Towards a controlled study of the QCD critical point

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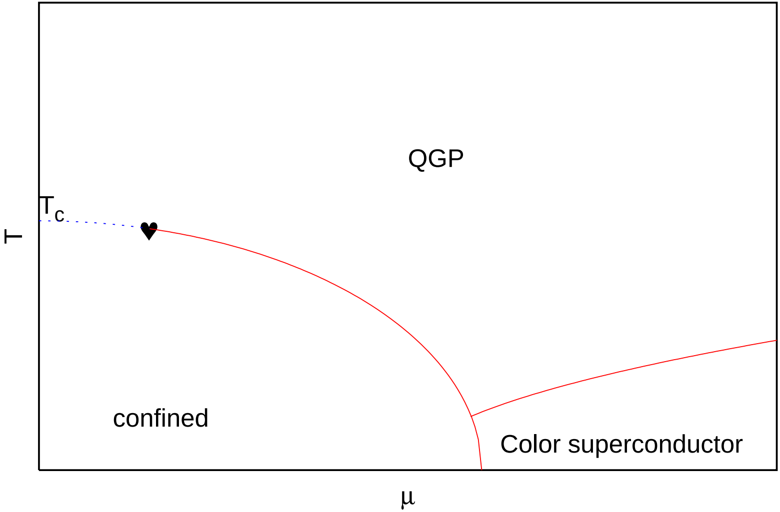
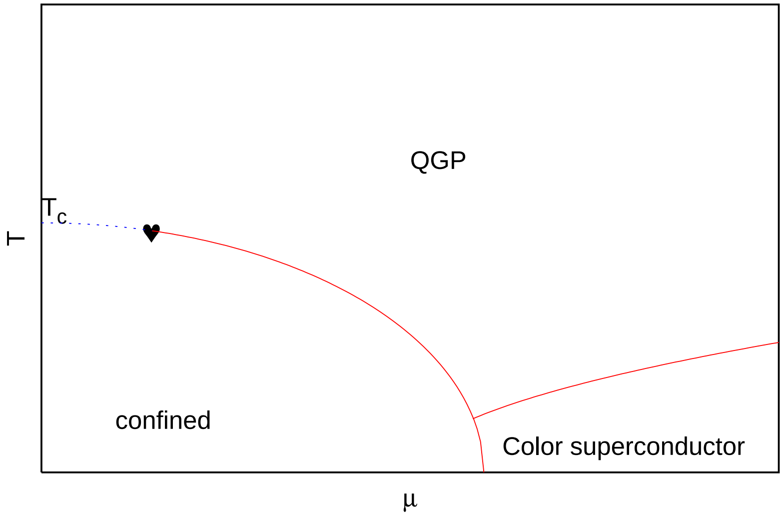
Conjectured QCD Phase diagram



Non-perturbative $\longrightarrow$ lattice simulations

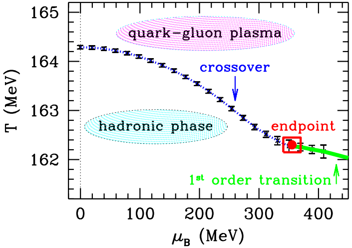
Conjectured QCD Phase diagram

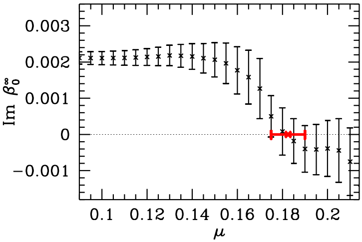


- Coordinates (μ_E, T_E) of critical point?
- Effect of changing the quark masses?

Critical point: already determined?

Fodor & Katz: hep-lat/0402006 ($\sim$ physical quark masses)





$$(\mu_E^q, T_E) = (120(13), 162(2)) \text{ MeV}$$

Legitimate **concerns**:

- Discretization error? $N_t = 4 \implies a \sim 0.3$ fm
- Abrupt qualitative change near μ_E:

  abrupt change of physics or breakdown of algorithm (Splittorff)?

Simulations at $\mu \neq 0$: "sign" problem

- $\det \not{D}(\mu \neq 0)$ is complex $\rightarrow$ not a probability distribution

- Fodor & Katz: rewrite as

$$\det \not{D}(\mu = 0) \quad \times \quad \frac{\det \not{D}(\mu)}{\det \not{D}(\mu{=}0)}$$

Sample at $(\mu = 0, \beta = \beta_c)$, reweight to $(\mu \neq 0, \beta \neq \beta_c)$

$\longrightarrow$ complex reweighting factor ρ

- $\langle \rho \rangle \sim \exp(-\mathrm{volume})$, each measurement $\mathcal{O}(\pm 1)$
$\Rightarrow$ need statistics $\sim \exp(+\mathrm{volume})$: sign problem

- Finite statistics:
breakdown without warning $\rightarrow$ wrong results ("overlap problem")

Controlled approach to $\mu \neq 0$

Taylor expand in $\frac{\mu}{T}$ about $\mu = 0$, and determine Taylor coefficients

1. By direct measurement at $\mu = 0$ of non-local operators

   Bielefeld-Swansea, Gavai & Gupta

2. **Better**: by fitting imaginary-μ (no sign pb.) simulation data

   de Forcrand & Philipsen, D'Elia & Lombardo, Azcoiti et al., etc...

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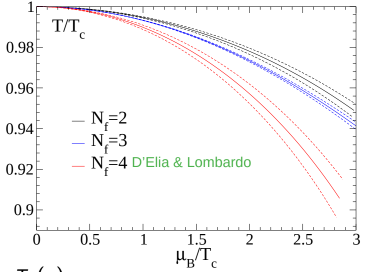


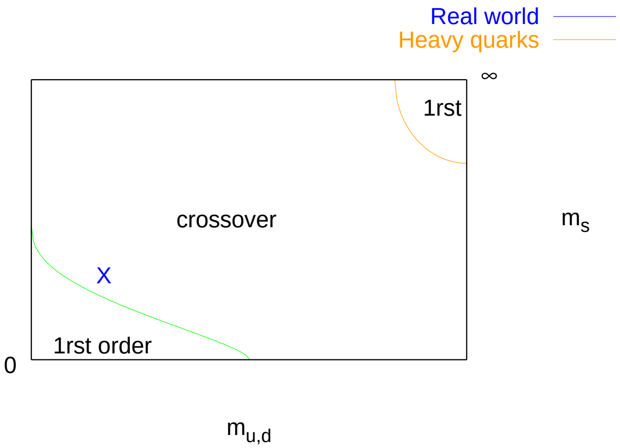
Pseudo-critical line $\frac{T_c(\mu)}{T_c(\mu=0)} = 1 - c(N_f, m_q)\left(\frac{\mu}{\pi T}\right)^2 + ..$

$c \approx 0.500(34), 0.602(9), 0.93(10)$ for $N_f = 2, 3, 4,\ m_q/T \ll 1 \quad (c \propto N_f/N_c)$

Extend to study critical point?

Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

$\mu = 0$



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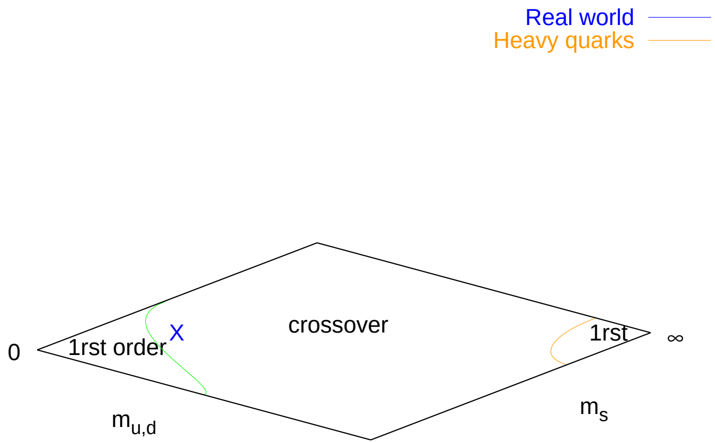
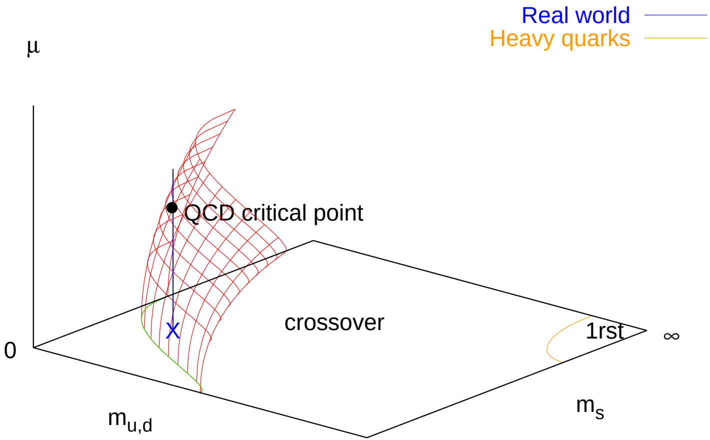


Now turn on μ

Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

$\mu \neq 0$



Conventional wisdom: first-order region expands with real $|\mu|$

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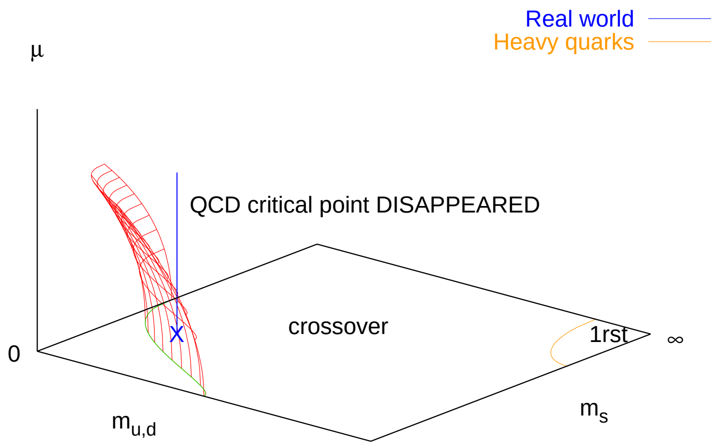
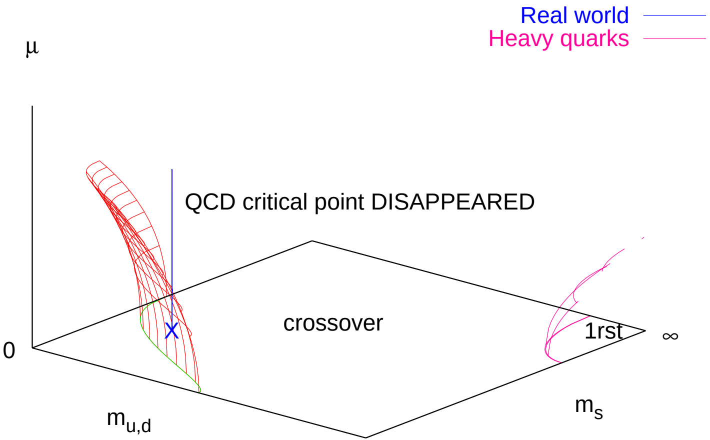


Exotic scenario: first-order region shrinks with real $|\mu|$ $\frac{d\ m_c}{d\mu^2}|_{\mu=0} < 0$

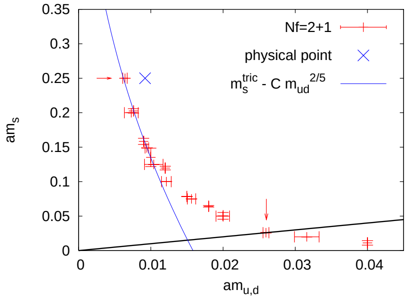
Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

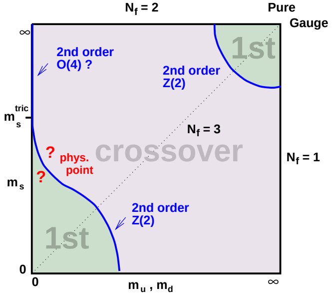


For heavy quarks, first-order region shrinks (PdF, Kim, Takaishi, hep-lat/0510069)

Lattice study with Owe Philipsen (hep-lat/0607017)

1. Line of second-order phase transitions in the quark mass plane $(m_{u,d}, m_s)$ via Binder cumulant $B_4 = \langle(\delta\bar{\psi}\psi)^4\rangle / \langle(\delta\bar{\psi}\psi)^2\rangle^2$



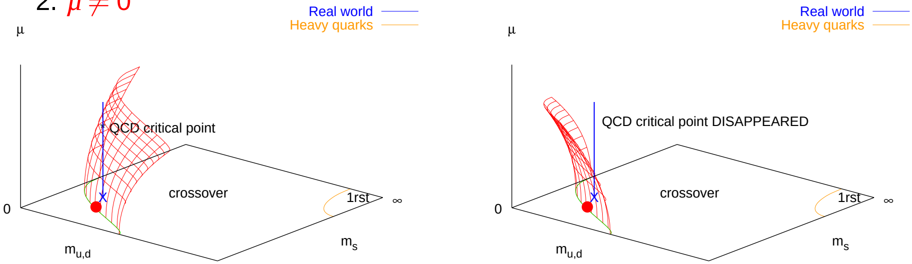


$\mu = 0$:

- data consistent with tricritical point at $m_{u,d} = 0,\ m_s \sim 2.8T_c$
- physical point in crossover region cf. Fodor & Katz

Lattice study with Owe Philipsen (hep-lat/0607017)

2. $\mu \neq 0$



Strategy: tune m_q for 2nd-order P.T. at $\mu = 0$, then turn on [imaginary] μ
Does the transition become 1rst-order (left) or crossover (right)?

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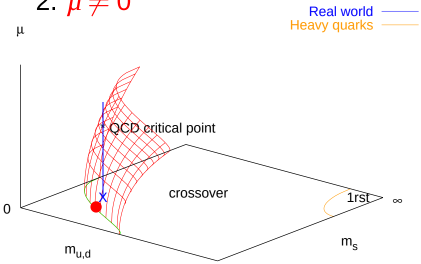

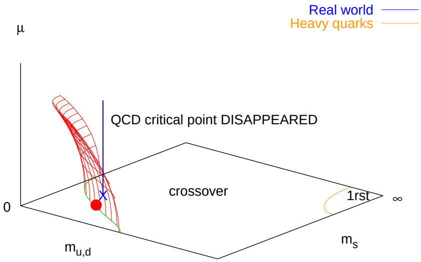

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Answer: very little change ($\longrightarrow$ surface almost vertical)

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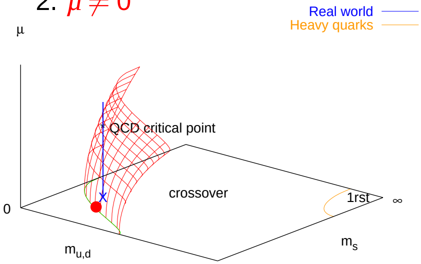

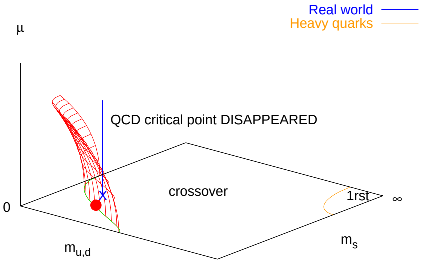

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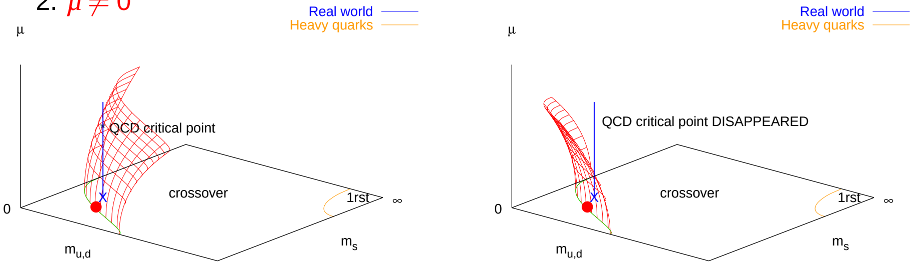

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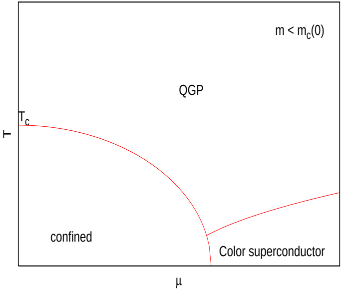
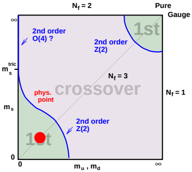
(preliminary) $\quad -12(6)\left(\frac{\mu}{\pi T}\right)^4$

Resulting phase diagram (simplest possibility)

Standard scenario





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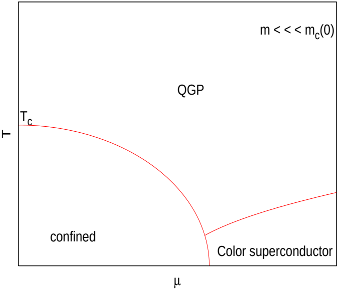

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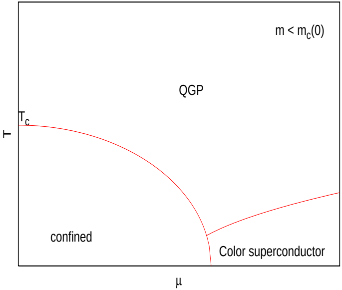

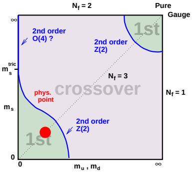

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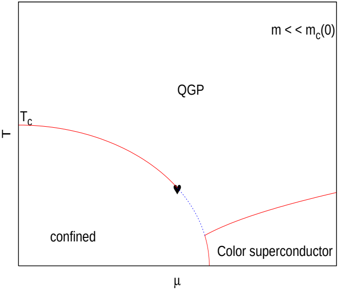

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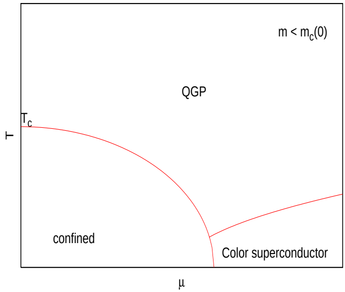

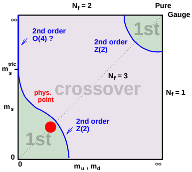

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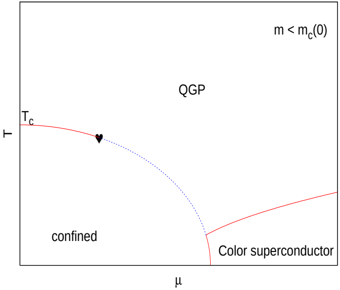

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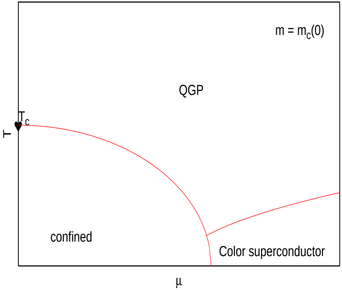

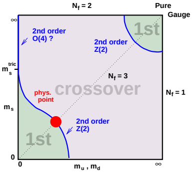

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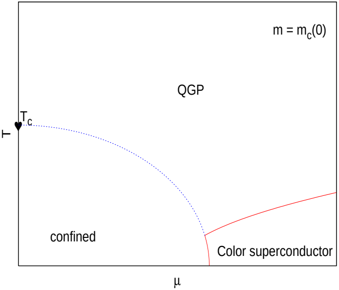

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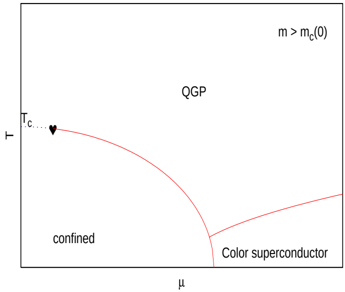

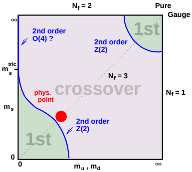

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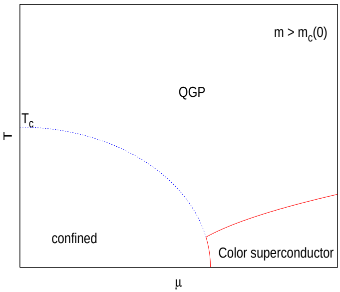

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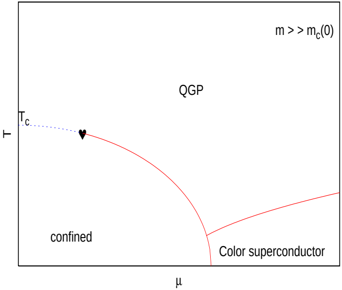

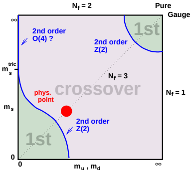

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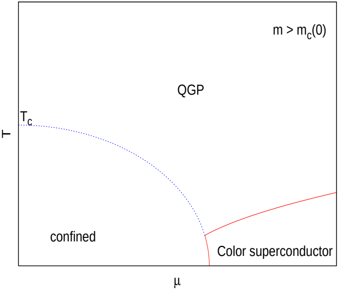

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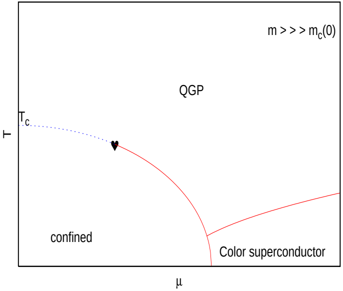

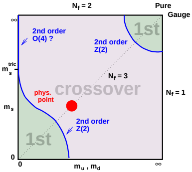

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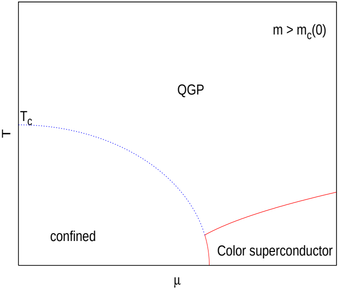
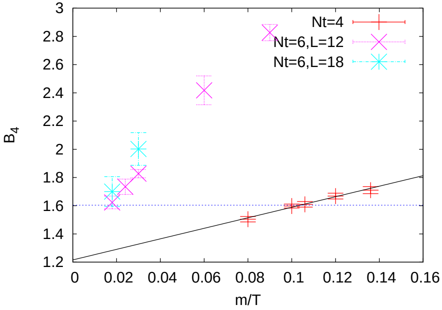


Discretization errors? Recall that $N_t = 4 \Rightarrow a \sim 0.3$ fm

Unknown, possibly large $\mathcal{O}(a^2)$ error on curvature $\rightarrow$ change sign?

- $N_t = 4$: curvature has same sign for $N_f = 3$ and $N_f = 2+1$
- $N_t > 4$, $\mu = 0$: it takes lighter quarks to have first-order transition

0711.0262, PdF & Philipsen; also 0710.0998, Fodor & Katz; Bielefeld, MILC



Critical quark mass m_c/T_c decreases by factor 5

Pion mass (measured at $T = 0$) decreases: $\frac{m_\pi}{T_c} \approx 1.6\ (N_t = 4) \rightarrow 0.95\ (N_t = 6)$

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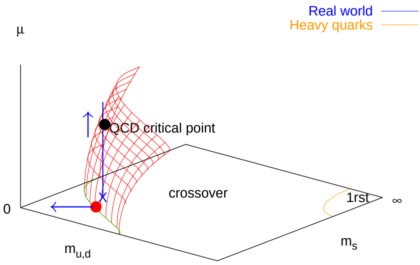
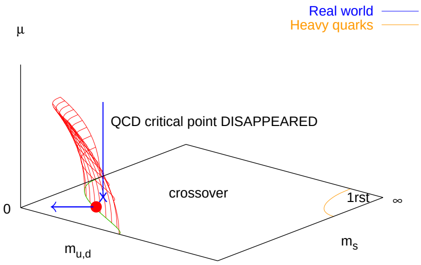




A critical point at "small" μ (ie. $\mu/T \lesssim 1$) would require curvature to

(i) change sign and (ii) become large

as $a \rightarrow 0$

Conclusions

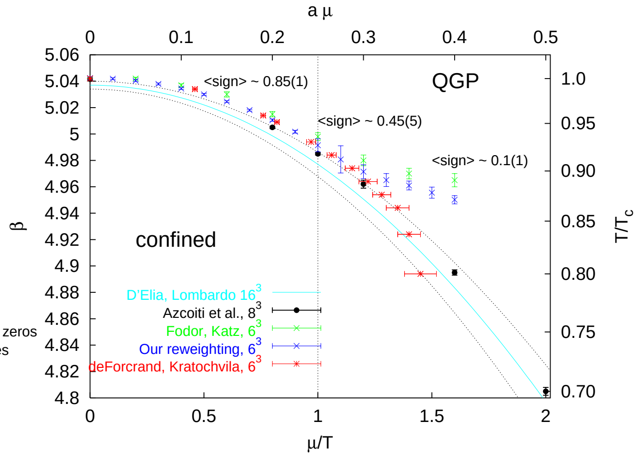
- Controlled approach $\Longrightarrow \frac{\mu}{T} \lesssim 1$
  OK for RHIC, LHC
- Critical point not settled: keep open mind
- $\mu = 0$, $a \to 0$: QCD with physical masses “deep” in crossover region, ie. far from critical surface
- $N_t = 4$: no critical point at small μ
- Large discretization errors $\to$ repeat study for $N_t = 6$: in progress

Phase Diagram $T - \mu$: comparing apples with apples

All with $N_f = 4$ staggered fermions, $am_q = 0.05, N_t = 4$ $(a \sim 0.3$ fm)

PdF & Kratochvila

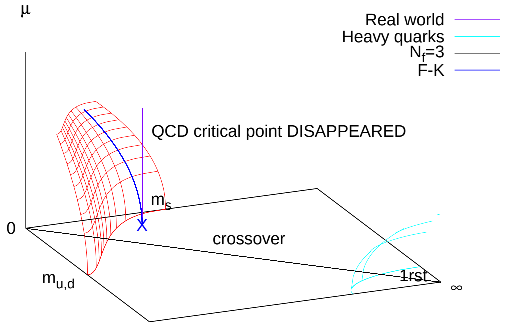
imaginary μ
2 param. imag. μ
dble reweighting, LY zeros
Same, susceptibilities
canonical



Agreement for $\mu/T \lesssim 1$

Contradiction with other lattice studies?

- Fodor & Katz: $(T_E, \mu_E) = (162(2), 120(13))$ MeV ?



F&K keep $m_q a$, ie. $\frac{m_q}{T}$ fixed, while $T_c(\mu)$ decreases with $\mu \Rightarrow$ non-const. physics
Lighter quarks at larger μ may cause the phase transition
(dominant effect in our study)

Fake critical point? In any case, strong underestimate of μ_E

Contradiction with other lattice studies?

- Gavai & Gupta: $\mu_E / T_E \lesssim 1$?
  different theory $N_f = 2$