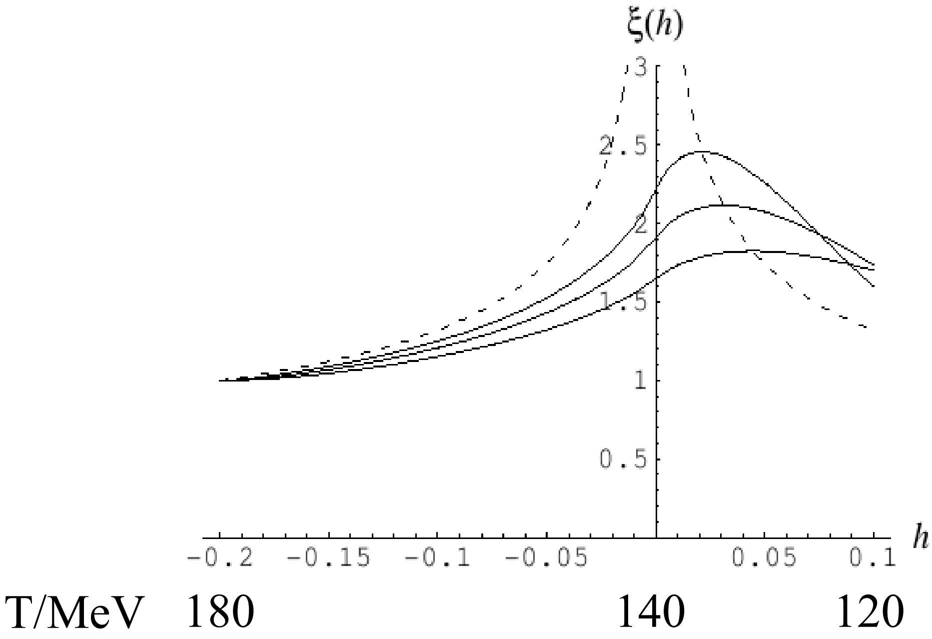
$$C_{BS} = -3\frac{\langle BS\rangle}{\langle S^2\rangle} \qquad C_{QS} = -3\frac{\langle QS\rangle}{\langle S^2\rangle}$$



Gavai,Gupta, hep-lat/0510044

Second order

correlation length ~$1/m_\sigma$



Bernikov, Rajagopal, hep-ph/9912274

- Critical slowing down
- limited sensitivity on model parameters
- Max. correlation length 2-3 fm
- Translates in **3-5%** effect in p_t-fluctuations