Multiplicities in Pb-Pb collisions at the LHC from non-linear QCD evolution

Javier L. Albacete

THE OHIO STATE UNIVERSITY

Quark Matter 08

Jaipur, India, February 4-10 2008

Outline

@ Energy dependence of nuclear gluon distributions: Balitsky-Kovchegov equation

⇒ Recent developments: Running coupling corrections

⇒ Strong reduction of the speed of evolution

@ Phenomenological consequences:

⇒ Energy dependence of multiplicity densities in A-A collisions

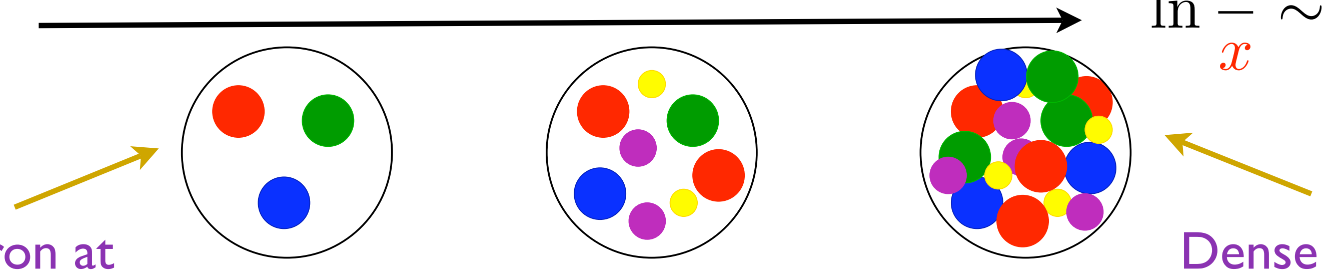
⇒ Determining initial conditions: RHIC @ $\sqrt{s}$=130 and 200 GeV

⇒ Extrapolation to LHC: Pb-Pb collisions @ $\sqrt{s}$=5.5 TeV

Motivation

- Color Glass Condensate: The high energy limit of QCD is governed by large gluon densities and gluon saturation

$$\ln\frac{1}{x} \sim \ln s \sim Y$$



Dilute hadron at moderate energy

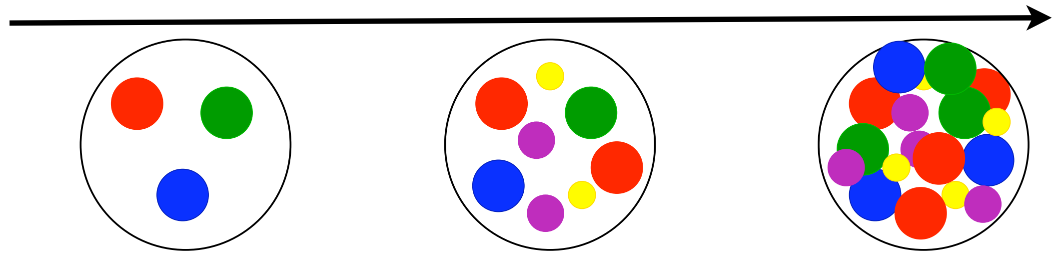
Dense hadron at high energy

- BK-JIMWLK equations: The evolution of gluon densities to high energy is driven by linear (radiation) and non-linear effects (recombination)

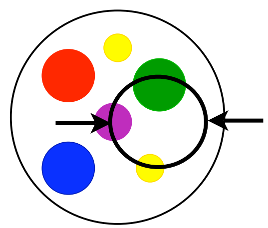
$$\frac{\varphi(x,k)}{\ln 1/x} \sim K \otimes \varphi(x,k) - \varphi^2(x,k)$$

Motivation

⇒ Charge density correlations define the saturation scale:



$$\ln\frac{1}{x} \sim \ln s \sim Y$$



$$\begin{cases} Q_s^2(Y) = Q_0^2 \, \exp \lambda Y \\ \lambda = \dfrac{d \ln Q_s^2(Y)}{dY} \end{cases}$$

⇒ LL-theory: $\lambda^{LL} \approx 4.8\alpha_s$ BK-JIMWLK at LL accuracy in $\alpha_s \ln 1/x$

⇒ Data: $\lambda^{data} \approx 0.288$ Fits to DIS and HIC data

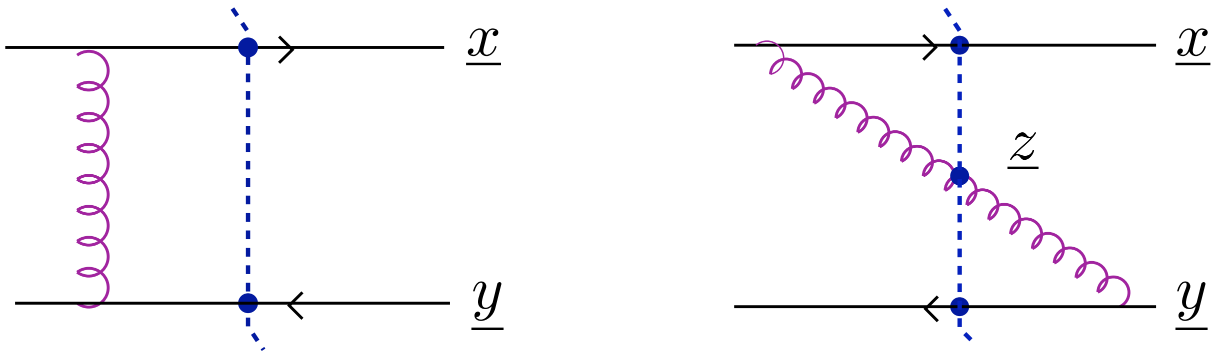
Experimental data demand a much slower evolution!
Need of higher order corrections to LL equations

Running coupling corrections (Kovchegov, Weigert, Balitsky, Gardi, Albacete ...

⇒ Strategy: resummation of quark loops to all orders, plus $N_f \longrightarrow -6\pi\beta$

$$\frac{\partial S(\underline{x},\underline{y};Y)}{\partial Y} = \int d^2z\, K^{LO}(\underline{r},\underline{r}_1,\underline{r}_2)\ \left[S(\underline{x},\underline{z})\,S(\underline{z},\underline{y}) - S(\underline{x},\underline{y})\right]$$

⇒ Leading log
(fixed coupling)



Running coupling corrections (Kovchegov, Weigert, Balitsky, Gardi, Albacete ...

⇒ Strategy: resummation of quark loops to all orders, plus $N_f \longrightarrow -6\pi\beta$

$$\frac{\partial S(\underline{x},\underline{y};Y)}{\partial Y} = \int d^2 z\, K^{LO}(\underline{r},\underline{r}_1,\underline{r}_2)\, \left[S(\underline{x},\underline{z})\, S(\underline{z},\underline{y}) - S(\underline{x},\underline{y})\right]$$

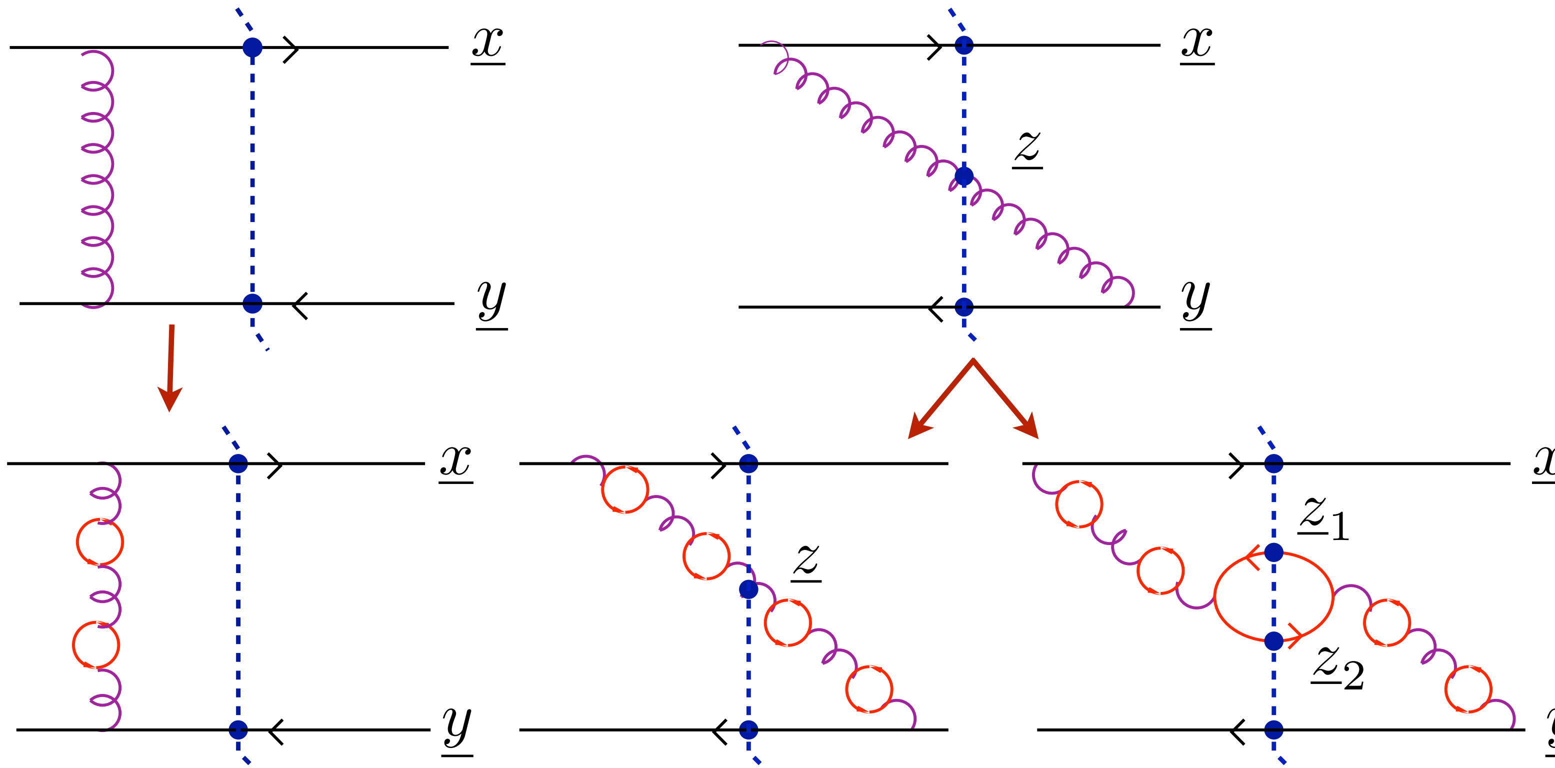
⇒ **Leading log**

(fixed coupling)

⇒ **All orders in $\alpha_s N_f$**

$N_f \longrightarrow -6\pi\beta$

(running coupling)



$$\frac{\partial S}{\partial Y} \propto \quad S(\underline{x},\underline{y}) \qquad S(\underline{x},\underline{z})S(\underline{z},\underline{y}) \qquad S(\underline{x},\underline{z}_1)S(\underline{z}_2,\underline{y})$$

"4-point" function

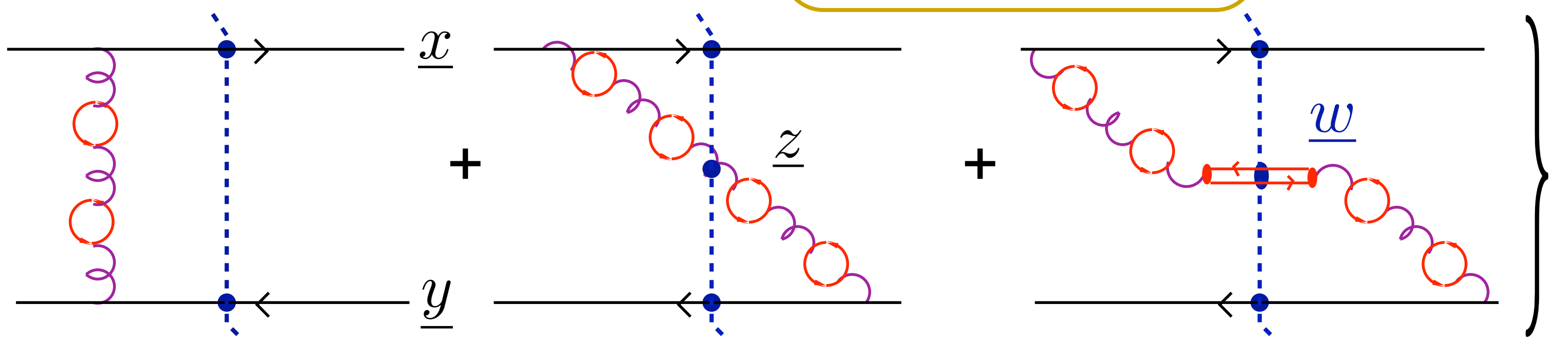
⇕

"3-point" function

⇒ The new physical channels modify the interaction structure of the LL equation.

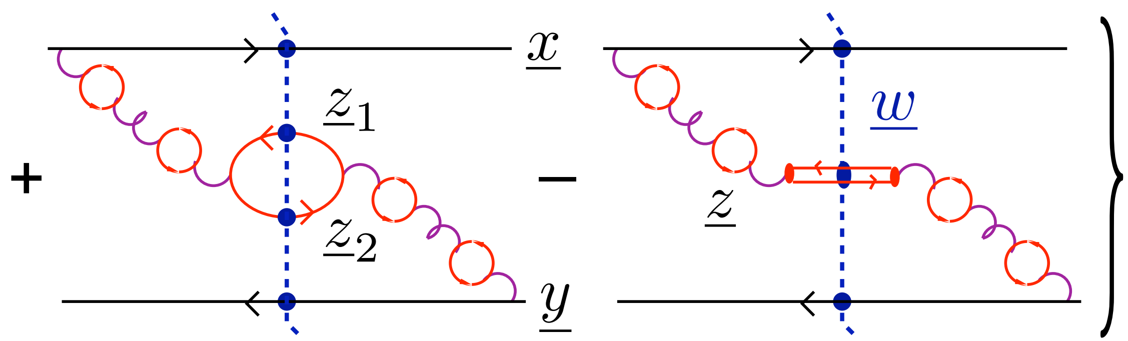
Complete in $\alpha_s N_f$ Evolution JLA and Y. Kovchegov PRD75 125021

$$\frac{\partial S}{\partial Y} = \mathcal{R}\,[S] - \mathcal{S}\,[S]$$



$\mathcal{R}[S]$

UV-divergent terms that contribute to the running of the coupling



$\mathcal{S}[S]$

Conformal, non running coupling terms. Neglected in previous calculations

⇒ ***Running*** term: $\mathcal{R}\,[S] = \int d^2 z\, \tilde{K}(\underline{r}, \underline{r}_1, \underline{r}_2)\, \left[S(\underline{x}, \underline{z}) S(\underline{z}, \underline{y}) - S(\underline{x}, \underline{y})\right]$

⇒ ***Subtraction*** term: $\mathcal{S}\,[S] = \int d^2 z_1 d^2 z_2\, K_{sub}(\underline{x}, \underline{y}, \underline{z}_1, \underline{z}_2)\, \left[S(\underline{x}, \underline{w}) S(\underline{w}, \underline{y}) - S(\underline{x}, \underline{z}_1) S(\underline{z}_2, \underline{y})\right]$

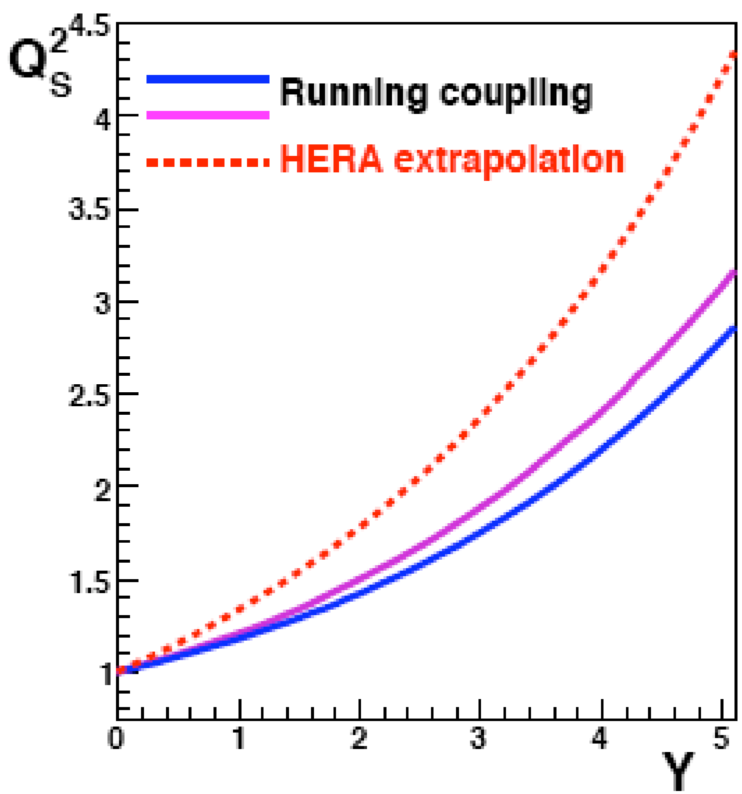
⇒ Running coupling comes in a "triumvirate": $K \sim \dfrac{\alpha_s(R_1)\,\alpha_s(R_2)}{\alpha_s(R_3)}$

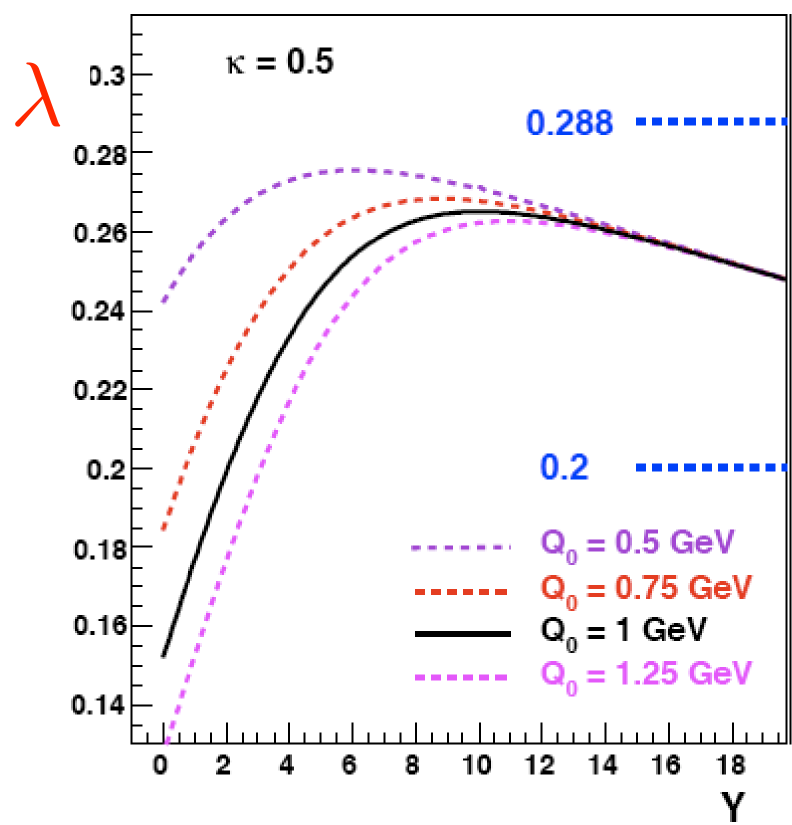
⇒ The running of the coupling reduces the speed of the evolution down to values compatible with experimental data (JLA PRL 99 262301 (07)):

⇒ Energy dependence of multiplicity in saturation models for particle production:

$$\left.\frac{dN_{AA}}{d\eta}\right|_{\eta=0} \sim \sqrt{s}^{\lambda} \qquad \text{where} \qquad Q_s^2(Y) = Q_0^2 \, \exp[\lambda Y]$$

$$\frac{\partial S}{\partial Y} = \mathcal{R}\left[S\right] - \mathcal{S}\left[S\right]$$





Fits to DIS HERA data

CGC +hydrodynamics at RHIC

Hirano-Nara (04)

Multiparticle production @ RHIC

k_t-factorization + saturation + local parton-hadron duality (Kharzeev-Levin-Nardi)

$$\frac{dN^g_{AB}}{d\eta} = \frac{4\pi N_c}{N_c^2 - 1} \int \frac{d^2p}{p^2} \int d^2k \, \alpha_s(Q^2) \, \varphi_A(x_1, k) \, \varphi_B(x_2, |p-k|)$$ with $$x_{1,2} = \frac{p}{\sqrt{s}} e^{\pm\eta}$$

⇒ rapidity ↔ pseudorapidity: average hadron mass

$$y(\eta, p_t, m) = \frac{1}{2} \ln \left[\frac{\sqrt{\frac{m^2+p_t^2}{p_t^2} + \sinh^2\eta} + \sinh\eta}{\sqrt{\frac{m^2+p_t^2}{p_t^2} + \sinh^2\eta} - \sinh\eta} \right]$$

• Running coupling:

$$Q = \max\left\{ \frac{|p_t \pm k_t|}{2} \right\}$$

⇒ $$\varphi(x, k) = \int \frac{d^2r}{2\pi^2 r^2} \exp\left[i \, \underline{k} \cdot \underline{r}\right] \mathcal{N}(Y, r)$$ Solutions of BK equation including all orders in $\alpha_s\beta_2$ corrections $\times (1-x)^4$

with $$Y = \ln\left(\frac{0.05}{x}\right) + \Delta Y_{ev}$$

⇒ Initial condition: MV model $$\mathcal{N}(r, Y_0) = 1 - \exp\left[-r^2 Q_0^2 \ln\frac{1}{r\lambda}\right]$$

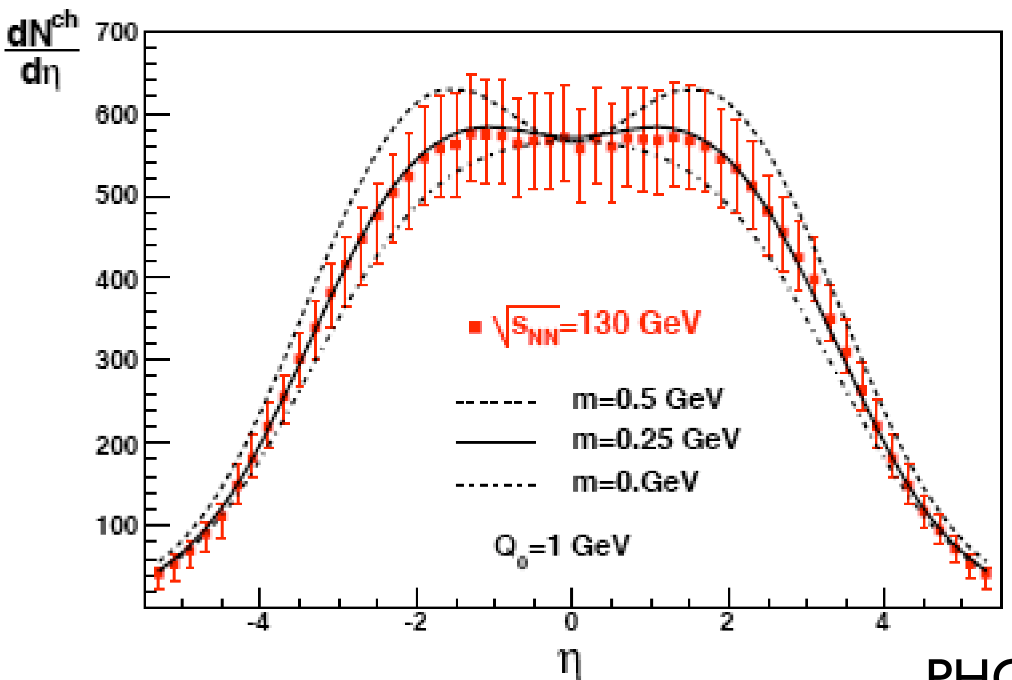
Free Parameters:

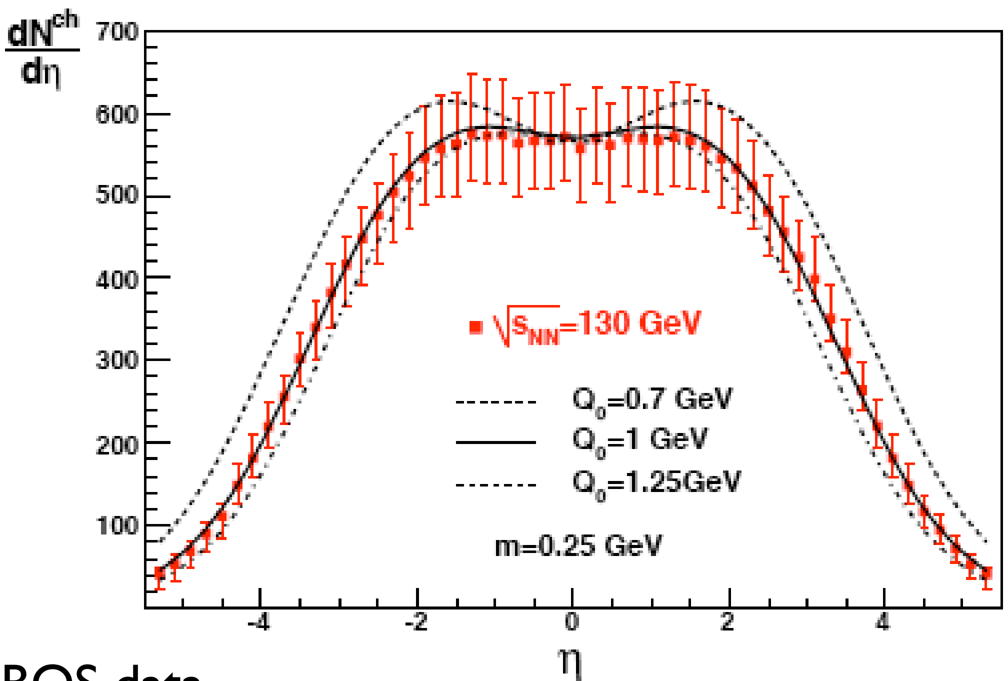
- ⇒ Average hadron mass, m
- ⇒ Initial saturation scale Q_0
- ⇒ Is there significant evolution prior to $\sqrt{s} = 130$?: ΔY_{ev}
- ⇒ Energy independent normalization

Initial conditions for evolution:
Au-Au central collisions at RHIC at $\sqrt{s}$ =130

Average hadron mass ~ 0.25 GeV

Initial saturation scale
Q_0 ~ 1 GeV





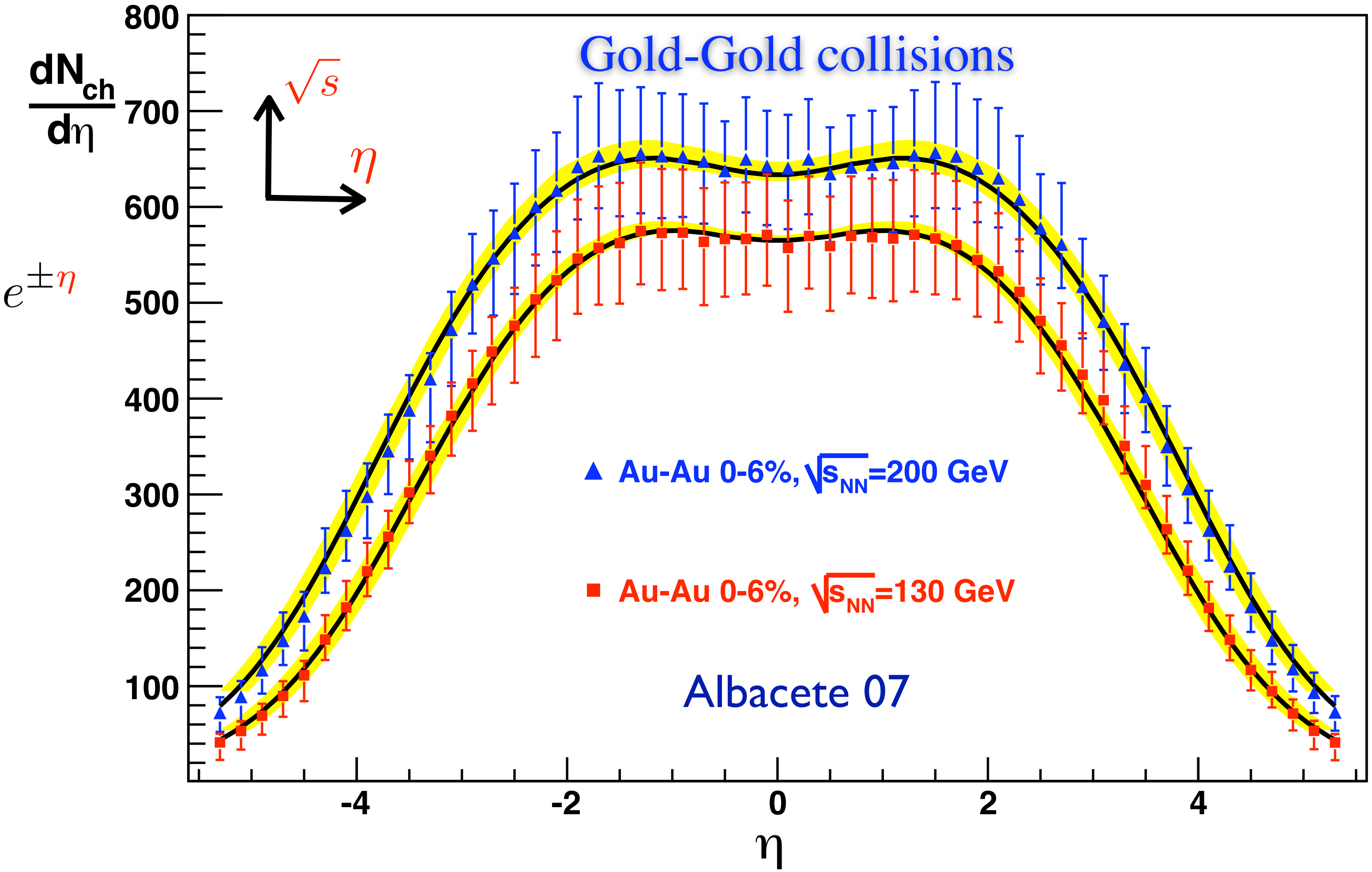
PHOBOS data

Multiparticle production @ RHIC

Excellent description of both energy and rapidity dependence of RHIC Au+Au multiplicity data using solutions of the BK equation with running coupling

$$m \approx 0.25\,\text{GeV}, \quad 0.75 \leq Q_0 \leq 1.25\,\text{GeV}, \quad 1 \leq \Delta Y_{ev} \leq 3$$

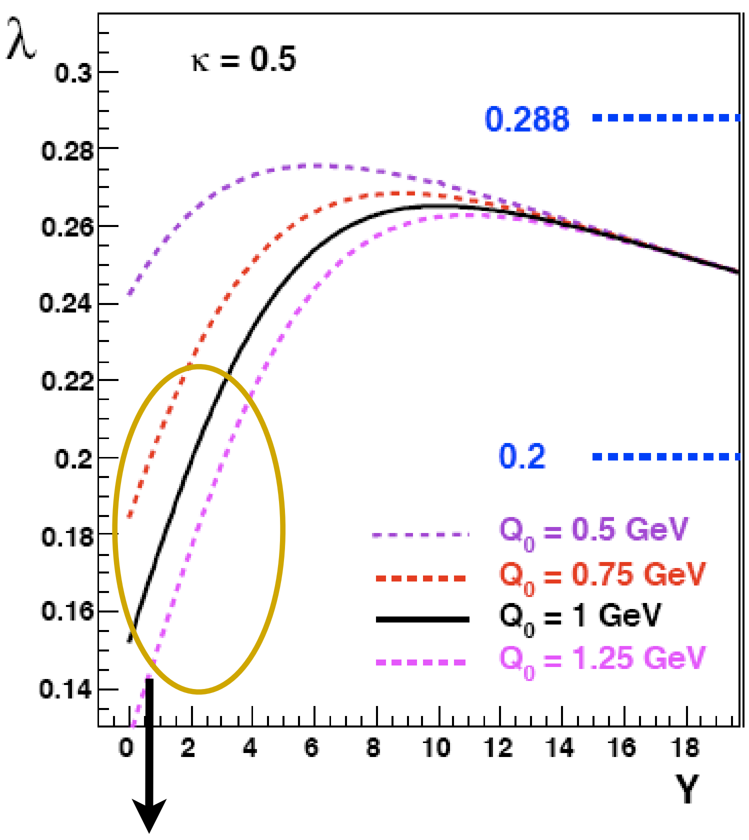
$$x_{1,2} = \frac{p}{\sqrt{s}}\, e^{\pm\eta}$$

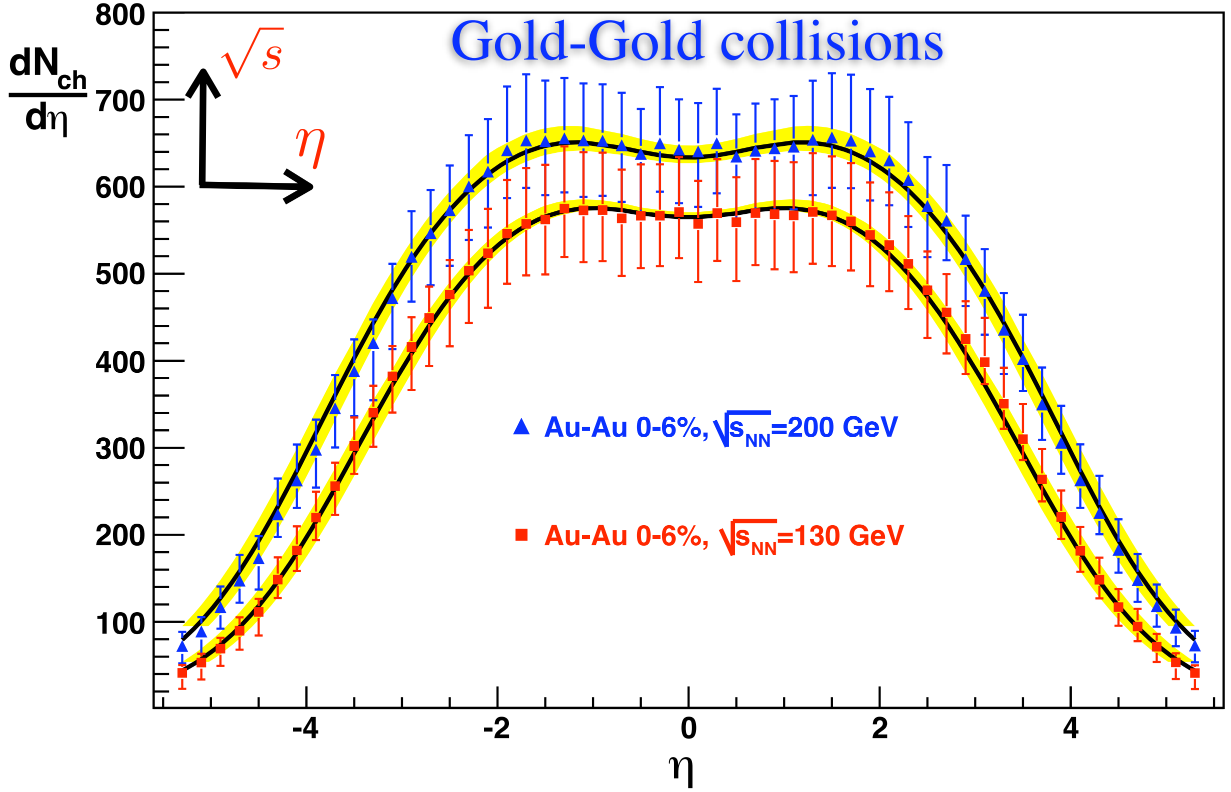


Multiparticle production @ RHIC

RHIC energies are governed by pre-asymptotics effects (MV model: good i.c.)
Solutions close to the scaling region fail to reproduce RHIC data: No universality

$$m \approx 0.25\,\mathrm{GeV}, \quad 0.75 \leq Q_0 \leq 1.25\,\mathrm{GeV}, \quad 1 \leq \Delta Y_{ev} \leq 3$$



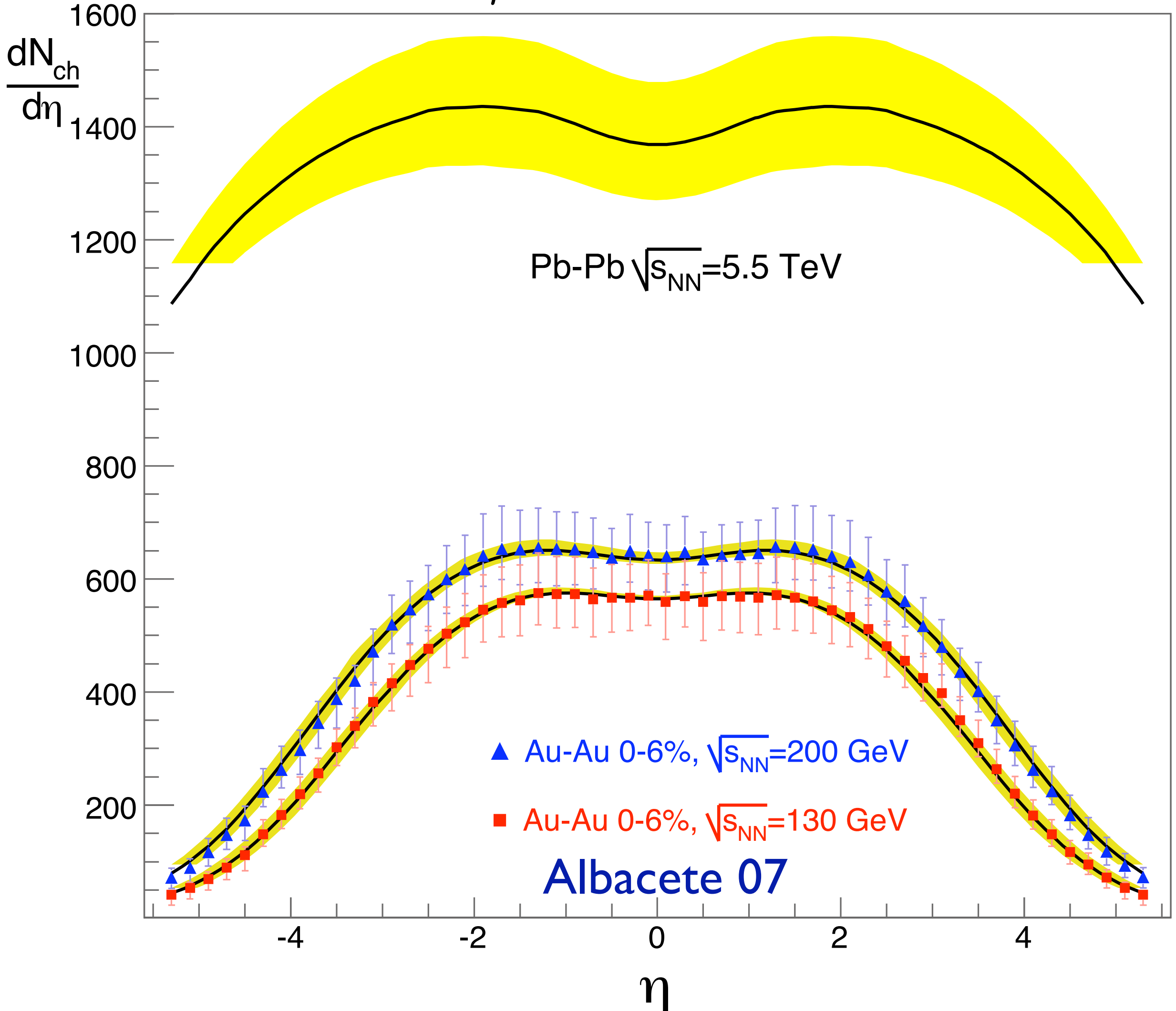


Nuclear gluon distributions probed at RHIC are in the pre-asymptotic regime

Multiparticle production @ LHC

⇒ The extrapolation to Pb-Pb collisions at the LHC is completely driven by the small-x evolution. Compared to previous calculations, it yields a reduced multiplicity:

$$\frac{dN_{ch}^{Pb-Pb}(\sqrt{s}=5.5\,\mathrm{TeV},\eta=0)}{d\eta} \approx 1290 \div 1480$$



- Other saturation-based calculations:

EHNRR arXiv:0705.1770	~ 2500
KLN[1] hep-ph/0408050	$\sim 2100 \div 1800$
ASW hep-ph/0407018	~ 1700
GSV arXiv:0707.1870	$\sim 1000 \div 1400$

- Empiric extrapolation from lower energies data (W Busza nucl-ex/0410035) ~ 1100

⇒ The extrapolation to Pb-Pb collisions at the LHC is completely driven by the small-x evolution

Modifications

a) $\varphi(x,k) \rightarrow h(x,k) = k^2\nabla_k^2\varphi$

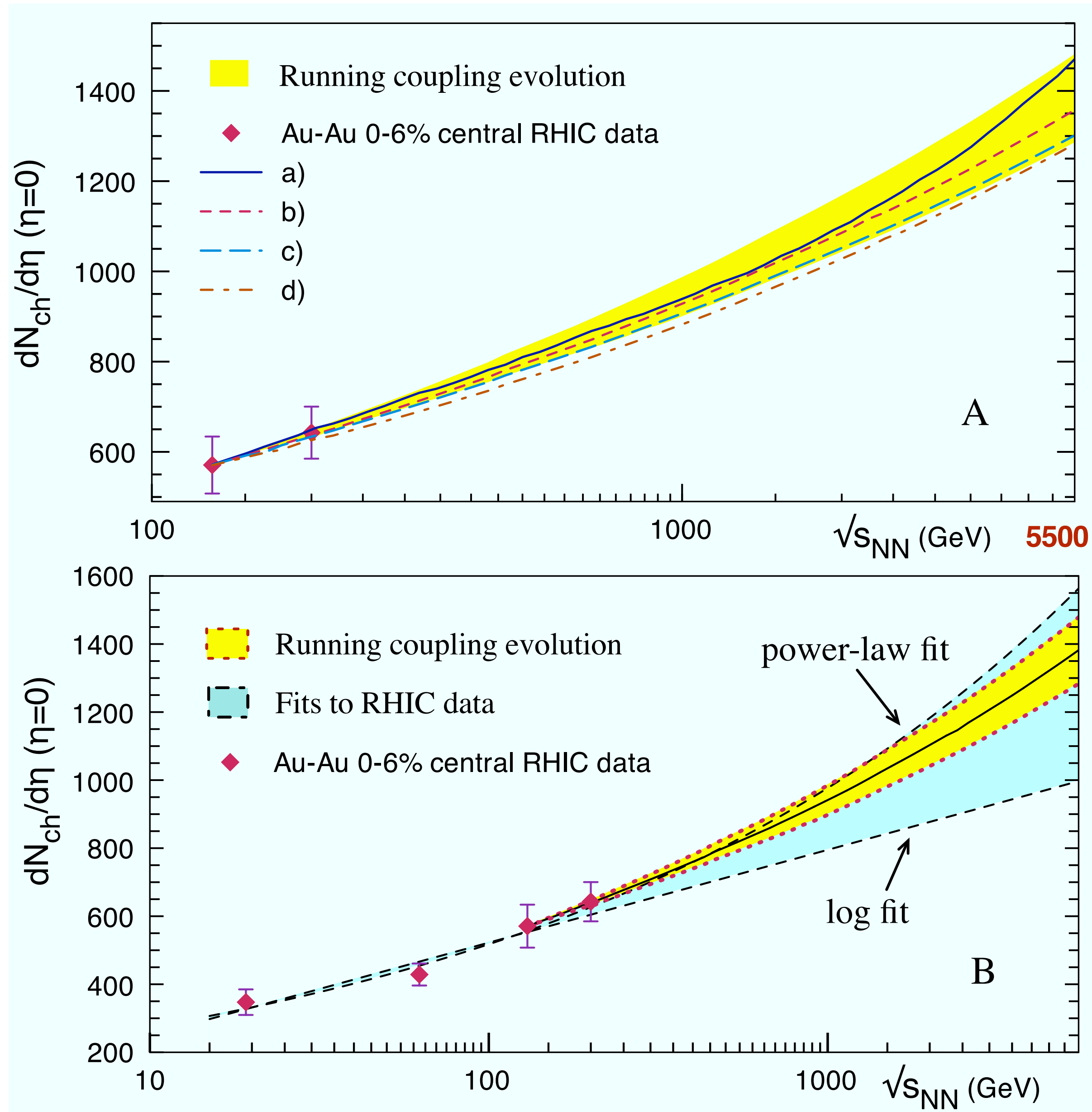
b) $\alpha_{fr} = 0.5$

c) $m = 0$

d) No $(1-x)^4$ corrections

- RHIC data does not discriminate power-law behavior from a logarithmic one

- Logarithmic behaviour seems to be dictated by lower energy data



CONCLUSIONS

⇒ Running coupling corrections to BK-JIMWLK equations considerably reduce the speed of non-linear evolution

⇒ Multiplicity densities at RHIC can be reproduced using kt-factorization + solutions of the evolution

⇒ hadron mass mass ≈ 0.2 ÷ 0.3 GeV

⇒ $Q_s(\sqrt{s}=130\text{ GeV}, \eta=0) \approx 0.9 \div 1.1$ GeV

⇒ Pre-asymptotic regime: strong scaling violations

⇒ Extrapolation to Pb-Pb central collisions at $\sqrt{s}$=5.5 TeV yields a central value:

$$\frac{dN^{\text{evol}}}{d\eta}(\sqrt{s} = 5.5\,\text{TeV})\Big|_{\eta=0} \approx 1400$$

⇒ Smaller than predictions based on HERA information

$$\frac{dN^{\lambda=0.288)}}{d\eta}(\sqrt{s} = 5.5\,\text{TeV})\Big|_{\eta=0} \approx 2100 \div 1700$$

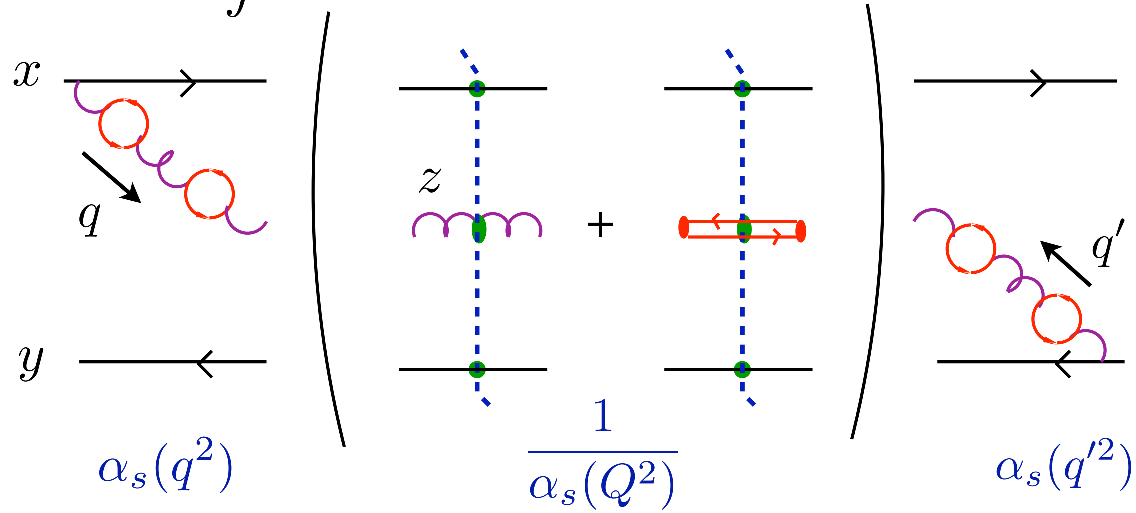
⇒ Larger than empiric extrapolations from lower energies data

$$\frac{dN^{\text{log ext}}}{d\eta}(\sqrt{s} = 5.5\,\text{TeV})\Big|_{\eta=0} \approx 1100$$

Back up slides

- Complete (all orders in $\alpha_s\beta_2$) evolution equation: $\Rightarrow$ $$\frac{\partial S}{\partial Y} = \mathcal{R}[S] - \mathcal{S}[S]$$ JLA and Y. Kovchegov PRD 75 125021 (07):

$\Rightarrow$ ***Running*** **term:** $$\mathcal{R}[S] = \int d^2z\, \tilde{K}(\underline{r}, \underline{r}_1, \underline{r}_2) \left[S(\underline{x}, \underline{z})S(\underline{z}, \underline{y}) - S(\underline{x}, \underline{y})\right]$$



+ virtual terms

$$\tilde{K}_{KW}(\underline{r}, \underline{r}_1, \underline{r}_2) = \frac{N_c}{2\pi^2}\left[\frac{\alpha_s(r_1^2)}{r_1^2} - 2\frac{\alpha_s(r_1^2)\alpha_s(r_2^2)}{\alpha_s(R^2)} + \frac{\alpha_s(r_2^2)}{r_2^2}\right]$$

$$\tilde{K}_{Bal}(\underline{r}, \underline{r}_1, \underline{r}_2) = \frac{N_c\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2\, r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right]$$

- The qq contribution ensures the renormalizability of the all orders in $\alpha_s\beta_2$ corrections and the right physical behavior of the running term:

$$\mathcal{R}[S] \to 0 \quad \text{for} \begin{cases} S \to 0 \\ S \to 1 \end{cases}$$

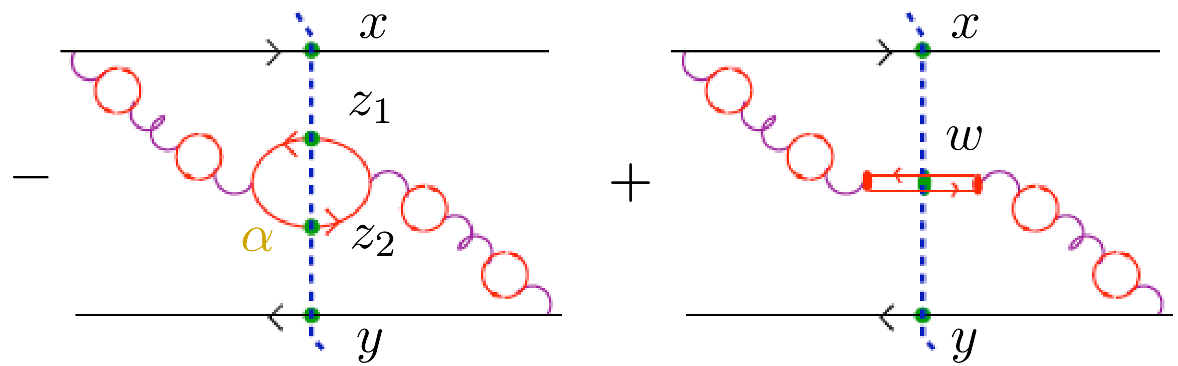
$S \to 0$ $\Rightarrow$ Probability conservation

$S \to 1$ $\Rightarrow$ Unitarity:

$$\frac{\partial S}{\partial Y} = \mathcal{R}[S] - \mathcal{S}[S]$$

⇒ ***Subtraction*** term:

$$\mathcal{S}[S] = \int d^2z_1 d^2z_2\, K_{sub}(\underline{x}, \underline{y}, \underline{z}_1, \underline{z}_2) \left[S(\underline{x}, \underline{w})S(\underline{w}, \underline{y}) - S(\underline{x}, \underline{z}_1)S(\underline{z}_2, \underline{y})\right]$$



$N_f \longrightarrow -6\pi\beta_2$

$$K_{sub}(\underline{x}, \underline{y}, \underline{z}_1, \underline{z}_2) = -\frac{3\beta_2}{2\pi^3}\int_0^1 d\alpha \frac{1}{[\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2]\,[\alpha(\underline{z}_1 - \underline{y})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2]\, z_{12}^4}$$

$$\{[-4\alpha\bar{\alpha}\, \underline{z}_{12} \cdot (\underline{z} - \underline{x})\, \underline{z}_{12} \cdot (\underline{z} - \underline{y}) + z_{12}^2(\underline{z} - \underline{x}) \cdot (\underline{z} - \underline{y})]\ \alpha_s(R_T(\underline{x}))\, \alpha_s(R_T(\underline{y}))$$

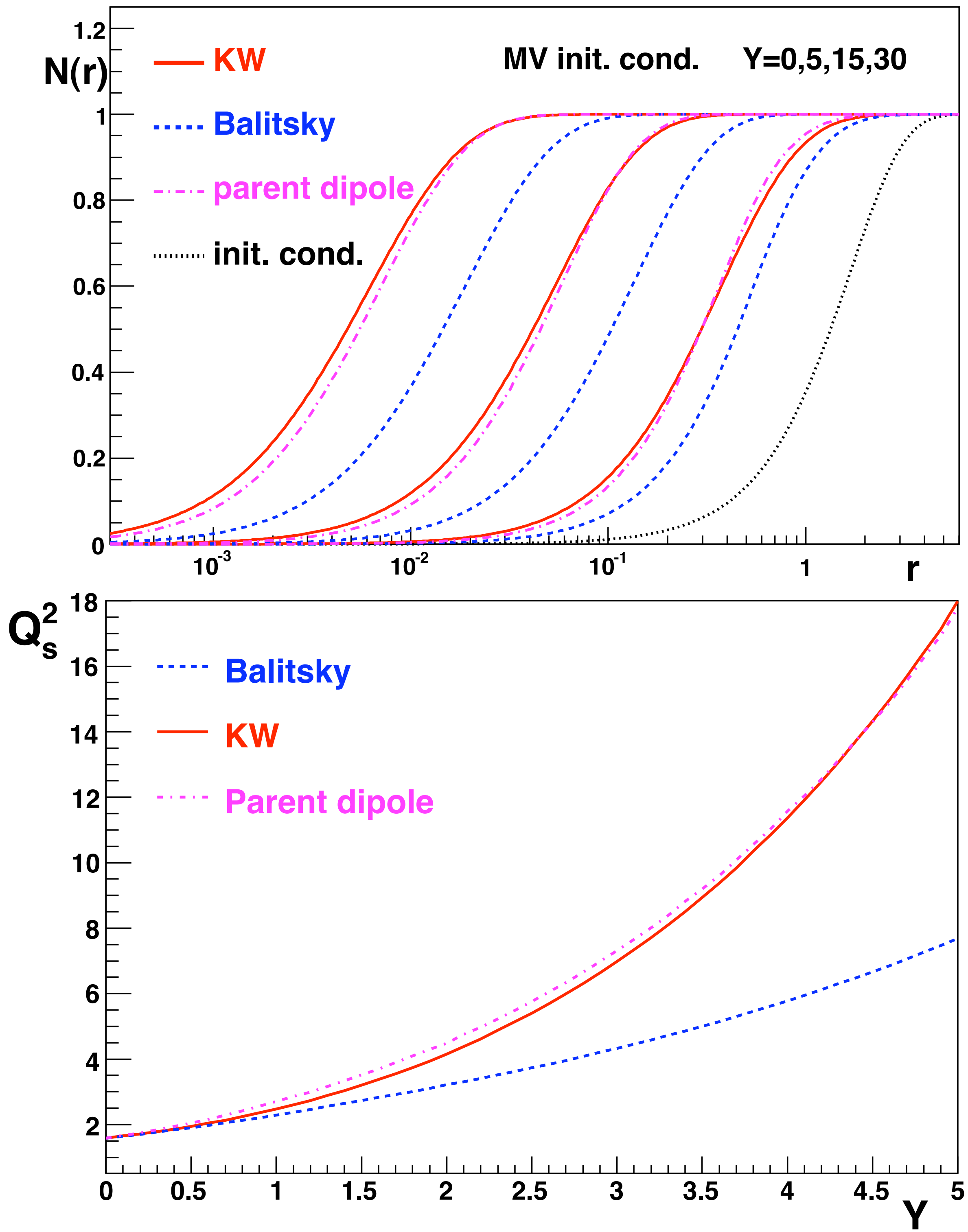
$$2\alpha\bar{\alpha}(\alpha - \bar{\alpha})z_{12}^2 \left[\underline{z}_{12} \cdot (\underline{z} - \underline{x})\, \alpha_s(R_T(\underline{x}))\, \alpha_s(R_L(\underline{y})) + \underline{z}_{12} \cdot (\underline{z} - \underline{y})\, \alpha_s(R_L(\underline{x}))\, \alpha_s(R_T(\underline{y}))\right]$$

$$4\alpha^2\bar{\alpha}^2 z_{12}^4\, \alpha_s(R_L(\underline{x}))\, \alpha_s(R_L(\underline{y}))\}$$

- It receives contributions from transverse (T) and longitudinal (L) gluon's polarization:

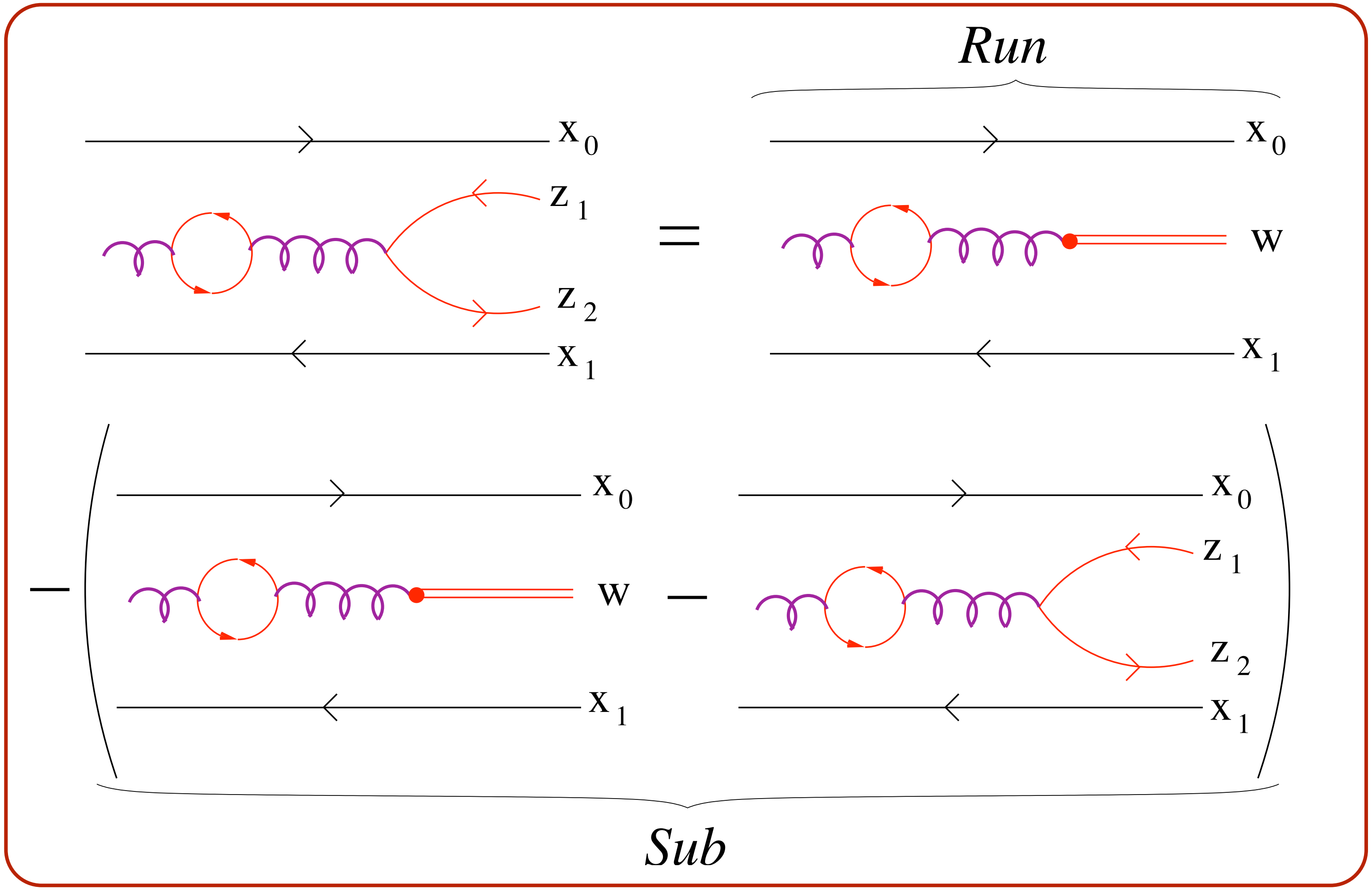
$$\ln\left(\frac{1}{R_T^2(\underline{x})\mu^2}\right) = \ln\left(\frac{4e^{-2\gamma-5/3}}{[\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2]\mu^2}\right) + \frac{\alpha\bar{\alpha}\, z_{12}^2}{(\underline{z} - \underline{x})^2} \ln\left(\frac{\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2}{\alpha\bar{\alpha} z_{12}^2}\right)$$

$$\ln\left(\frac{1}{R_L^2(\underline{x})\mu^2}\right) = \ln\left(\frac{4e^{-2\gamma-5/3}\, \alpha\bar{\alpha}\, z_{12}^2}{[\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2]^2\mu^2}\right)$$



N(r)
KW
Balitsky
parent dipole
init. cond.
MV init. cond.
Y=0,5,15,30
1.2
1
0.8
0.6
0.4
0.2
0
10-3
10-2
10-1
1
r
Q_s^2
Balitsky
KW
Parent dipole
18
16
14
12
10
8
6
4
2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Y

- The separation procedure is similar in both calculations:



$$\mathcal{S}[S] = \int d^2z_1\, d^2z_2\, K^{sub}\,[S(\underline{x}_0, \underline{\mathbf{w}}, Y)\, S(\underline{\mathbf{w}}, \underline{x}_1, Y) - S(\underline{x}_0, \underline{z}_1, Y)\, S(\underline{z}_2, \underline{x}_1, Y)]$$

- The differences between the two approaches stem from the choice of the subtraction point, w

- In Balitsky's scheme: w = z_1 (or z_2), the quark's (anti-q) transverse position :

$$\mathcal{S}^{Bal}[S] = \int d^2z_1\, d^2z_2\, K^{sub}\, [S(\underline{x}_0, \underline{\mathbf{z}_1}, Y)\, S(\underline{\mathbf{z}_1}, \underline{x}_1, Y) - S(\underline{x}_0, \underline{z}_1, Y)\, S(\underline{z}_2, \underline{x}_1, Y)]$$

An expansion in term of N's result in just non-linear terms (N^2<<N at small-r)

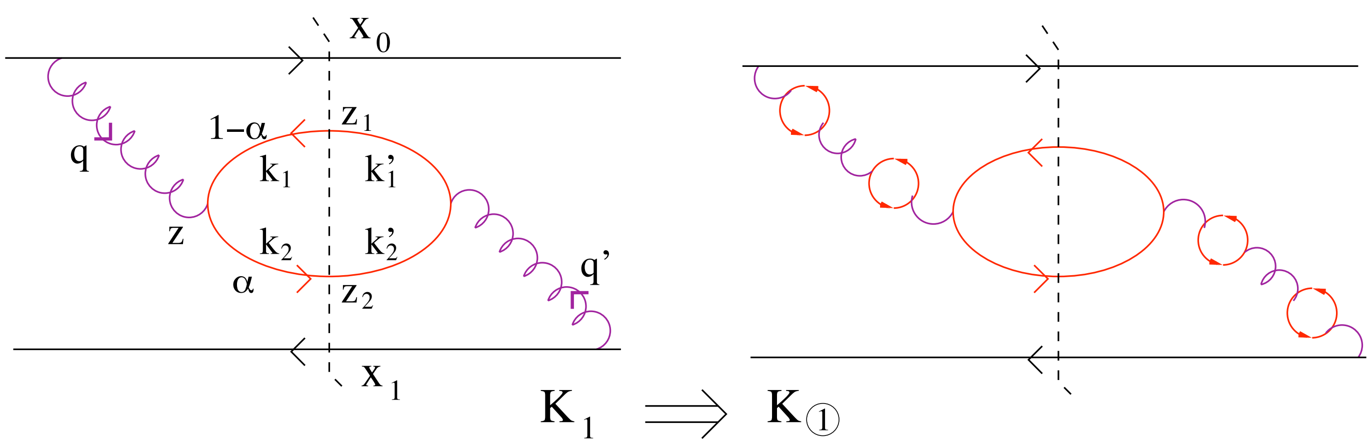
- In KW scheme: w = z =, the gluon's transverse position:

$$\mathcal{S}^{KW}[S] = \int d^2z_1\, d^2z_2\, K^{sub}\, [S(\underline{x}_0, \underline{\mathbf{z}}, Y)\, S(\underline{\mathbf{z}}, \underline{x}_1, Y) - S(\underline{x}_0, \underline{z}_1, Y)\, S(\underline{z}_2, \underline{x}_1, Y)]$$

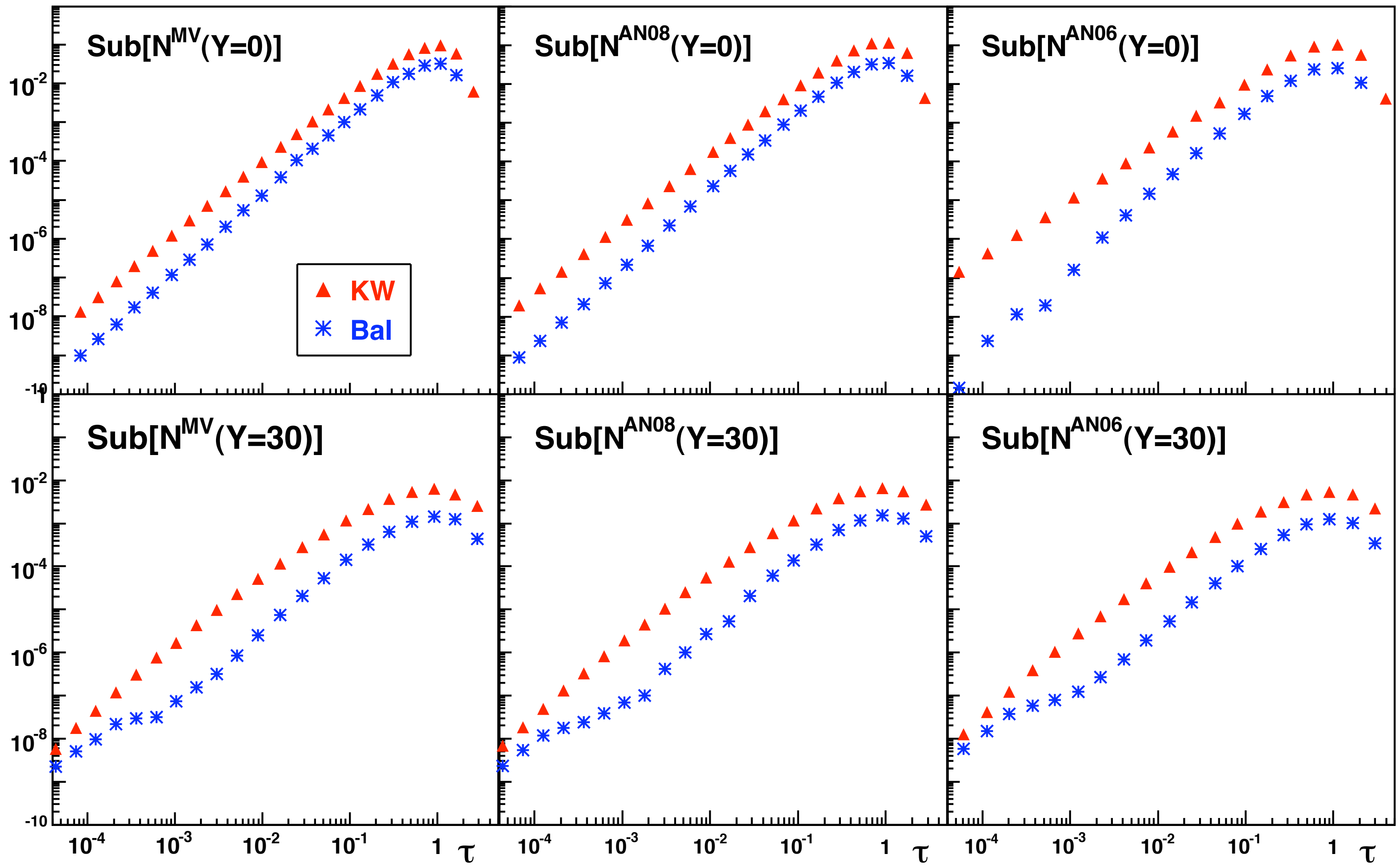
An expansion in term of N's also includes linear terms.

- The kernel of the subtraction contribution is the same in both cases:

Leading-Nf ⇒ All orders Nf (LCPT) ⇒ Fourier transform to coordinate space



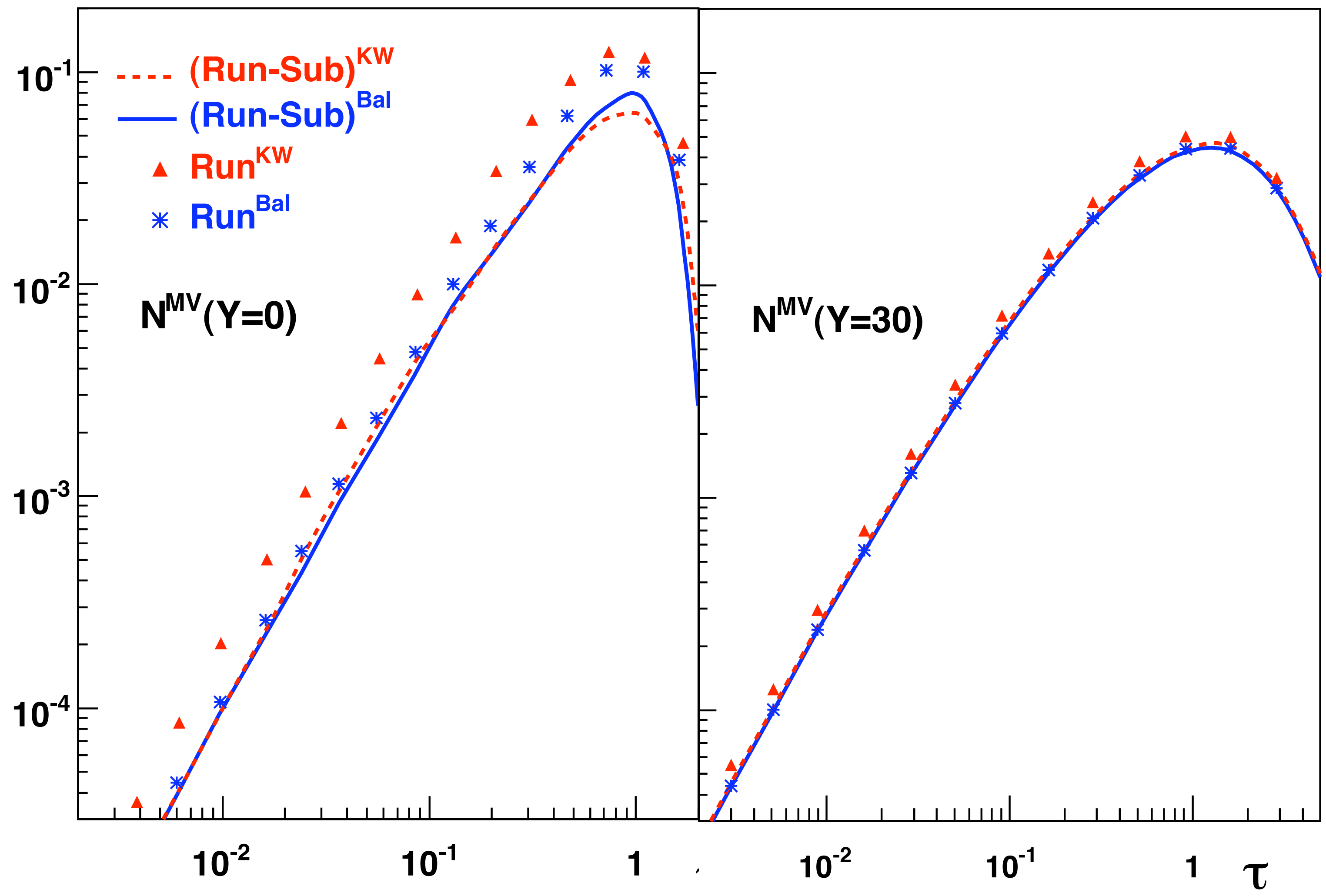
- The subtraction term is larger in KW's scheme than in Balitsky's:



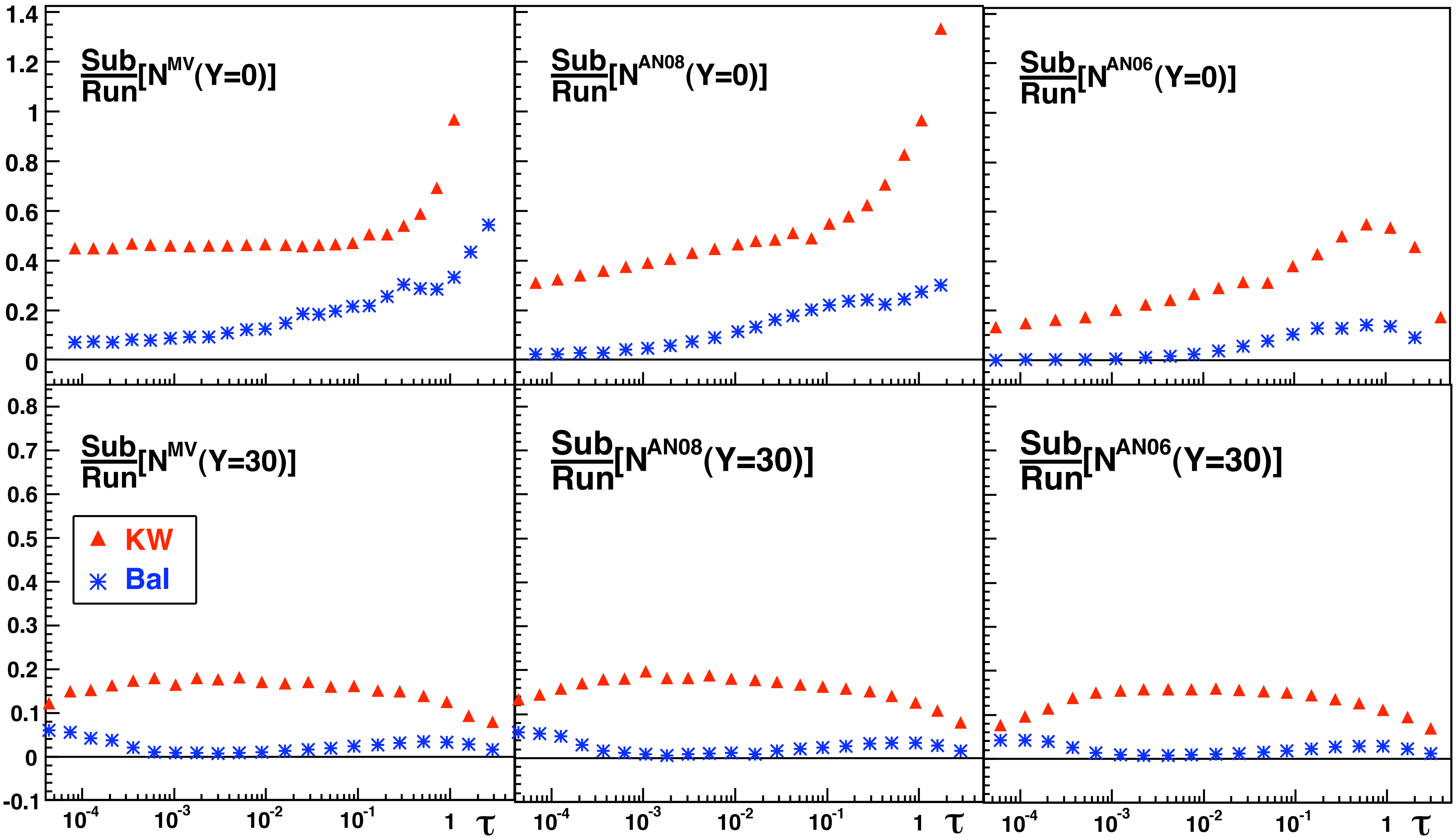
- It has the same sign as the running term: It slows down the evolution

$$\mathcal{F} = \mathcal{R} - \mathcal{S}$$

Once the subtraction term is added back, the two approaches agree:



- The subtraction term is larger in KW's scheme than in Balitsky's:



- The relative contribution of the subtraction term to the evolution fades away at large rapidity