2008

Jaipur India

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H-QM | Helmholtz Research School
Quark Matter Studies

Thermalization beyond the Bottom-Up picture

Jaipur, India, 5th February 2008

Andrej El

Zhe Xu, Carsten Greiner

arXiv: 0712.3734 (hep-ph)

Outline

> Introduction and motivation

> Parton cascade **BAMPS** & Initial conditions (CGC)

> **Bottom-Up** scenario of thermalization

> Results :

Thermalization of a CGC, comparison with Bottom Up,

shear viscosity to entropy density

> Summary

Intro and Motivation

> Success of **ideal hydrodynamics** describing **v_2** in Au+Au collisions at RHIC → **equilibration on a short time scale**

> Mechanism of thermalization not completely understood, however

pQCD bremsstrahlung processes are essential for isotropization of quark gluon matter **[**Xu, Greiner, PR C 76 (2007) 024911**]**

> The importance of pQCD bremsstrahlung raised in the

"Bottom-Up" scenario **[**Baier, Mueller, Schiff, Son PL B 502 (2001) 5158**]**

> Parton Cascade BAMPS with **2↔3 processes included**:

>> Thermalization of CGC initial condition & Comparison with Bottom-Up

>> Transport coefficients and $\frac{\eta}{s}$ can be calculated using Navier-Stokes and Israel-Stewart hydro equations

BAMPS

Z.Xu, C.Greiner, A.El, O.Fochler

[Xu, Greiner, PR C 71 (2005) 064901]

Boltzmann
Approach for
Multi-
Parton
Scatterings

> Solving the Boltzmann equation

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}_1}{E_1}\vec{\nabla}\right)f_1(\vec{r},\vec{p},t)=C_{22}+C_{23}$$

for on-shell gluons (and quarks)

using Monte Carlo techniques

> **2↔3 processes included** (detailed balance)

stochastic interpretation of collision rates

====== in particular for this work ======

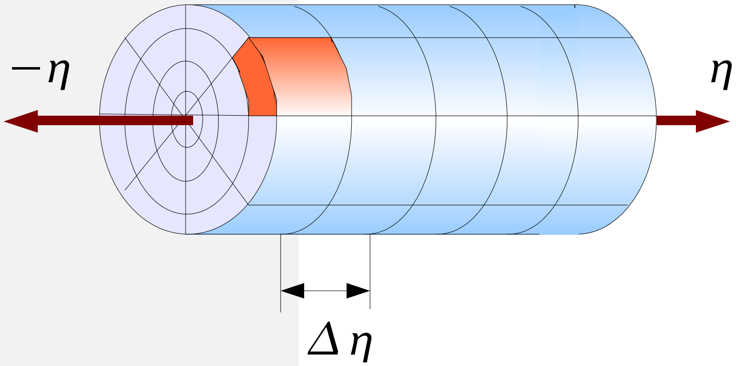
>> Boltzmann gas of gluons(only)

>> **1-Dim Bjorken expanding geometry:**

in longitudinal direction: equidistant η -bins

>> **pQCD** calculated crossections

“tube” geometry



Initial Condition:

Color Glass Condensate [McLerran, Venugopalan , PR D 49 (1994) 2233]

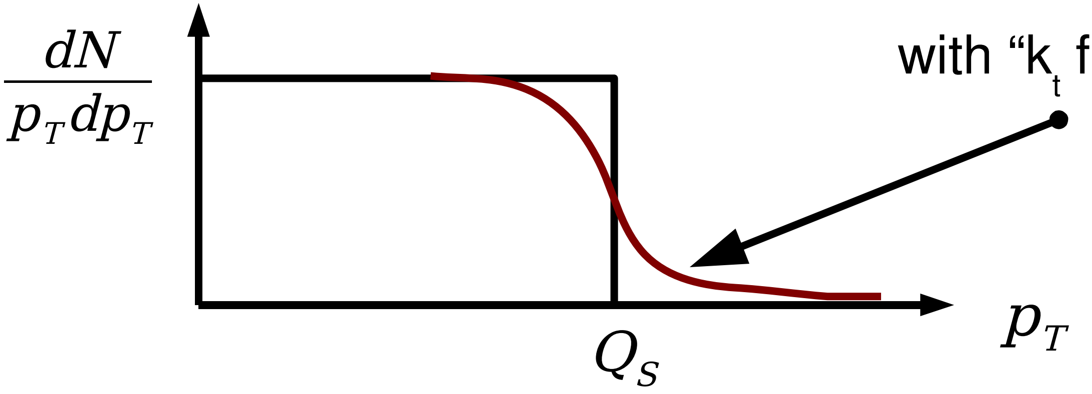
A simple, „idealistic“ initial parton distribution

$$f(x,p)_{x=0} = \frac{c}{\alpha_s N_c}\frac{1}{\tau_0}\,\delta(p_z)\,\Theta(Q_s^2 - p_t^2) \quad \mathit{with} \quad t_{ini} = \frac{c}{\alpha_s N_c}\frac{1}{Q_S}$$

$$\frac{dN}{d\eta} = c\ \pi R^2 \frac{N_c^2-1}{4\pi^2\alpha_s N_c}\,Q_s^2 \qquad \mathit{with} \quad c=0.4$$ (Q_S – saturation momentum)

> Boost-invariant initial condition.

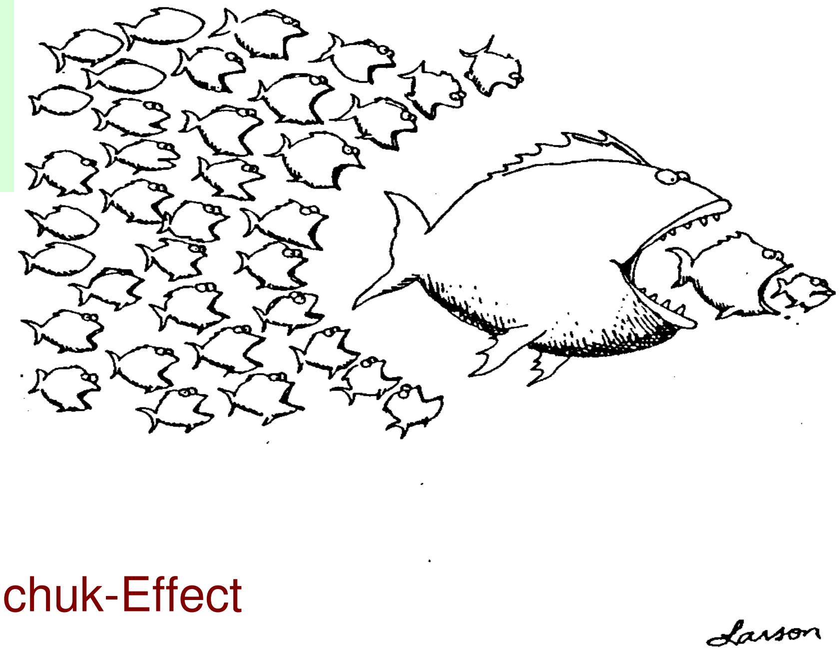
> Saturation for $p_t<Q_S$, high occupation number; for gluons with higher transverse momenta occupation number ~0.



with “k_t factorization” (Kharzeev, Levin, Nardi approach)

Bottom-Up Scenario of Thermalization

[Baier, Mueller, Schiff, Son, PL B 51(2001) 502]



1. $Q_S^{-1} \ll t \ll \alpha^{-3/2} Q_S^{-1}$

Initial system is dominated by „hard“ gluons with $p_t \sim Q_S$.

2. $\alpha^{-3/2} Q_S^{-1} \ll t \ll \alpha^{-5/2} Q_S^{-1}$

In **inelastic collisions** soft gluons are produced.

Production of hard suppressed by the Landau-Pomeranchuk-Effect

Number of soft gluons increases.

Soft gluons thermalize among themselves and build up a **thermal bath**.

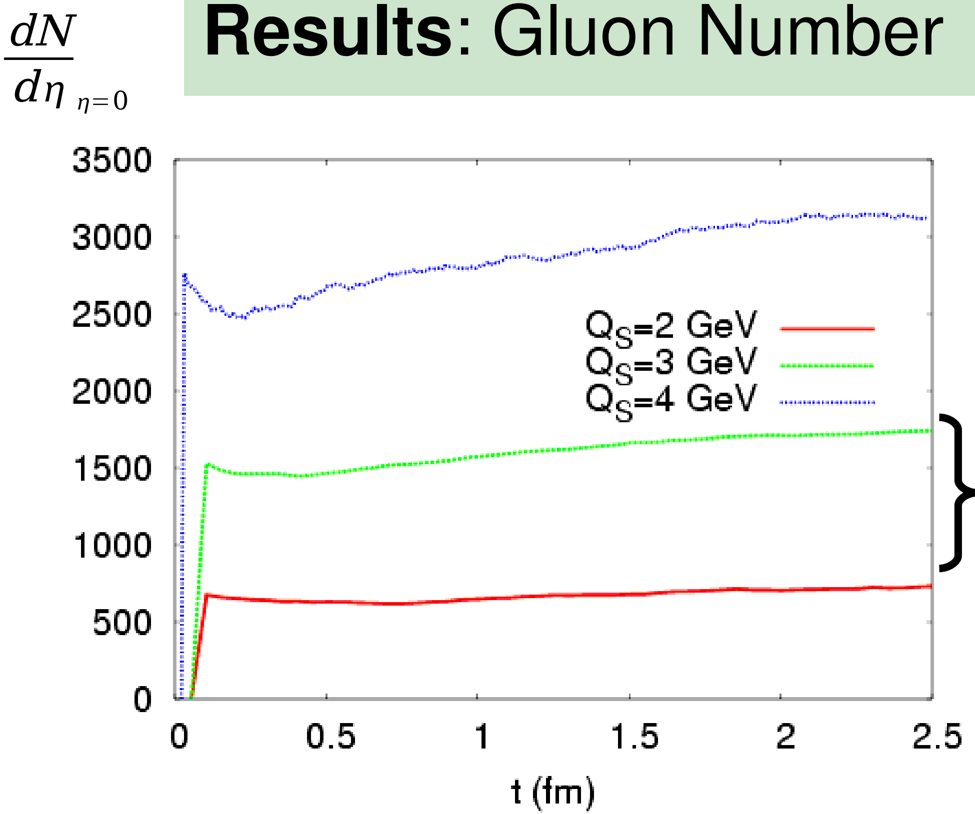
3. $\alpha^{-5/2} Q_S^{-1} \ll t < \alpha^{-13/5} Q_S^{-1}$

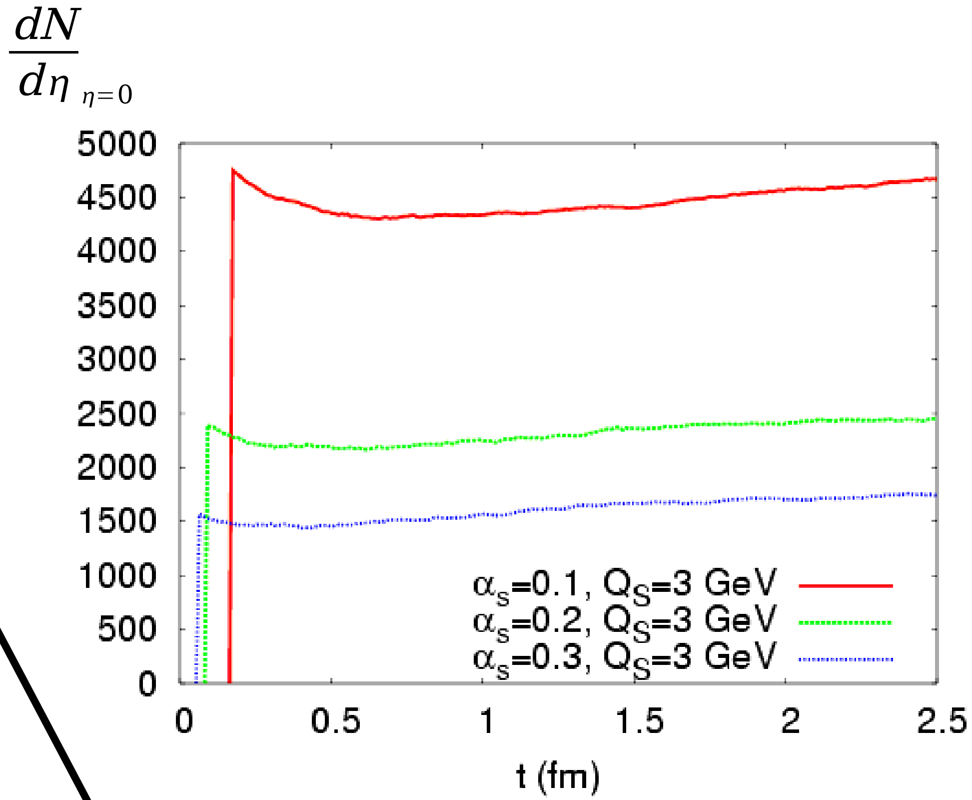
Hard gluons lose their entire energy to the thermal bath, built up by soft gluons.

The system thermalizes.

Estimated timescale for thermalization $\tau_{th} \sim \alpha_s^{-13/5} Q_S^{-1}$

Results: Gluon Number





$$\frac{dN_{ch}}{d\eta} \quad in \quad most \quad central \quad Au+Au@\sqrt{s}=130-200\mathrm{AGeV} \quad (RHIC)$$

> during the first 0.3-0.75 fm/c: gluon annihilation (~10% of initial number).

3→2 processes dominant at early times

> parametric enhancement of total gluon number <u>not observed</u> (contrary to “Bottom-Up”)

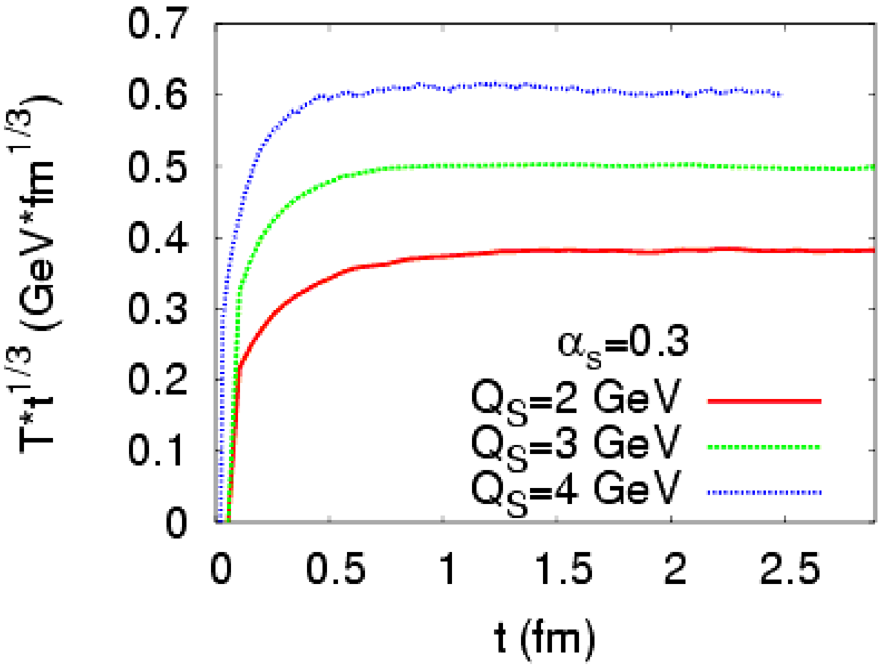
> the initial CGC is oversaturated.

Thermalization

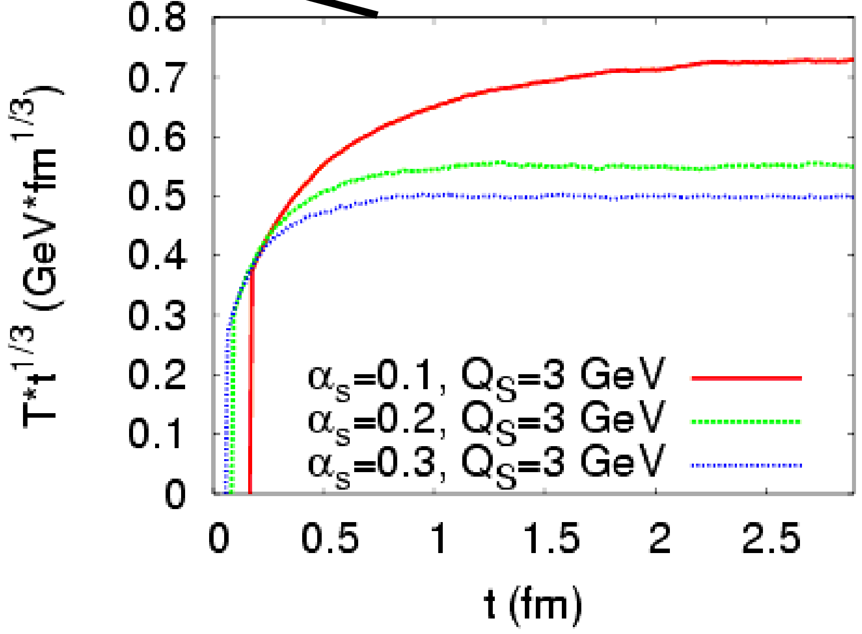
$T(t)\cdot t^{\frac{1}{3}}$ saturates ⇒ thermalized $\quad T=\frac{e}{3n}$

(quasi-ideal hydro)

$$t_{th}(\alpha_s=0.3, Q_S=2\ GeV)=1.2\ fm$$
$$t_{th}(\alpha_s=0.3, Q_S=3\ GeV)=0.75\ fm$$
$$t_{th}(\alpha_s=0.3, Q_S=4\ GeV)=0.55\ fm$$



$$t_{th}(\alpha_s=0.1, Q_S=3\ GeV)=1.75\ fm$$
$$t_{th}(\alpha_s=0.2, Q_S=3\ GeV)=1.0\ fm$$
$$t_{th}(\alpha_s=0.3, Q_S=3\ GeV)=0.75\ fm$$



Comparison with Bottom-Up:

$t_{th}\sim\alpha_s^{-\frac{13}{5}}$ **wrong** $\qquad t_{th}\sim\frac{1}{Q_S}$ **fullfilled**

BAMPS: $t_{th}\sim\alpha_s^{-2}(\ln\alpha_s)^{-2}Q_S^{-1}$

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Quicker equilibration than in Bottom-Up.

Reason:

> The gluon bremsstrahlung favors large-angle radiation

(due to the LPM suppression)

Z.Xu, C.Greiner, PR C 76 (2007) 024911

=> “hard” and “soft” gluons thermalize almost simultaneously

For more details see:

Plenary VI, 6th Feb,

Talk by Zhe Xu (C.Greiner)

Shear viscosity from Navier-Stokes equations

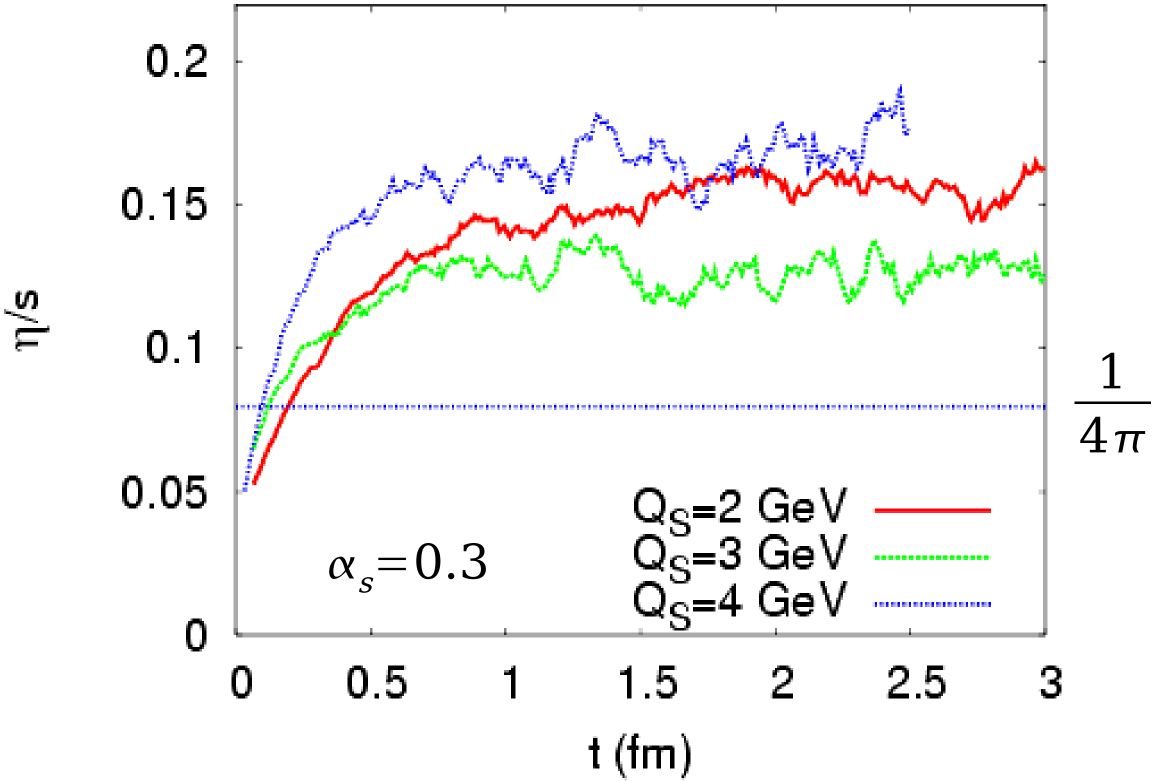
Navier-Stokes in Bjorken geometry:

$$\eta=\frac{\tau}{4}\left(T_{xx}+T_{yy}-2\mathrm{T}_{zz}\right) \quad with \quad T_{ii}=\int\frac{d^3p}{(2\pi)^3}p_ip_if(x,p)$$

$$s \simeq 4\mathrm{n}-n\cdot\ln(\lambda) \quad with \quad \lambda=\frac{n}{n_{eq}}$$

In BAMPS η/s proves to be a universal number, if going from RHIC to LHC energy

(dependence of α_s on Q_s NOT considered in simulations)



(average over 10 runs)

Shear viscosity from 2nd order equations

[Israel, Stewart Ann.Phys. 118 (1979) 341, Muronga, PR C 76 (2007) 014910]

i. Assume a small departure of distribution function from equilibrium:

$$f(x,p)=e^{-\beta p_\mu u^\mu}(1+\epsilon+\epsilon_\mu p^\mu+\epsilon_{\mu\nu}p^{\mu\nu})$$

2nd order relativistic dissipative hydro then can be derived from the Boltzmann equation:

ii. → Grad's method: Calculate momenta of BE and take projections:

$$-\frac{\eta}{\beta}\underbrace{\left(\frac{1}{2}\Delta^\alpha_\mu\Delta^\beta_\nu+\frac{1}{2}\Delta^\beta_\mu\Delta^\alpha_\nu-\frac{1}{3}\Delta_{\mu\nu}\Delta^{\alpha\beta}\right)P_{\alpha\beta}}_{\text{projection of:}}=J_{42}\underbrace{\left(\frac{1}{2}\Delta^\alpha_\mu\Delta^\beta_\nu+\frac{1}{2}\Delta^\beta_\mu\Delta^\alpha_\nu-\frac{1}{3}\Delta_{\mu\nu}\Delta^{\alpha\beta}\right)T_{\alpha\beta}}_{\pi_{\mu\nu}}$$

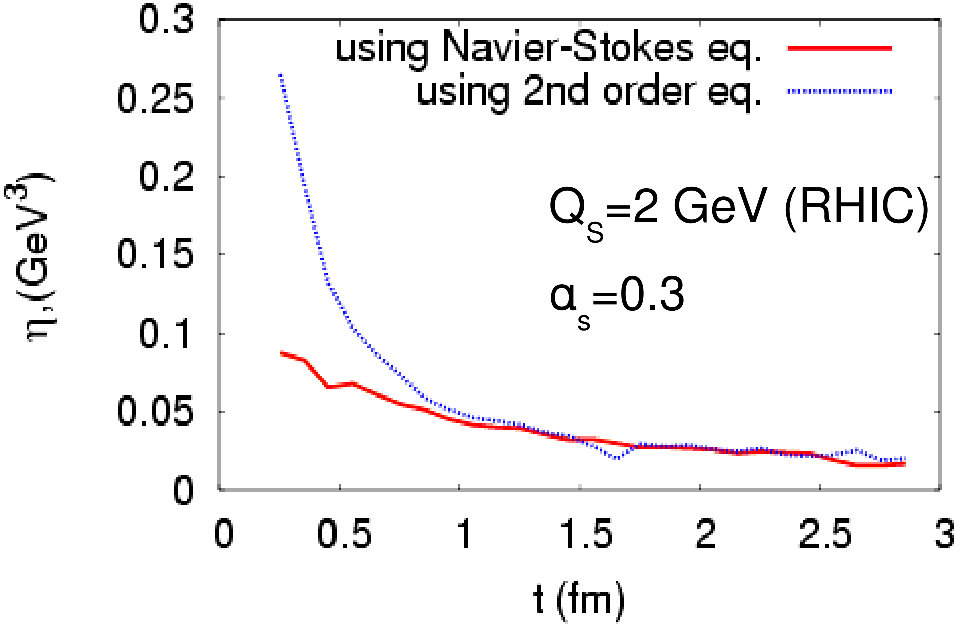
$$P_{\alpha\beta}=\int\frac{d^3p}{(2\pi)^3}p_\alpha p_\beta C\,[f\,]$$

with

$$J_{42}=\frac{8}{\pi^2}\frac{1}{15}\Gamma(6)\,T^6$$

$$T=\frac{1}{\beta}=\sqrt[4]{\frac{\pi^2\cdot e(t)}{48}}$$

$$\Delta_{\mu\nu}=g_{\mu\nu}-u_\mu u_\nu$$



Summary

> **Equilibration** of quark and gluon matter at RHIC and LHC **on a shorter time scale** than the one suggested by "Bottom-Up" scenario is observed in BAMPS

> **no enhancement of total particle multiplicity** is observed until thermalization

> production of soft ($p_T < \alpha_s Q_S$) gluons in gg→ggg processes is hindered by the reverse channel

> **"hard"** and **"soft"** gluons thermalize almost **simultaneously**

> thermalization times are **≤ 1.2 fm/c** for RHIC and **~0.6 fm/c** for LHC regimes

> thermalization time is proportional to $\alpha_s^{-2} (\ln \alpha_s)^{-2} Q_S^{-1}$,
thus the dependence on α_s is weaker than the "Bottom-Up" result

Thermalization process differs from the "Bottom-Up" scenario

> η /s ~ 0.16 proves to be a universal number in BAMPS , if going from RHIC to LHC energy

Outlook

Second order relativistic dissipative hydrodynamics can be derived from the Boltzmann equation: Grad's method

[Israel, Stewart Ann.Phys. 118 (1979) 341,
Muronga, PR C 76 (2007) 014910]

>> Second order transport coefficients, relaxation times and entropy production $\partial_\mu S^\mu$ can be calculated from the cascade simulations.

>> Results from hydrodynamic equations are to be compared with those from the cascade.

>> Validity limits of hydrodynamical equations have to be investigated

Thank you for your attention!

Thanks to:

Zhe Xu, Carsten Greiner

&

Organizers of QM 08

&

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Shear viscosity to entropy density from 2nd order equations

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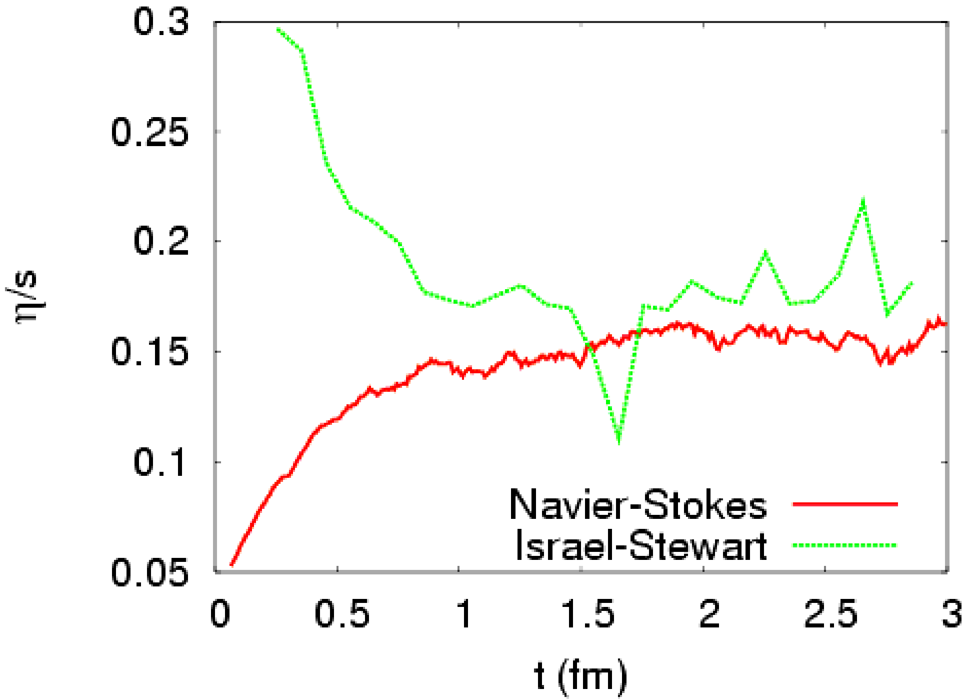
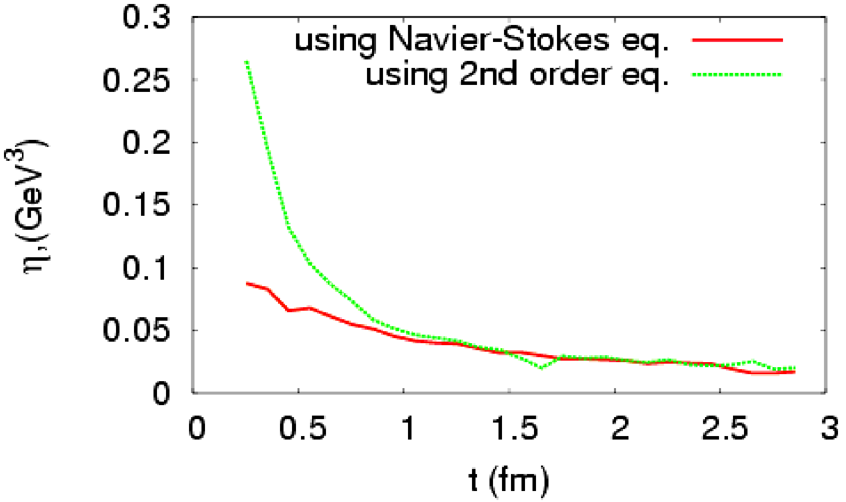
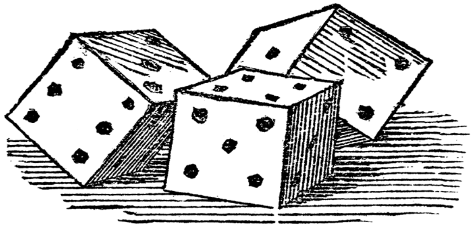




Collision Terms in the Boltzmann Equation



$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}_1}{E_1}\vec{\nabla}\right)f_1(\vec{r},\vec{p},t)=C_{22}+C_{23}$$

No Bose Enhancement factors in collision terms

$$\sigma_{23}=\frac{1}{2s}\frac{1}{(2\pi)^9}\int\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3p_3}{2E_3}|M_{1'2'\to 123}|^2\cdot(2\pi)^4\delta^{(4)}(p'_1+p'_2-p_1-p_2-p_3)$$

Matrix element 2->3: Gunion-Bertsch formula

$$|M_{gg\to ggg}|^2=\frac{9g^4}{2}\frac{s^2}{(\vec{q}_t^{\,2}+m_D^2)^2}\cdot\frac{12g^2\vec{q}_t^{\,2}}{\vec{k}_t^{\,2}[(\vec{k}_t-\vec{q}_t)^2+m_D^2]}\cdot\boxed{\Theta(k_t\Lambda_g-\cosh y)}$$

LPM Effect:

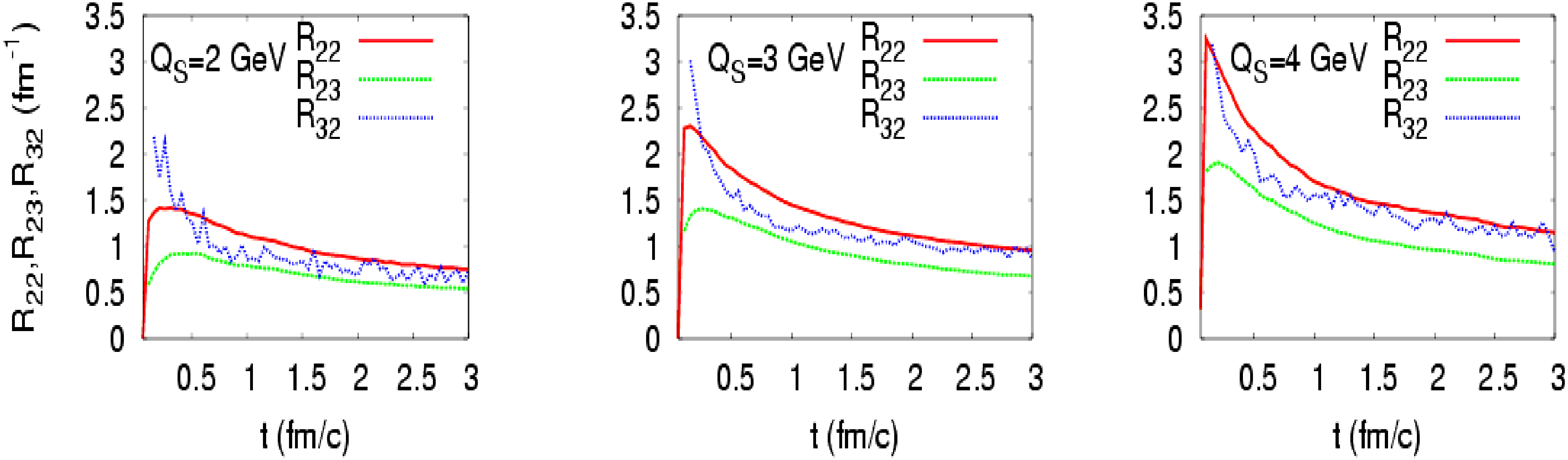
(formation time < mean free pass Λ_g)

$$|M_{123\to 1'2'}|^2=\frac{1}{16}|M_{1'2'\to 123}|^2$$

$$I_{32}=\frac{1}{2}\frac{1}{(2\pi)^6}\int\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}|M_{1'2'3'\to 12}|^2\cdot(2\pi)^4\delta^{(4)}(p'_1+p'_2+p'_3-p_1-p_2)$$

Collision rates

BACKUP



define three energy scales:

p_T<1.5 GeV ---> soft gluons

1.5GeV<p_T<Q_S ---> medium

p_T>Q_S ---> hard gluons

>> 2->3 collisions create soft gluons (in middle 1.5/collision) from medium&hard

>> 3->2 create hard gluons annihilating soft and medium

>> 2->2 smaller contribution to soft and hard gluon production

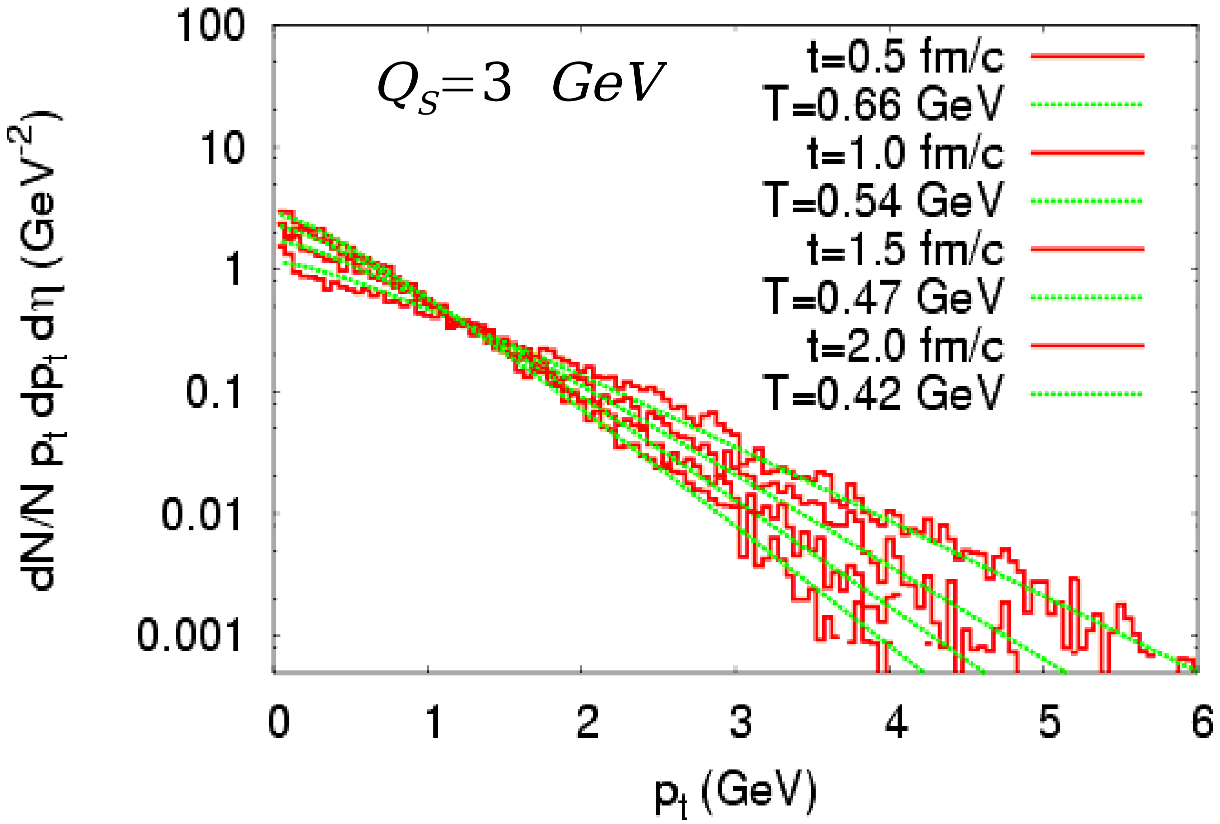
3->2 collisions hinder a strong increase of total particle number due to soft gluon production (contrary to “Bottom-Up”)

Transverse momentum spectra

B
A
C
K
U
P

typical hydro behavior:

spectrum becomes “steeper”



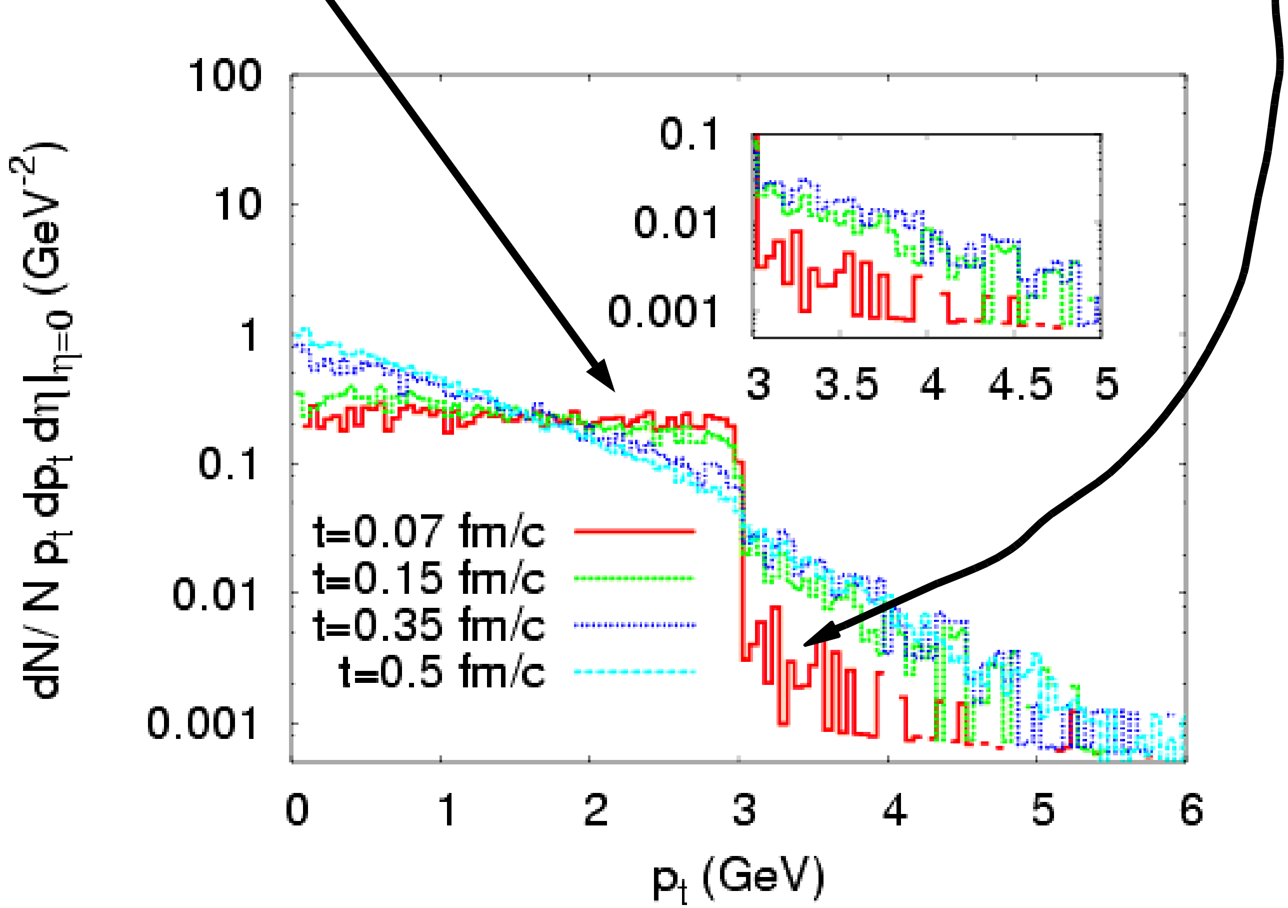
Transverse momentum spectra: avalanche

B
A
C
K
U
P

spectra achieve a thermal shape almost simultaneously in “soft” and “hard” sectors

production of “hard” gluons

degradation of “medium” gluons



Number of soft, medium, hard particles

B
A
C
K
U
P

System thermalized at t=0.75 fm/c (Q_s=3 GeV)

ongoing degradation of "medium" sector

particle production in hard sector within first 0.3 fm/c

particle number in soft sector increases slowly: hindered by 3->2 channel