

EARLY EVOLUTION OF TRANSVERSALLY THERMALIZED PARTONS

based on: [A. Bialas, M.Ch., and W. Florkowski](#), arXiv:0708.1076 (nucl-th)
and [M.Ch., and W. Florkowski](#), arXiv:0710.5871 (nucl-th)

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Quark Matter 2008

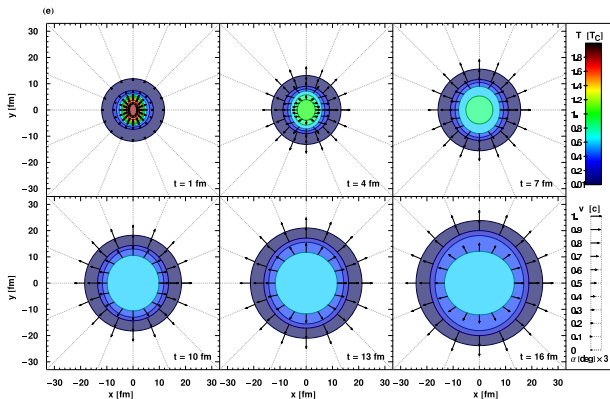
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Parallel Session VI



Motivation

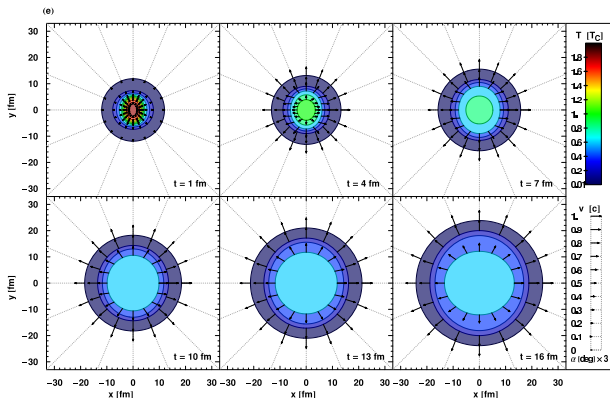
contesting the hydrodynamical picture of heavy-ion collisions at RHIC



Motivation

contesting the hydrodynamical picture of heavy-ion collisions at RHIC

- system created in heavy-ion collisions at RHIC energies is best described by hydrodynamics of an almost ideal fluid
- in particular, the particle transverse-momentum spectra and v_2 are very well reproduced by the hydrodynamic approach



Heinz, Huovinen, Kolb, Hirano, Shuryak, Teaney, Bass, Nonaka, Hama, Kodama, Ruuskanen,



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- hydrodynamic evolution requires very **early thermalization** of the system, since the asymmetry of the transverse flow is produced most effectively at the very early stage of the evolution, to obtain v_2 consistent with data it is necessary to start the hydrodynamic evolution at the time below 1 fm after the collision takes place



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- **in this talk the possibility is explored that, at its early stages, the hydrodynamic evolution applies only to transverse degrees of freedom of the partonic system created in high-energy collisions,**

the idea pioneered by Heinz and Wong, PRC 66 (2002) 014907



Outline

1. Ansatz for the phase-space distribution function
2. Moments of the phase-space distribution function
3. Transverse hydrodynamics
4. Initial conditions and Cooper-Frye prescription
5. Results
6. Conclusions



1.1. Longitudinal vs. transverse dynamics

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the early equilibration, if at all possible, is particularly difficult to achieve in longitudinal direction elastic collisions do not change significantly the direction of the colliding partons and thus it requires very many interactions to produce a locally isotropic distribution from the initially strongly anisotropic one

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transverse momentum spectra observed in nucleon-nucleon collisions are well described by the Boltzmann distribution the partonic system produced in hadronic collisions emerges already in a state close enough to equilibrium in the transverse direction

Hagedorn, ... , Heinz+Becattini, Bialas, Kharzeev, Florkowski, ...



1.2. Factorization of the distribution function

- our main assumption: the 3D phase-space distribution function $f(x, p)$ is factorized into the longitudinal and transverse part

$$f(x, p) = f_{\parallel} g_{\text{eq}}$$

- f_{\parallel} - non-equilibrium longitudinal part, describes essentially **free-streaming**
- g_{eq} - equilibrium transverse part, describes **2D hydrodynamic expansion**



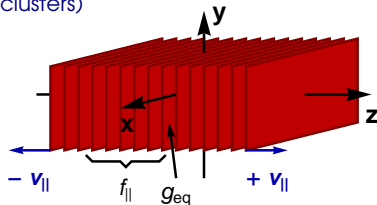
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- useful visualization in terms of discrete independent transverse layers (clusters)



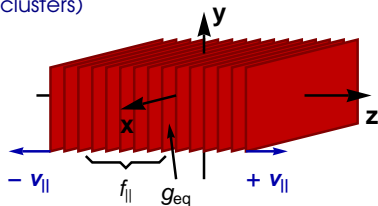
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- with standard definitions of rapidity y and spacetime rapidity η

$$E = m_{\perp} \cosh y, \quad p_{\parallel} = m_{\perp} \sinh y$$

$$t = \tau \cosh \eta, \quad z = \tau \sinh \eta$$

$$\tau = \sqrt{t^2 - z^2}, \quad m_{\perp} = \sqrt{m^2 + p_{\perp}^2}$$



1.3. Longitudinal part

partons originate at the spacetime point $t, z = 0$ and reach the point z after time t , $v_{\parallel} = \frac{z}{t} = \frac{p_{\parallel}}{E}$, to satisfy this condition and have the **correct dimension** we define the longitudinal part as

$$f_{\parallel} = n_0 \delta(p_{\parallel} t - E z) = n_0 \frac{\delta(y - \eta)}{\tau m_{\perp}}$$

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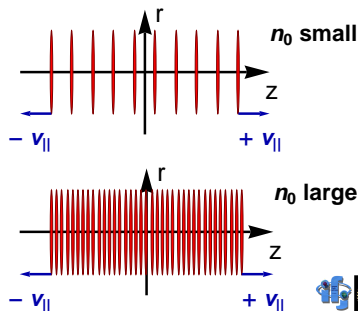
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1.4. Transverse part

g_{eq} has the form of the **two-dimensional equilibrium distribution function convoluted with the transverse flow**, for simplicity we use the Boltzmann statistics and neglect the chemical potential

$$g_{\text{eq}} = \exp \left(- \frac{m_{\perp} u_0 - \vec{p}_{\perp} \cdot \vec{u}_{\perp}}{T} \right)$$

the transverse flow u^{μ} has the structure

$$u^{\mu} = (u_0, u_x, u_y, 0) = \left(u_0, \vec{u}_{\perp}, 0 \right)$$

$$u_0^2 - \vec{u}_{\perp}^2 = 1$$

g_{eq} depends on the spacetime coordinates $\tau, \eta, \vec{x}_{\perp}$ via temperature T and transverse flow u^{μ}



2.1. Particle current and energy-momentum tensor

we define the particle current and energy-momentum tensor in the standard way, as the first and second moment of the distribution function

$$N^\mu = n_0 \nu_g \int \frac{dy d^2 p_\perp}{(2\pi)^2} p^\mu \frac{\delta(y - \eta)}{\tau m_\perp} g_{\text{eq}} = \frac{n_0 \nu_g T^2}{2\pi\tau} U^\mu$$

$$T^{\mu\nu} = n_0 \nu_g \int \frac{dy d^2 p_\perp}{(2\pi)^2} p^\mu p^\nu \frac{\delta(y - \eta)}{\tau m_\perp} g_{\text{eq}} = \frac{n_0 \nu_g T^3}{2\pi\tau} (3U^\mu U^\nu - g^{\mu\nu} - V^\mu V^\nu)$$

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$$U^\mu = (\cosh \eta u_0, u_x, u_y, \sinh \eta u_0)$$

$$V^\mu = (\sinh \eta, 0, 0, \cosh \eta)$$

the presence of the four-vector V^μ in the energy-momentum tensor is related with special role of the longitudinal direction



2.2. Entropy current

entropy current (based on the Boltzmann nonequilibrium definition)

$$S^\mu = -n_0 \nu_g \int \frac{dy d^2 p_\perp}{(2\pi)^2} p^\mu \frac{\delta(y - \eta)}{\tau m_\perp} g_{\text{eq}} (\ln g_{\text{eq}} - 1) = \frac{3n_0 \nu_g T^2}{2\pi\tau} U^\mu$$

explicit calculation shows $U_\nu \partial_\mu T^{\mu\nu} = T \partial_\mu S^\mu$ – the energy-momentum conservation implies the entropy conservation, as in the standard 3D hydrodynamics



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$$\int_0^\infty dr r \int_0^{2\pi} d\phi T^2(\tau, r, \phi) u_0(\tau, r, \phi) = \text{const}$$



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for 2D hydro the initial thermal energy may decrease only at the expense of increasing transverse flow, for 3D boost-invariant hydro, even without the transverse expansion temperature drops, as is well known from the famous Bjorken model



2.3. Relation to 2D thermodynamic quantities

in the local rest frame, the densities of our 3D system are simply related to 2D thermodynamic variables:

$$N^0 = \frac{n_0}{\tau} n_2(T), \quad T^{00} = \frac{n_0}{\tau} \varepsilon_2(T), \quad S^0 = \frac{n_0}{\tau} s_2(T)$$

the appropriate two-dimensional densities are defined by the equations following from the two-dimensional potential Ω_2 , they satisfy all required thermodynamic identities

$$n_2 = \nu_g \int \frac{d^2 p}{(2\pi)^2} g_{\text{eq}} = \frac{\nu_g T^2}{2\pi}$$

$$\varepsilon_2 = \nu_g \int \frac{d^2 p}{(2\pi)^2} p_{\perp} g_{\text{eq}} = \frac{\nu_g T^3}{\pi}$$

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$$\begin{aligned} n_2 &= \nu_g \int \frac{d^2 p}{(2\pi)^2} g_{\text{eq}} = \frac{\nu_g T^2}{2\pi} \\ \varepsilon_2 &= \nu_g \int \frac{d^2 p}{(2\pi)^2} p_{\perp} g_{\text{eq}} = \frac{\nu_g T^3}{\pi} \\ s_2 &= -\nu_g \int \frac{d^2 p}{(2\pi)^2} g_{\text{eq}} (\ln g_{\text{eq}} - 1) = \frac{3\nu_g T^2}{2\pi} \end{aligned}$$

thermodynamic relation $\varepsilon_2 + P_2 = T s_2$ gives pressure and sound velocity

$$P_2 = \nu \frac{T^3}{2\pi} = n_2 T = \frac{\varepsilon_2}{2}, \quad c_s^2 = \frac{1}{2}$$



3.1. Derivation of hydrodynamic equations

the hydrodynamic equations are obtained from the energy and momentum conservation laws

$$\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = \frac{n_0 \nu_g T^3}{2\pi\tau} (3U^\mu U^\nu - g^{\mu\nu} - V^\mu V^\nu)$$

the energy-momentum conservation laws are consistent with the entropy conservation law

$$U_\nu \partial_\mu T^{\mu\nu} = T \partial_\mu S^\mu = 0, \quad S^\mu = \frac{3n_0 \nu_g T^2}{2\pi\tau} U^\mu$$

in the derivation of hydro equations the following 3 combinations are used

$$\partial_\mu S^\mu = 0$$

$$U_1 \partial_\mu T^{\mu 1} + U_2 \partial_\mu T^{\mu 2} = 0$$

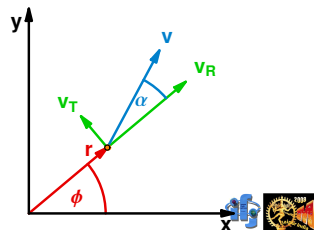
$$U_2 \partial_\mu T^{\mu 1} - U_1 \partial_\mu T^{\mu 2} = 0$$

as the result we obtain
hydrodynamic equations for
2D perfect fluid

$$s(T)$$

$$v_x$$

$$v_y$$



3.2. Boost non-invariance

Lemma (one of many beautiful features of this approach)

If $T^{\mu\nu}$ is conserved, then $T^{\mu\nu}$ multiplied by any function of η is also conserved

$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow \partial_\mu [f(\eta)T^{\mu\nu}] = 0$$

in particular n_0 may be a rapidity dependent function



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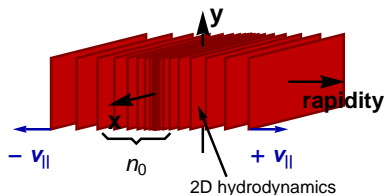
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→ hydro equations $\partial_\mu T^{\mu\nu} = 0$ do not specify the η -dependence, the hydrodynamic evolution is determined separately for each transverse layer, the η -dependence is determined by the initial conditions



4.1. Initial conditions

similarly to other hydrodynamic calculations we assume that the initial energy density at the transverse position point \vec{x}_\perp is proportional to the wounded-nucleon density ρ_{WN} at this point, namely

$$\varepsilon_2(\vec{x}_\perp) = \frac{n_0 \nu_g T^3(\vec{x}_\perp)}{\pi} \propto \rho_{WN}(\vec{x}_\perp).$$

this assumption used for a 2D system is equivalent to the assumption $s_3 \propto \rho_{WN}$ used in 3D hydrodynamic codes,



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$$T(\tau_{\text{init}}, \vec{x}_\perp) = T_i \left[\frac{\rho_{WN}(\vec{x}_\perp)}{\rho_{WN}(0)} \right]^{1/3}$$

where the parameter T_i is the initial central temperature



4.2. Cooper-Frye prescription

transverse-momentum spectra are obtained with the standard Cooper-Frye prescription

$$\frac{dN}{dyd^2p_{\perp}} = \frac{n_0\nu_g}{(2\pi)^2} \int d\Sigma^{\mu} p_{\mu} \frac{\delta(\eta - y)}{\tau m_{\perp}} g_{\text{eq}}$$



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for cylindrically asymmetric collisions and midrapidity, $y = 0$, the transverse-momentum spectrum has the following expansion in the azimuthal angle of the emitted particles

$$\frac{dN}{dyd^2p_{\perp}} = \frac{dN}{dy 2\pi p_{\perp} dp_{\perp}} (1 + 2v_2(p_{\perp}) \cos(2\phi_p) + \dots)$$

this equation defines the elliptic flow coefficient v_2 ,



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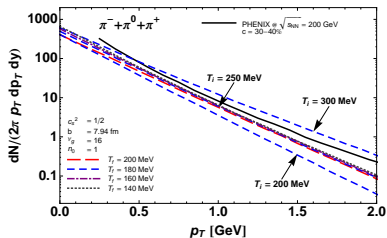
this equation defines the elliptic flow coefficient v_2 , which may be calculated from the asymmetry of the momentum spectrum

$$v_2(p_{\perp}) = \frac{1}{2} \frac{f_N(p_{\perp}, \phi_p = 0) - f_N(p_{\perp}, \phi_p = \frac{\pi}{2})}{f_N(p_{\perp}, \phi_p = 0) + f_N(p_{\perp}, \phi_p = \frac{\pi}{2})}$$

with f_N being a shorthand notation for $dN/(dyd^2p_{\perp})$



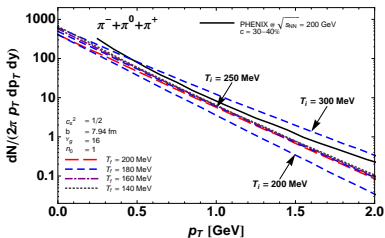
5.1. Transverse-momentum spectra and v_2



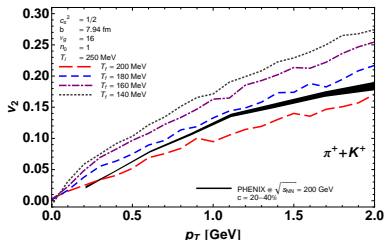
- solid line: transverse-momentum spectra of positive pions ($\times 3$) measured by PHENIX, centrality class 30-40%, $\sqrt{s_{NN}} = 200$ GeV
- dashed and dotted lines: model spectra of gluons for various choices of the initial and final temperature, $n_0 = 1$



5.1. Transverse-momentum spectra and v_2



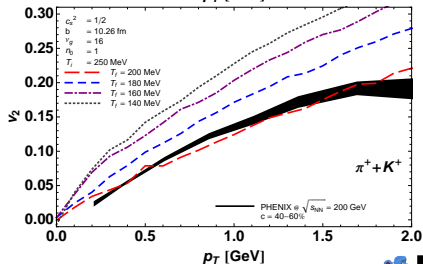
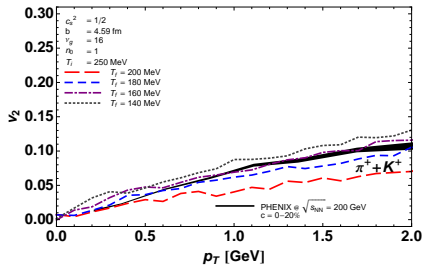
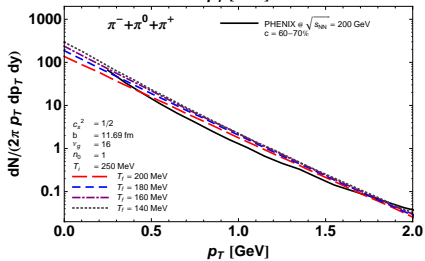
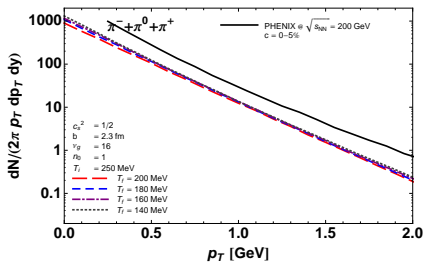
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- solid line: $v_2(p_{\perp})$ measured by PHENIX (data for pions and kaons), centrality class 20-40% , $\sqrt{s_{NN}} = 200$ GeV
- dashed and dotted lines: model calculations for $T_i = 250$ MeV and for four final temperatures $T_f = 200, 180, 160$ and 140 MeV.



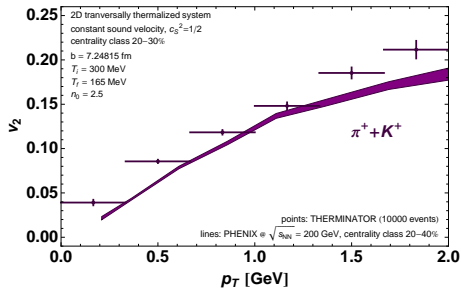
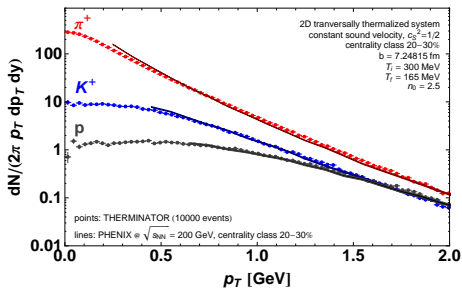
5.2. Central vs. peripheral



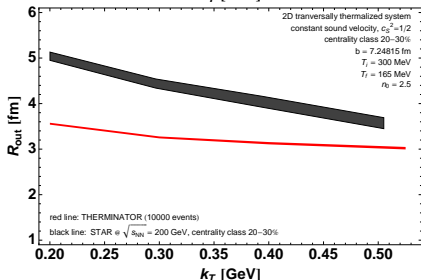
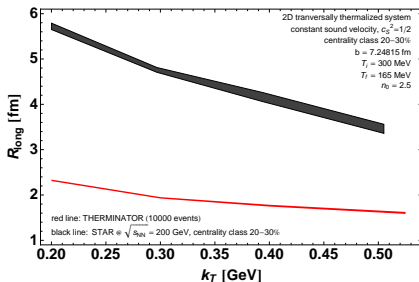
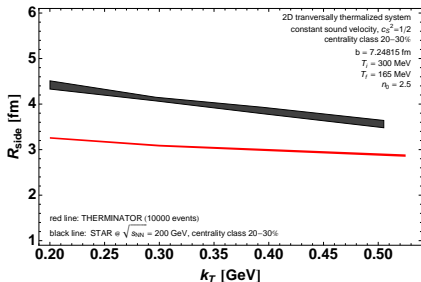
5.3. Freeze-out with THERMINATOR

(A. Kisiel et al., Comput. Phys. Commun. 174 (2006) 669, modified version)

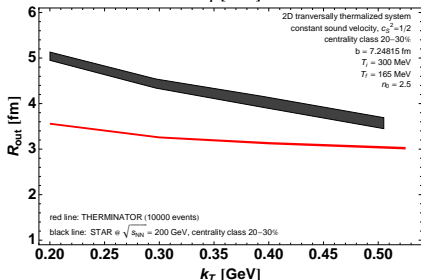
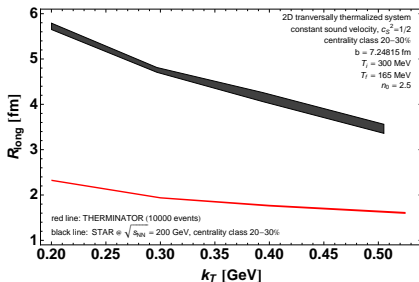
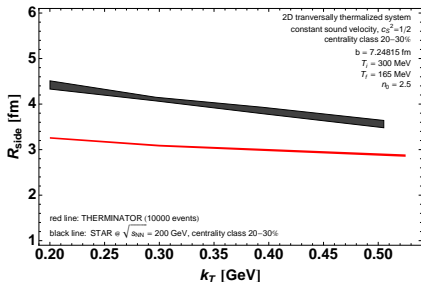
the two-dimensional gluon system is replaced by the two-dimensional hadron gas, hadronic resonances decay



HBT two-particle method



HBT two-particle method



further work on smooth merging of the 2D gluon system and the 2D hadron system should be done (modeling 2D phase transition)



6. Conclusions

1. the idea that the parton system created in relativistic heavy-ion collisions emerges in a state with transverse momenta close to thermodynamic equilibrium and its evolution at early times is dominated by the 2-dimensional (transverse) hydrodynamics is investigated and shown to be consistent with data (spectra and v_2)
2. this mechanism does not require early 3D equilibration, strong v_2 is produced in non-completely thermalized matter
3. this mechanism may also help to solve the HBT puzzle, but this needs extra work

