SEARCH FOR SQUEEZED-PAIR CORRELATIONS AT RHIC

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Outline

- Introduction and motivation
- Brief review and previous results (infinite systems)
- Focus on finate expanding system, non-relativistic approach →illustration: $\phi\phi$ BBC pairs
- How to search for squeezed BBC pairs in experiments → suitable variables
- Modified-mass effects and squeezing on BBC and HBT correlations
- Summary and conclusions

Brief Introduction

- Late 90's: Back-to-Back Correlations (BBC) among boson-antiboson pairs → shown to exist if the masses of the particles were modified in a hot and dense medium

  [Asakawa, Csörgo" & Gyulassy, P.R.L. 83 (1999) 4013].

- Shortly after → similar BBC existed among fermion-antifermion pairs with medium modified masses

  [Panda, Csörgo", Hama, Krein & SSP, P. L. B512 (2001) 49].

- Some properties:
  - Similar formalism for both bosonic (bBBC) and fermionic (fBBC) Back-to-Back Correlations
  - Similar (and unlimited) intensity of fBBC and bBBC
  - Expected to appear for $p_T \leq 1- \ 2 \ \mathrm{GeV}/c$

Similarities

- Maximum of the squeezed correlation function vs. M_*
  → small mass shifts seem to induce stronger correlations

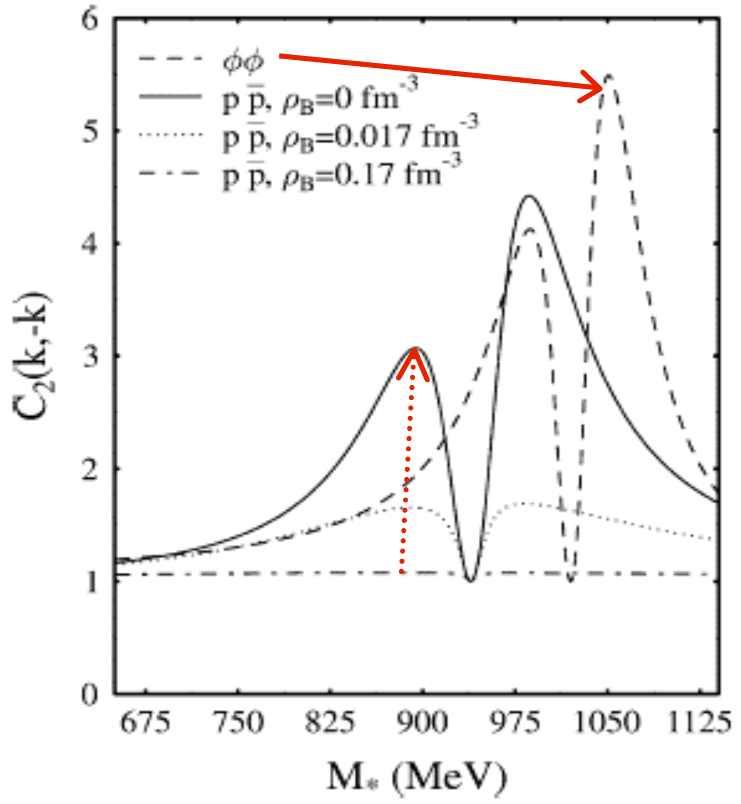


Fig. 1. Back-to-back correlations of proton–anti-proton pairs and ϕ-meson pairs, for $T = 140$ MeV, $\Delta t = 2$ fm/c and $|\mathbf{k}| = 800$ MeV/c.

Full Correlation Function

$$\langle a^\dagger_{k_1} a^\dagger_{k_2} a_{k_1} a_{k_2} \rangle = \langle a^\dagger_{k_1} a_{k_1} \rangle \langle a^\dagger_{k_2} a_{k_2} \rangle \pm \langle a^\dagger_{k_1} a_{k_2} \rangle \langle a^\dagger_{k_2} a_{k_1} \rangle + \langle a^\dagger_{k_1} a^\dagger_{k_2} \rangle \langle a_{k_1} a_{k_2} \rangle$$

NOTATION

$$N_1(\vec{k}_i) = \omega_{k_1} \frac{d^3N}{d^3k} = G_c(\vec{k}_i,\vec{k}_i) \equiv G_c(i,i) = \omega_{k_1} \langle a^\dagger_{k_i} a_{k_i} \rangle$$ → **Spectra**

$$G_c(\vec{k}_1,\vec{k}_2) \equiv G_c(1,2) = \sqrt{\omega_{k_1}\,\omega_{k_2}}\,\langle a^\dagger_{k_1} a_{k_2} \rangle$$ → **Chaotic amplitude**

$$G_s(\vec{k}_1,\vec{k}_2) \equiv G_s(1,2) = \sqrt{\omega_{k_1}\,\omega_{k_2}}\,\langle a_{k_1} a_{k_2} \rangle$$ → **Squeezed amplitude**

$$C_2(\vec{k}_1,\vec{k}_2) = 1 \pm \frac{\left|G_c(1,2)\right|^2}{G_c(1,1)\,G_c(2,2)} + \frac{\left|G_s(1,2)\right|^2}{G_c(1,1)\,G_c(2,2)}$$

HBT

BBC

In-medium & asymptotic operators

– $a_k\,(a^\dagger{}_k)$ → annihilation (creation) operator of the asymptotic quanta with 4-momemtum p^μ;

– $b_k\,(b^\dagger{}_k)$ → in-medium annihilation (creation) operator

(a-quanta → observed; b-quanta → thermalized in medium)

They are related by the Bogoliubov transformation:

$$\begin{cases} a^\dagger{}_k = c_k^*\, b^\dagger{}_k + s_{-k}\, b_{-k} \\ a_k = c_k\, b_k + s^*_{-k}\, b^\dagger{}_{-k} \end{cases} \quad ; \quad \boxed{c_k = \cosh[f_k]} \; ; \; \boxed{s_k = \sinh[f_k]}$$

– $\boxed{f_k = \frac{1}{2}\ln(\omega_k \,/\, \Omega_k)}$ → squeezing parameter (Bogoliubov transformation is equivalent to a squeezing operation)

bBBC & fBBC - formalism summary

- Bosonic BBC

$c_k = \cosh[f_k]$; $s_k = \sinh[f_k]$

$$\begin{cases} a^\dagger{}_k = c_k\, b^\dagger{}_k + s_{-k}\, b_{-k} \\ a_k = c_k\, b_k + s^*_{-k}\, b^\dagger{}_{-k_1} \end{cases}$$

$$\begin{cases} f_k \equiv r_k^{ACG} = \frac{1}{2}\log\left(\frac{\omega_k}{\Omega_k}\right) \\ \omega_k^2 = m^2 + \vec{k}^2 \\ \Omega_k^2 = \omega_k^2 - \delta M^2(|k|) \\ m_*^2 = m^2 - \delta M^2(|k|) \end{cases}$$

- Fermionic BBC

$c_k = \cos[f_k]$; $s_k = \sin[f_k]$

$$\begin{pmatrix} a_{\lambda,k} \\ \tilde{a}^\dagger_{\lambda',-k} \end{pmatrix} = \begin{pmatrix} c_k & \frac{f_k}{|f_k|} s_k A \\ -\frac{f_k^*}{|f_k|} s_k^* A^\dagger & c_k^* \end{pmatrix} \begin{pmatrix} b_{\lambda,k} \\ \tilde{b}^\dagger_{\lambda',-k} \end{pmatrix}$$

$$A = [\chi^\dagger_\lambda (\sigma.\hat{k}) \tilde{\chi}_{\lambda'}] \ ; \ A^\dagger = [\tilde{\chi}^\dagger_{\lambda'} (\sigma.\hat{k})^\dagger \chi_\lambda]$$

$$\begin{cases} \tilde{\chi}_{\lambda'} = -i\sigma^2 \chi_{\lambda'} \ ; \ \hat{k} = \vec{k}/|\vec{k}| \\ \rightarrow \text{ is a Pauli spinor} \\ \tan(2f_k) = -\frac{|k|\Delta M(k)}{\omega_k^2 - \Delta M(k) M} \\ m_*(k) = m - \Delta M(k) \\ \omega_k^2 = m^2 + \vec{k}^2 \ ; \ \Omega_k^2 = m_*^2 + \vec{k}^2 \end{cases}$$

Finite expanding systems

- Does the BBC survive
  - Finite medium (volume V) ?
  - Flow ?

- Squeezed correlations were shown
  - to survive both (more realistic) conditions, still with sizeable strength
  - non-relativistic treatment with flow-independent squeezing parameter → $\phi\phi$ squeezed correlations

(partial results shown @ Quark Matter 2005 →
http://qm2005.kfki.hu/ ; see also www.wpcf2007.llnl.gov)

… brief reminder of main results follows →

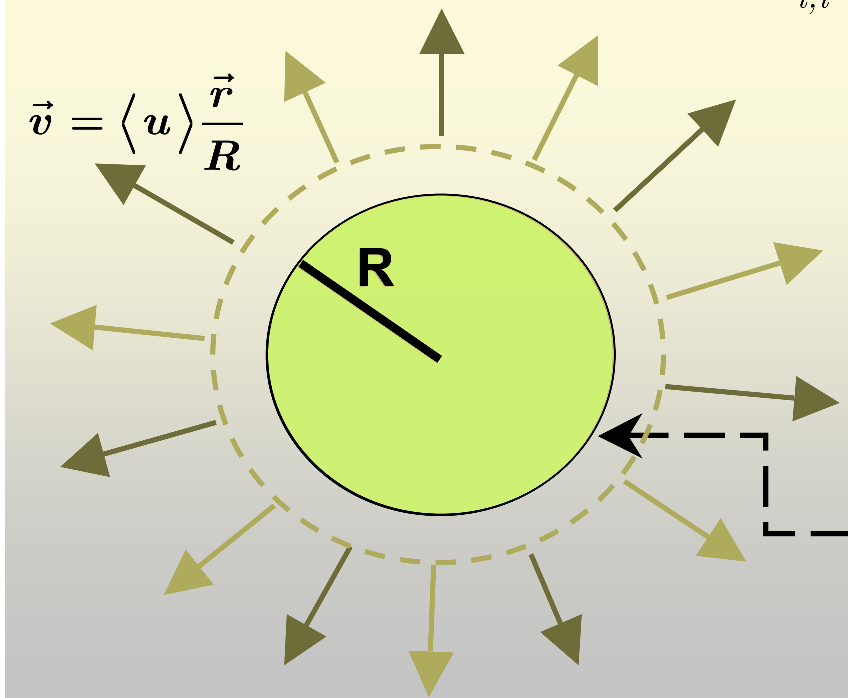
Expanding non-relativistic finite system

- For large mass m and small mass shifts [$(m_* - m)\ /\ m \ll m$]

→ flow effects on squeezing parameter $f_{i,j}$ are negligible :

$c_{i,j}$ and $s_{i,j}$ → flow independent

- Finite volume V
  - $s_{i,i} = 0$ outside mass-shift region (ΔM=0)



- Simplest V profile → analytical calculations:

← **Cross-sectional area → Gaussian**

$$\approx \exp[-\vec{r}^{\,2}\ /\ (2R^2)]$$

Region where mass-shift is non-vanishing

Additional hypotheses

- $n_{i,j}$ → Boltzmann limit of Bose-Einstein distribution:

$$n_{i,j}(x) \sim \exp\left[-\left(K^{\mu}_{i,j}u_{\mu} - \mu(x)\right) / T(x)\right]$$

Hydro parameterization → $$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T(x)} - \frac{\vec{r}^2}{2R^2}$$

- Freeze-out:

Sudden freeze-out → $$\int dt\, E_{i,j} e^{-2iE_{i,j}.\tau}\, \delta(\tau - \tau_0)\, d\tau_f = E_{i,j} e^{-2iE_{i,j}.\tau_0}$$

Finite emission interval → $$\int dt\, E_{i,j}\, F(\tau_f) e^{-iE_{i,j}(\tau-\tau_0)}\, d\tau_f = \frac{E_{i,j}}{[1+(E_{i,j}\Delta\tau)^2]}$$

$$F(\tau) = \frac{\theta(\tau-\tau_0)}{\Delta\tau}\, e^{-(\tau-\tau_0)/\Delta\tau}$$

- Non-relativistic limit:

$$u^{\mu} = \gamma(1,\vec{v}) \quad ; \quad \vec{v} = \langle u \rangle \frac{\vec{r}}{R}$$

$$\gamma = (1-\vec{v}^2)^{-1/2} \approx 1 + \frac{1}{2}\vec{v}^2 \qquad [\mathcal{O}(v^2)]$$

Summary of the previous results

- Previous results showed:
  - $C_s(k, -k)$ survives both
    - Finite emission times ($\Delta t = 2\mathrm{fm}/c$)
    - Moderate flow (could enhance signal at small $\underline{k}$)
  - However, only the behavior of the maximum value of $C_s(k,-k)$ vs. m_* vs. k was studied before (not useful for looking for the signal)
- Which would be the basic signal to be searched for? → better look for different values of k_1, k_2, i.e., $C_s(k_1,k_2)$

Squeezed Correlation vs. k_1 & k_2

$2 * \vec{K} = \vec{k}_1 + \vec{k}_2$ $\vec{q} = \vec{k}_1 - \vec{k}_2$

$$G_s(k_1, k_2) = \frac{E_{1,2}}{(2\pi)^{3/2}} c_{12} s_{12} \left\{ \begin{array}{l} R^3 \exp\left(-\frac{R^2 (k_1 + k_2)^2}{2} \right) + 2\, n_0^* R_*^3 \exp\left(-\frac{(k_1 - k_2)^2}{8 m_* T} \right) \times \\ \exp\left[\left(-\frac{im\langle u \rangle R}{2 m_* T_*} - \frac{1}{8 m_* T_*} - \frac{R_*^2}{2} \right) (k_1 + k_2)^2 \right] \end{array} \right\}$$

$2 * \vec{K} = \vec{k}_1 + \vec{k}_2$

Remember: $2 * \vec{K} = \vec{k}_1 + \vec{k}_2$, $\vec{q} = \vec{k}_1 - \vec{k}_2$

$$G_c(k_i) = \frac{E_{i,i}}{(2\pi)^{3/2}} \left\{ |s_{ii}|^2 R^3 + n_0^* R_*^3 \left(|c_{ii}|^2 + |s_{ii}|^2 \right) \exp\left(-\frac{k_i^2}{2 m_* T_*} \right) \right\}$$

$$R_* = R \sqrt{\frac{T}{T_*}}$$

$$C_s(\vec{k}_1, \vec{k}_2) = 1 + \frac{|G_s(1,2)|^2}{G_c(1,1) G_c(2,2)}$$

$$T_* = \left(T + \frac{m^2}{m_*} \langle u \rangle^2 \right)$$

Suitable variables

- Two main possibilities:

  1. Combining particle-antiparticle pairs (k_1, k_2) → theory ↔ simulation

  2. Rewriting $C_s(k_1, k_2)$ in terms of K and q :

- $$2 * \vec{K}_{i,j} = (\vec{k}_i + \vec{k}_j) \qquad \vec{q}_{i,j} = (\vec{k}_i - \vec{k}_j)$$

- The effect is maximum for $\vec{k}_1 = -\vec{k}_2 = \vec{k}$
  i.e., for $\vec{K} = 0$ → study for different values of q

Relativistic extension of $2 * \vec{K}_{i,j} = (\vec{k}_i + \vec{k}_j)$

- If we define (suggested by M. Nagy)

$$Q_{inv}^{back} = (\omega_1 - \omega_2, \vec{k}_1 + \vec{k}_2) = (q_{12}^0, 2\vec{K}_{12})$$

  - Where

$$2K^{\mu} = [(k_1^0 + k_2^0), (\vec{k}_1 + \vec{k}_2)] \quad ; \quad q^{\mu} = [(k_1^0 - k_2^0), (\vec{k}_i - \vec{k}_j)]$$

- However, even better: define a new variable, such as

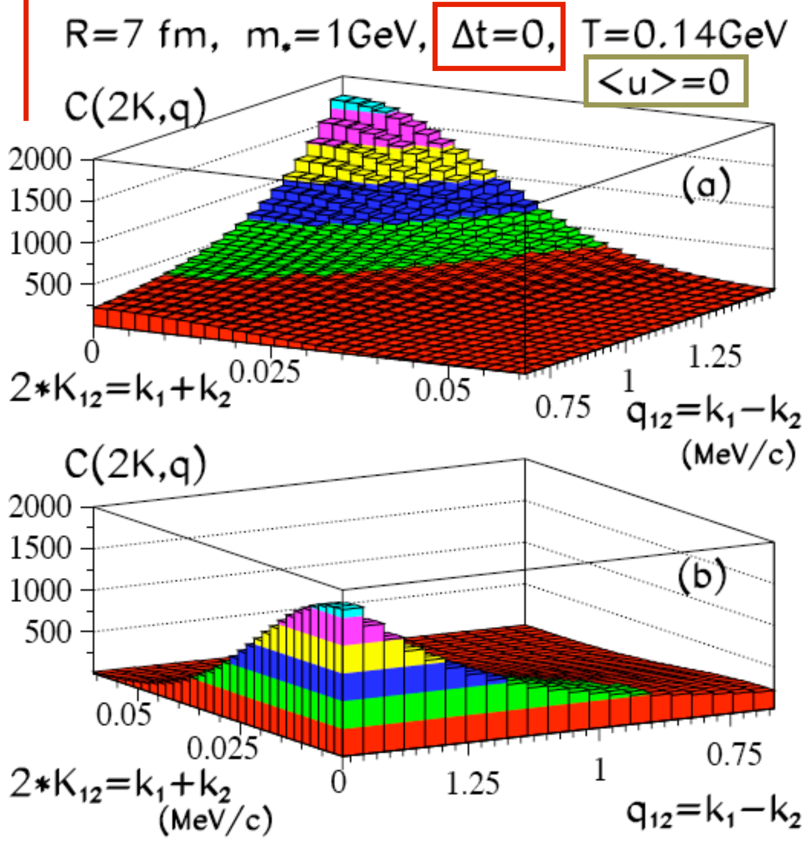
$$Q_{bbc}^2 = -(Q_{inv}^{back})^2 = 4(\omega_1\omega_2 - K^{\mu}K_{\mu})$$

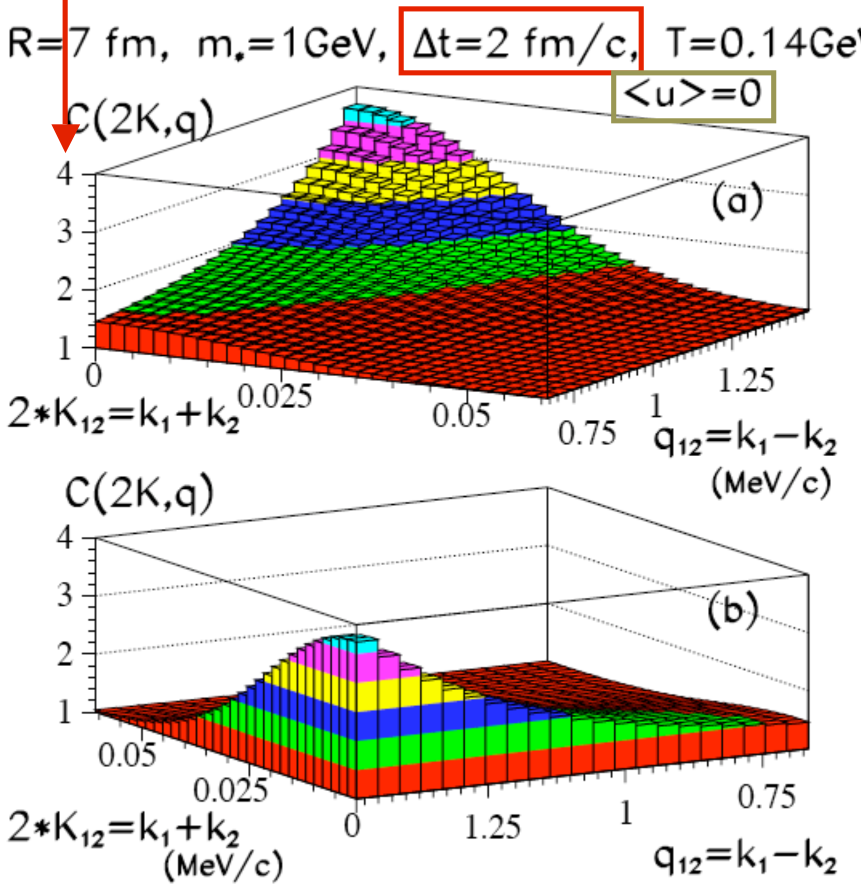
  - Then, its non-relativ. limit ($\omega_i = \sqrt{m^2 + \vec{k}_i^{\,2}} \approx m + \frac{\vec{k}_i^{\,2}}{2m}$) is

$$Q_{bbc}^2 \approx (2\vec{K}_{12})^2$$

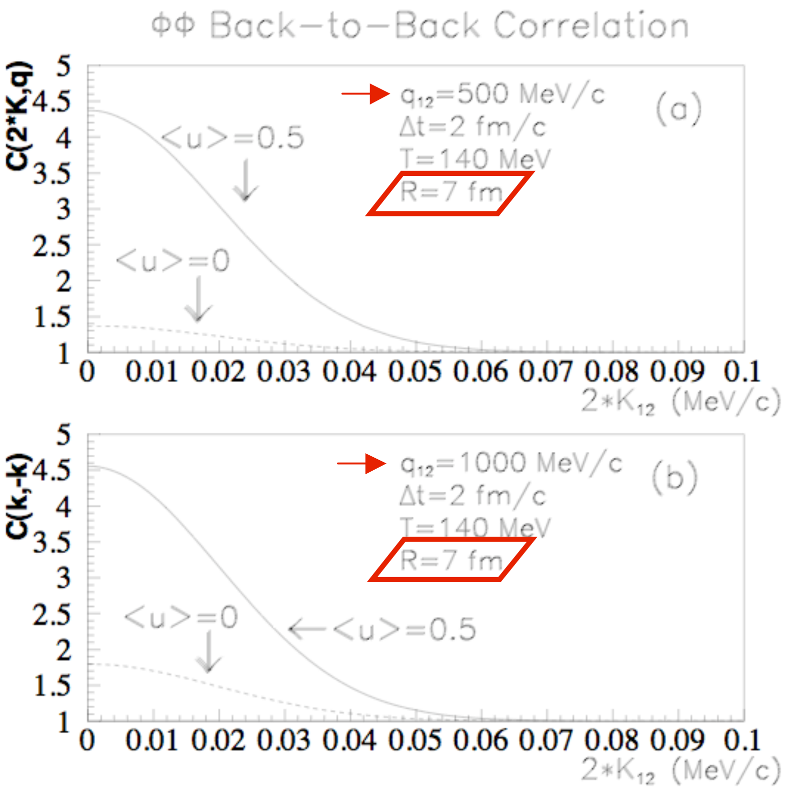
$C_s\ (K_{12},\ q_{12})$ vs. $(2{*}K_{12})$ vs q_{12} - no flow

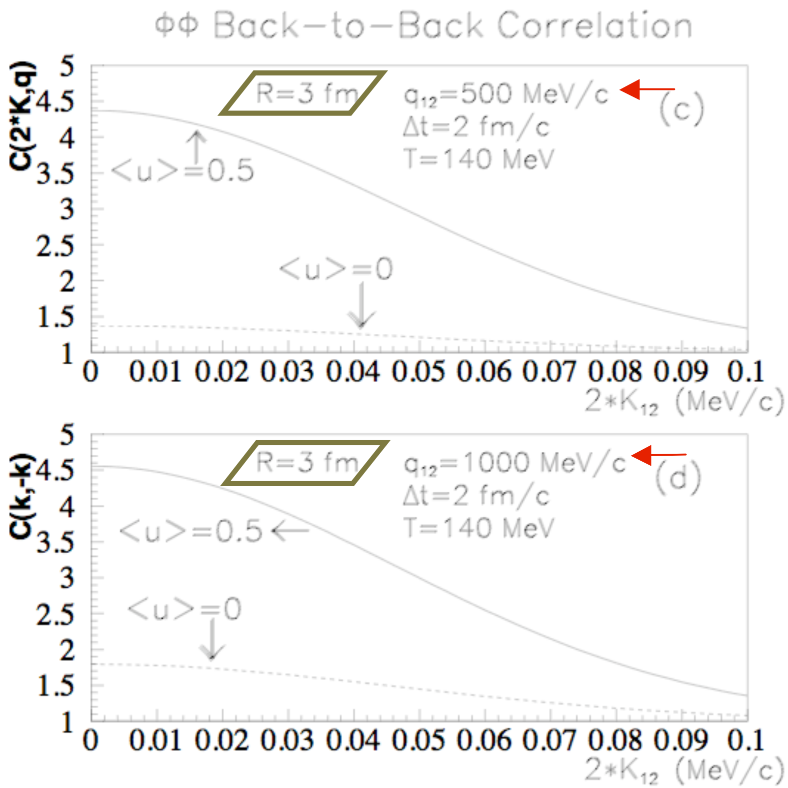
time reduction factor: $\dfrac{E_{i,j}}{[1+(E_{i,j}\Delta\tau)^2]}$



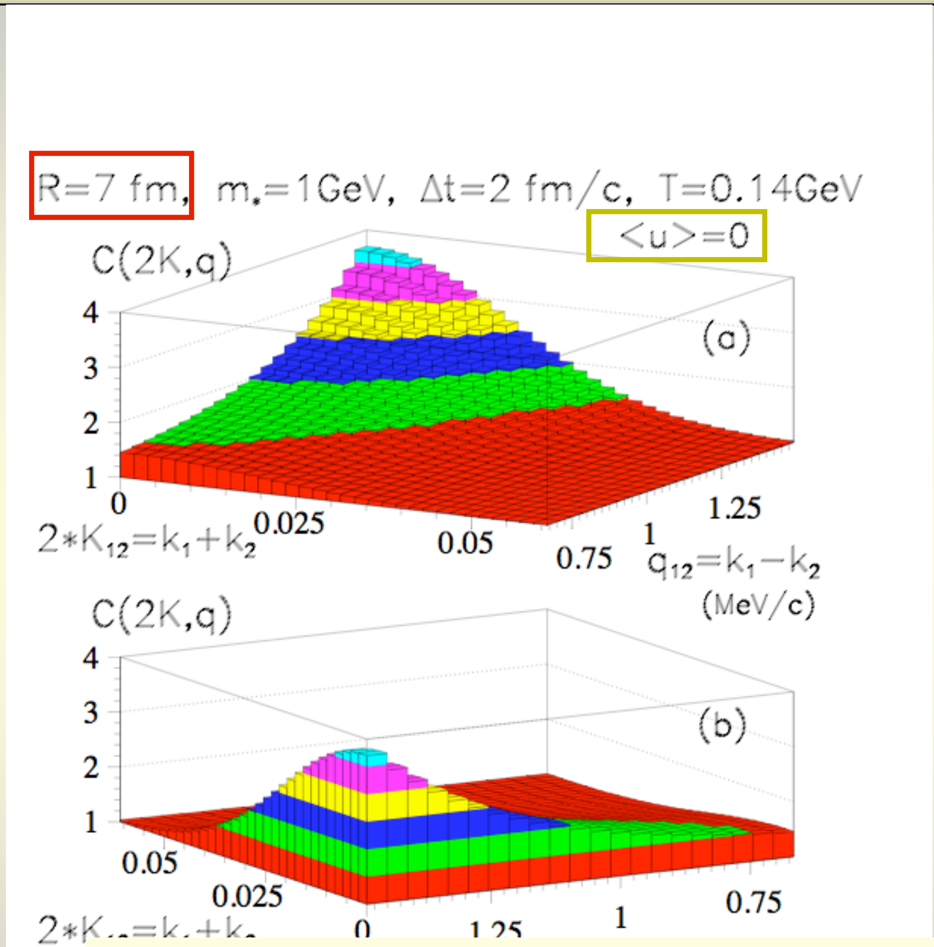


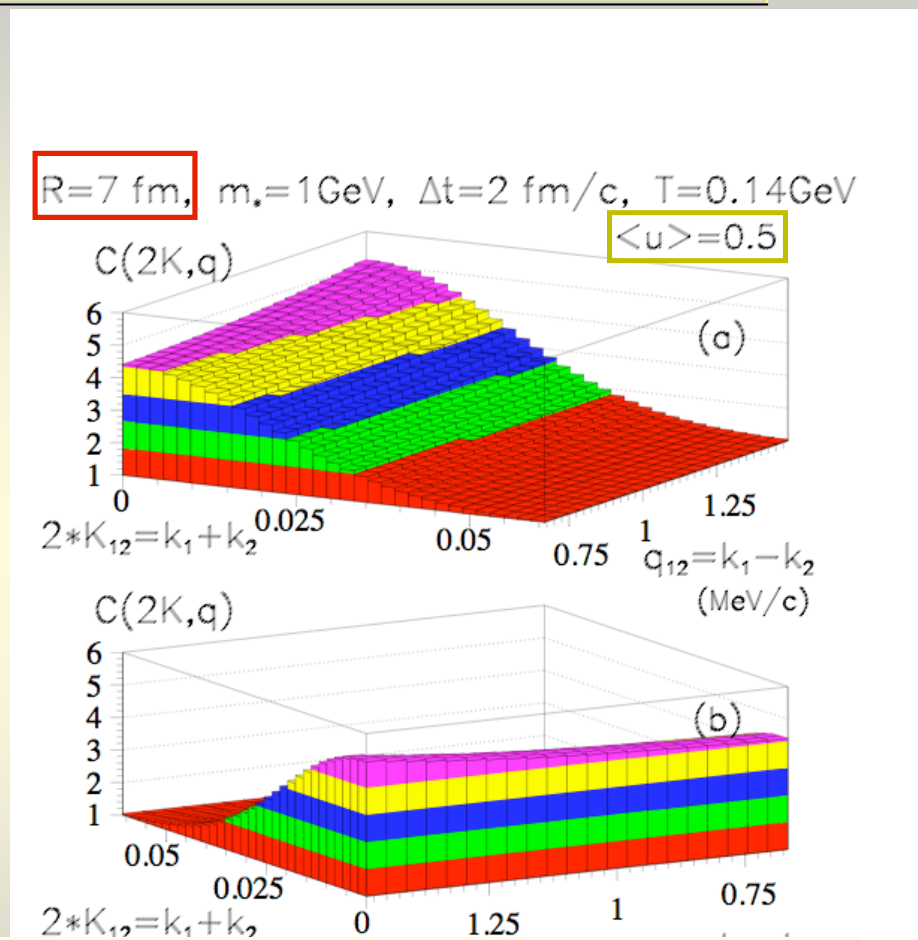
Effect of radial flow @ RHIC (<*u*> ~ 0.5)





$C_{sq}(K_{12},q_{12})$ vs. K_{12} vs. q_{12} - flow effects



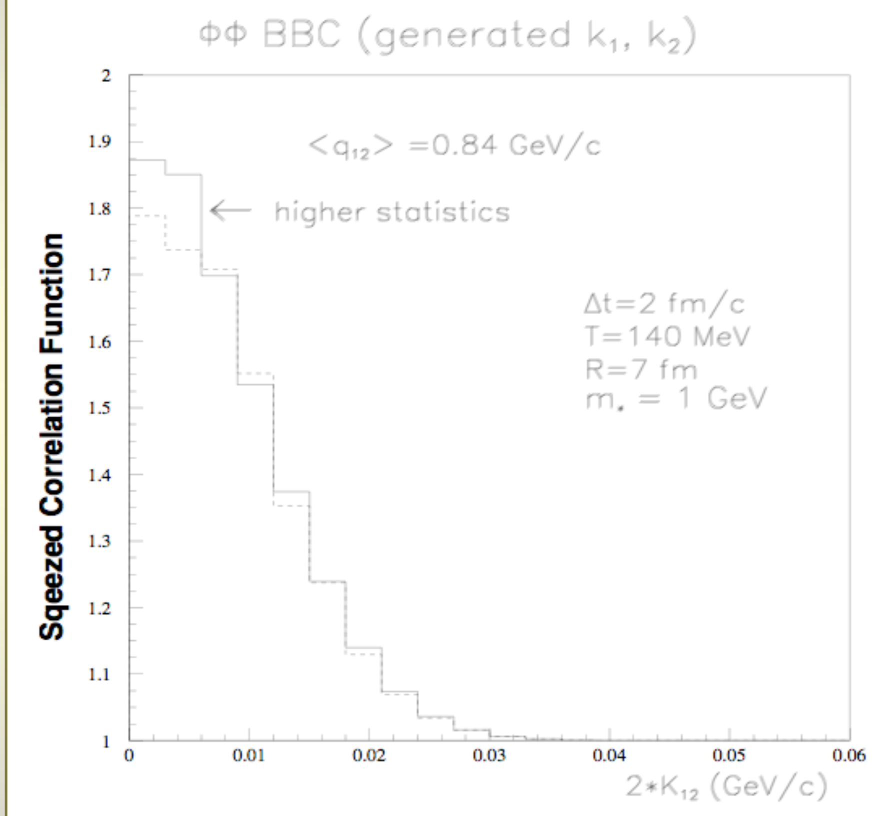


- Flow clearly has an effect:
  - For $<u>=0$ → C_s decreases fast for increasing q_{12} ;
    $<u>=0.5$ → C_s decreases more slowly
  - Flow enhances and extends the signal to broader region (K_{12},q_{12})

Simulation: $C_s(k_1, k_2)$ - preliminary

- Squeezed Correlation as function of $2{*}K_{12}$:

- $\Delta t = 2$ fm/c
- $T = 140\ MeV$
- $R = 7$ fm/c
- m_*



Bose-Einstein Correlations

- The complete correlation function of ϕ's have an identical-particle term ($\phi\ \phi$), reflecting their Bose-Einstein nature
- In certain regions of the $(\vec{K}_{12}, \vec{q}_{12})$ → B-E correlation dominates
- Would the mass-shift have any effect in the ϕ-ϕ identical particle correlation? A: YES! (although weaker than in the particle-antiparticle case)

HBT correlation function

- Effects of squeezing on the Chaotic (HBT) Correlation Function

$\vec{q} = \vec{k}_1 - \vec{k}_2$ $\quad 2 * \vec{K} = \vec{k}_1 + \vec{k}_2$

$$G_c(k_1,k_2) = \frac{E_{1,2}}{(2\pi)^{3/2}} \left\{ R^3 \left|s_{12}\right|^2 \exp\left(-\frac{R^2(k_1-k_2)^2}{2}\right) + n_0^* R_*^3 \left(\left|c_{12}\right|^2 + \left|s_{12}\right|^2\right) \exp\left(-\frac{(k_1+k_2)^2}{8m_*T_*}\right) \times \exp\left[\left(-\frac{im\langle u\rangle R}{2m_*T_*}\right)(k_1^2-k_2^2)\right] \exp\left[-\left(\frac{1}{8m_*T} + \frac{R_*^2}{2}\right)(k_1-k_2)^2\right] \right\}$$

(between $\vec{K}$ and $\vec{q}$) Θ ← $2 * \vec{K} \bullet \vec{q}$ $\quad \vec{q} = \vec{k}_1 - \vec{k}_2$

$2 * \vec{K} = \vec{k}_1 + \vec{k}_2$, $\vec{q} = \vec{k}_1 - \vec{k}_2$

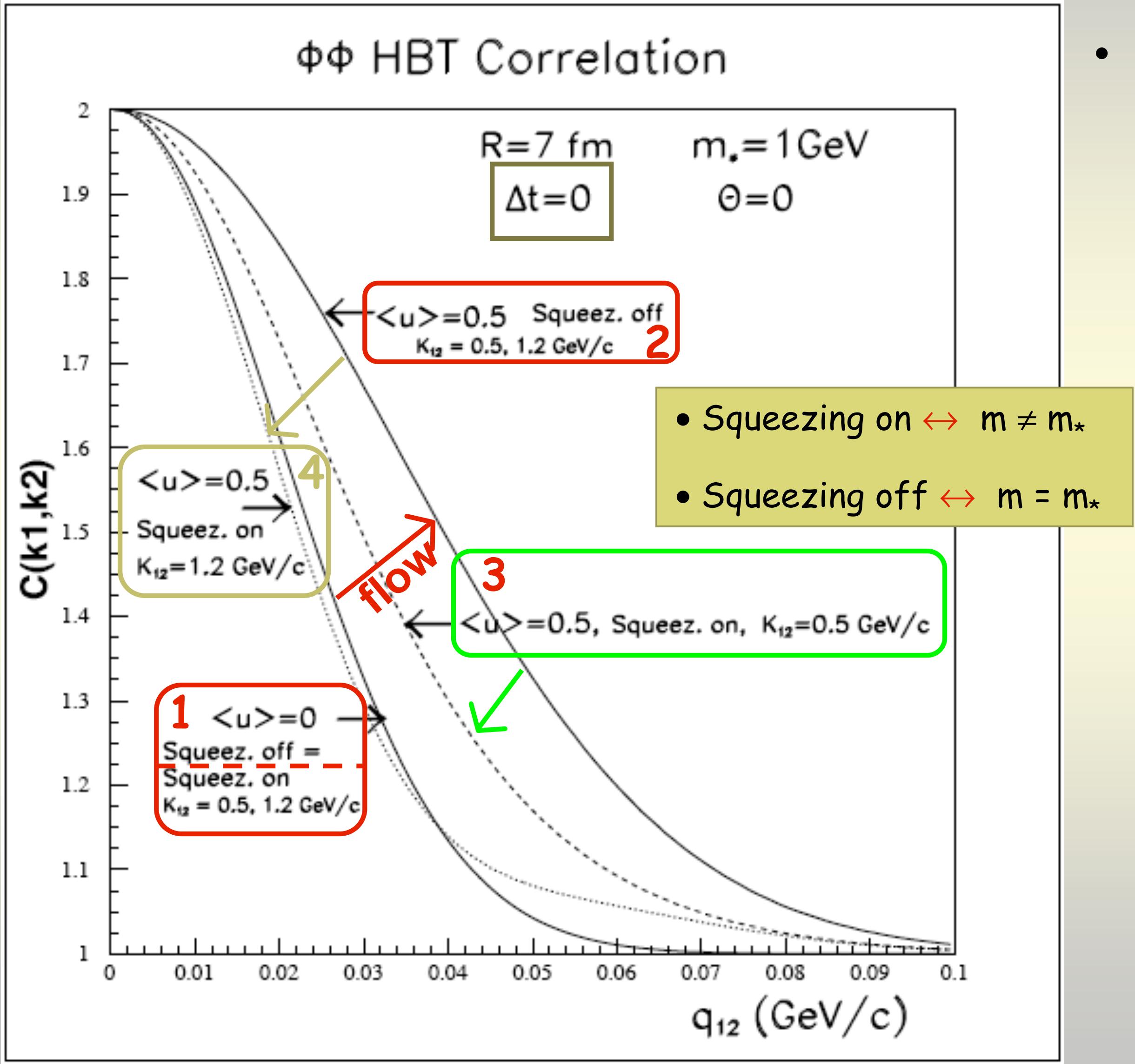
$$G_c(k_i) = \frac{E_{i,i}}{(2\pi)^{3/2}} \left\{ \left|s_{ii}\right|^2 R^3 + n_0^* R_*^3 \left(\left|c_{ii}\right|^2 + \left|s_{ii}\right|^2\right) \exp\left(-\frac{k_i^2}{2m_*T_*}\right) \right\}$$

$$R_* = R\sqrt{\frac{T}{T_*}}$$

$$C_c(\vec{k}_1,\vec{k}_2) = 1 + \frac{\left|G_c\left(\vec{k}_1,\vec{k}_2\right)\right|^2}{G_c\left(\vec{k}_1,\vec{k}_1\right)G_c\left(\vec{k}_2,\vec{k}_2\right)}$$

$$T_* = \left(T + \frac{m^2}{m_*}\langle u\rangle^2\right)$$

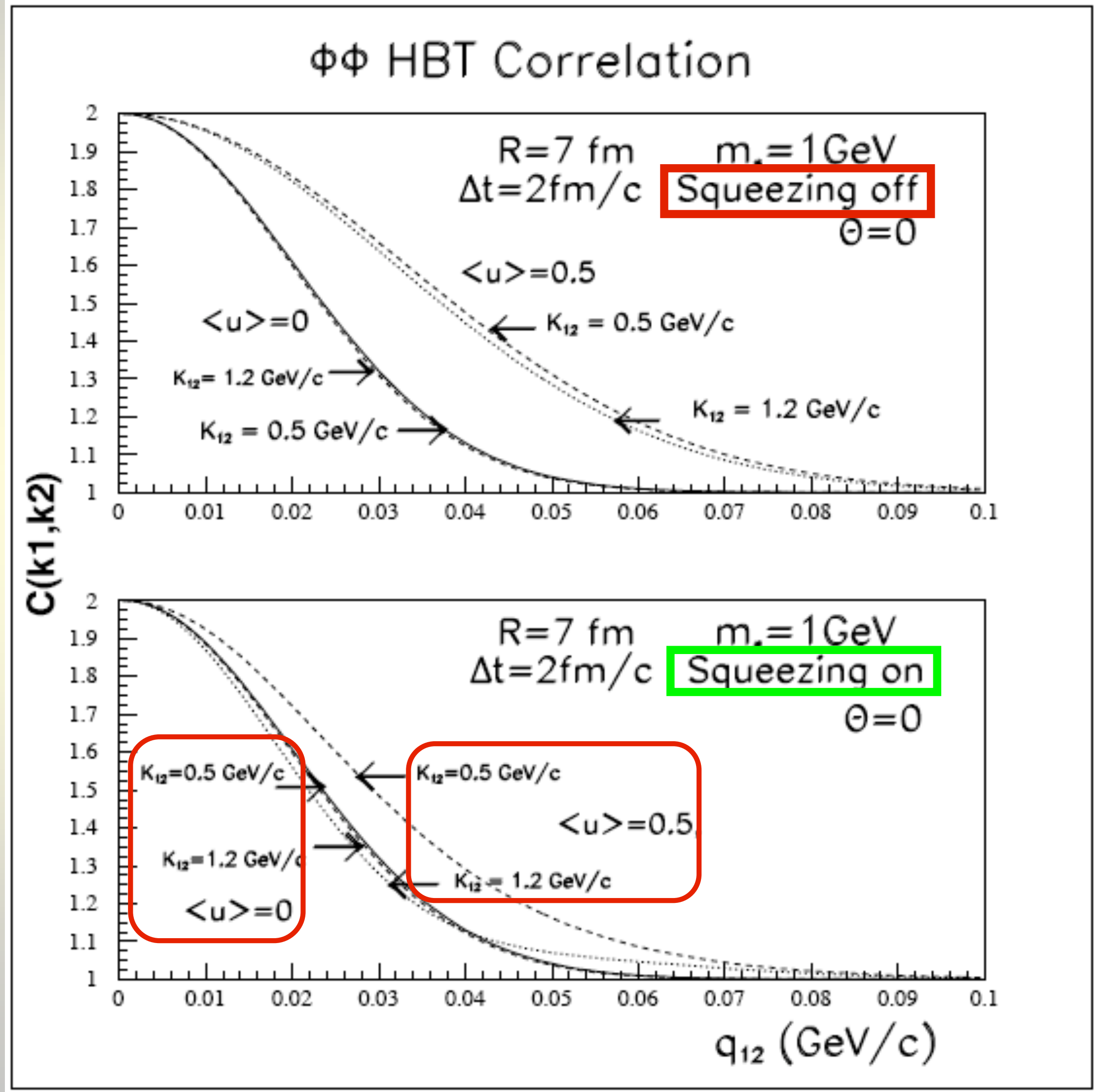
ϕϕ-HBT Correlations - $\Delta t=0$ - dependence on the average energy K_{12}



- For illustration:
  - Instant freezout ($\Delta t=0$)
  - No squeezing → correlation width increases (curve broadens)
  - Effects of squeezing:
    - Opposes those of flow (curves narrower)
    - effects more pronounced for increasing K_{12}

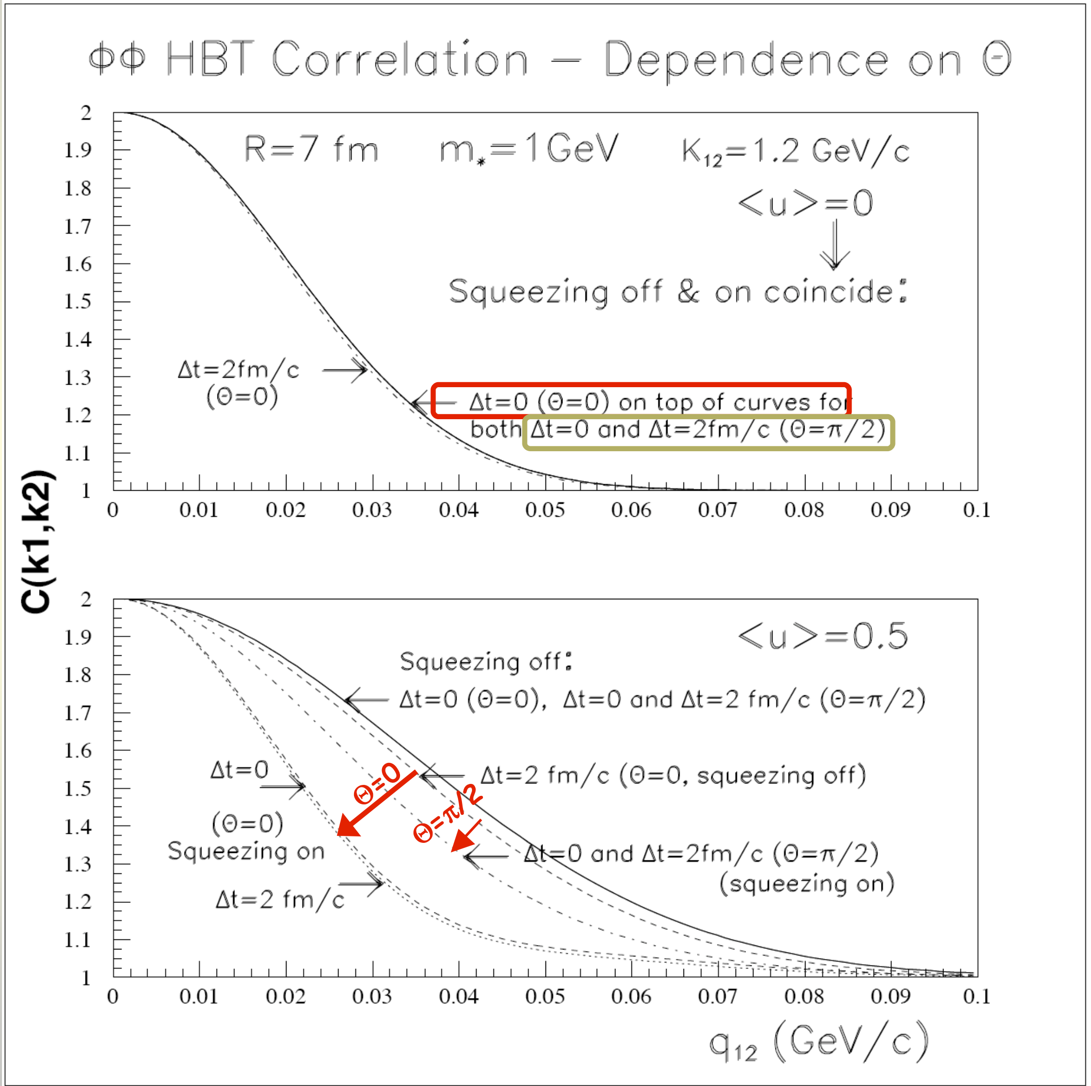
φφ-HBT Correlations - $\Delta t=2$ fm/c - dependence on the average energy K_{12}

- Similar to previous:
  - But finite freezout ($\Delta t=2$ fm/c)
  - Slight difference even at $<u>=0$
  - Same qualitative difference as for $\Delta t=0$: squeezing opposes to the flow effect, reducing the width



Dependence on Θ - angle($\vec{K}_{12}, \vec{q}_{12}$)

- Conclusions:
  - Very small sensitivity to squeezing at Θ=0 and $<u>=0$
  - Flow amplifies the differneces → sizeable for $<u>=0.5$
  - No sensitivity to time for Θ=π/2 (as expected)
  - Average over Θ → significant difference between no squeezing and squeezing on



Summary and Conclusions

- Brief review of squeezed correlations
- And of the most important results of the model (in a non-relativistic treatment of expanding finite systems)
- Suggestion of suitable variables to use in the experimental search of the BBC's:
  $C_s\ (K_{12},\ q_{12})$ vs. (2^*K_{12}) vs q_{12} or in invariant tersm:

$$Q^2_{bbc} = -\,(Q^{back}_{inv})^2 = 4(\omega_1\omega_2 - K^{\mu}K_{\mu})$$

- Showed some preliminary results on the expected behavior of the $C_s(k_1,k_2)$ & $C_c(k_1,k_2)$ vs. (2^*K_{12}) vs q_{12}

  Just a detail missing: experimental discovery!

- Let's find it now! And show it at the next QM 2009!!

Acknowledgments

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EXTRAS

Formalism (bosons)

- Infinite medium

$$H = H_0 - \frac{1}{2}\int d\vec{x}\, d\vec{y}\, \phi(\vec{x})\, \delta M^2(\vec{x}-\vec{y})\phi(\vec{y})$$ → In-medium Hamiltonian

$$H_0 = \frac{1}{2}\int d\vec{x}\, (\dot{\phi}^2 + |\nabla\phi|^2 + m^2\phi^2)$$ → Asymptotic (free) Hamiltonian, in the rest frame of matter

- Scalar field $\phi(x)$ → quasi-particles propagating with momentum-dependent medium-modified effective mass, m_*, related to the vacuum mass, m, by

$$m_*^2\left(\left|\vec{k}\right|\right) = m^2 - \delta M^2\left(\left|\vec{k}\right|\right)$$

- Consequently:

Ω_k → frequency of the in-medium mode with momentum $\vec{k}$

$$\Omega_k^2 = m_*^2 + \vec{k}^2 = \omega_k^2 - \delta M^2\left(\left|\vec{k}\right|\right)$$

Formalism (fermions)

$$H = H_0 + H_I \quad ; \quad H_0 = \int d\vec{x} : \bar{\psi}(x)(-i\vec{\gamma}.\vec{\nabla} + M)\psi(x):$$

$$\psi(x) = \frac{1}{V} \sum_{\lambda,\lambda',\vec{k}} (u_{\lambda,\vec{k}} a_{\lambda,\vec{k}} + v_{\lambda',-\vec{k}} a^{\dagger}_{\lambda',-\vec{k}}) e^{i\vec{k}.\vec{x}}$$

$$\langle a^{\dagger}_{k_1} a^{\dagger}_{k_2} a_{k_1} a_{k_2} \rangle = \langle a^{\dagger}_{k_1} a_{k_1} \rangle \langle a^{\dagger}_{k_2} a_{k_2} \rangle - \langle a^{\dagger}_{k_1} a_{k_2} \rangle \langle a^{\dagger}_{k_2} a_{k_1} \rangle + \langle a^{\dagger}_{k_1} a^{\dagger}_{k_2} \rangle \langle a_{k_1} a_{k_2} \rangle$$

- System described by quasi-particles → medium effects taken into account through self-energy function
- For spin-1/2 particles under mean fields in a many body system:

$$\sum{}^{s} + \gamma^0 \sum{}^{0} + \gamma^i \sum{}^{i}$$

→ **to be determined by detailed calculation**

- Σ^s → notation: $\Sigma^s(k) = \Delta M(k)$
- Σ^1 → very small → neglected
- Σ^0 → weakly-dependent on momentum → totally thermalized medium: $\mu_* = \mu - \Sigma^0$ → (results for net barion number)
- Hamiltoniana H_1 → describes a system of quasi-particles with mass- dependent momentum $m_* = m - \Delta M(k)$

Correlation for strict BBC pairs

- Momenta of the pair

Remember:

$$k_2 = -k_1 = k$$

$$2 * K^{\mu}_{i,j} = (k_i + k_j) \quad ; \quad q^{\mu}_{i,j} = (k_i - k_j)$$

- Back-to-Back correlation function

$$C_s(k,-k) = 1 + \left\{ |c_0||s_0| \left[R^3 + 2\left(\frac{R^2}{(1+\frac{m^2\langle u\rangle^2}{m_* T})} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T} \right) \right] \right\}^2 \times$$

$$\left\{ |s_0|^2 R^3 + \left(|c_0|^2 + |s_0|^2 \right) \left(\frac{R^2}{(1+\frac{m^2\langle u\rangle^2}{m_* T})} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T} + \frac{m^2\langle u\rangle^2 k^2 / m_*^2}{(1+\frac{m^2\langle u\rangle^2}{m_* T})2T^2} \right) \right\}^{-2}$$

Formalism for treating finite expanding systems

- For a hydrodynamical ensemble → amplitudes can be written as
  [Makhlin & Sinyukov, N.P. A566 (1994) 598c]

$$G_c(1,2) = \frac{1}{(2\pi)^3} \int d^4\sigma_\mu(x)\, K^\mu_{1,2}\, e^{iq_{1,2}.x} \left[\left|c_{1,2}\right|^2 n_{1,2} + \left|s_{-1,-2}\right|^2 \left(n_{-1,-2} + 1\right) \right]$$

$$G_s(1,2) = \frac{1}{(2\pi)^3} \int d^4\sigma_\mu(x)\, K^\mu_{1,2}\, e^{i2K_{1,2}.x} \left[s^*_{-1,2}\, c_{2,-1}\, n_{-1,2} + c_{1,-2}, 1\, s^*_{-2,1} \left(n_{1,-2} + 1\right) \right]$$

$$2 * K^\mu_{i,j} = (k_i + k_j)$$

$$q^\mu_{i,j} = (k_i - k_j)$$

Full correlation function - 1

- Estimate for Gaussian-type momentum-dependent mass shift

(by Asakawa, Csörgő and Gyulassy)

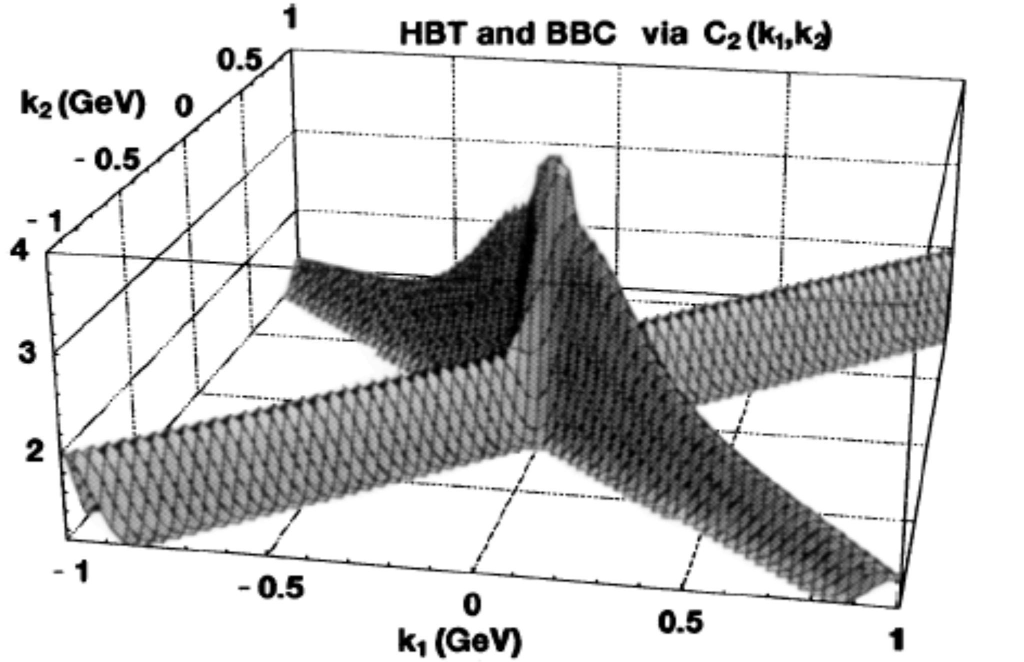

FIG. 2. Schematic illustration of the new kind of correlations for mass-shifted π^0 pairs, assuming $T = 140$ MeV, $G_c \sim \exp[-q_{12}^2 R_G^2/2]$, $G_s \sim \exp[-2K_{12}^2 R_G^2]$, with $R_G = 2$ fm. The fall of the BBC for increasing values of $|\mathbf{k}|$ is controlled here by a momentum-dependent effective mass, $m_\pi^* = m_\pi[1 + \exp(-\mathbf{k}^2/\Lambda_s^2)]$, with $\Lambda_s = 325$ MeV in the sudden approximation. Without the Λ_s cutoff, the BBC would increase indefinitely as $|\mathbf{k}| \longrightarrow \infty$.

- Scalar field $\phi(x)$ →

Full correlation function - 2

- Expectation with the simple momentum-independent model discussed here (squeezed correlation is enhanced at large values of the individual momenta)

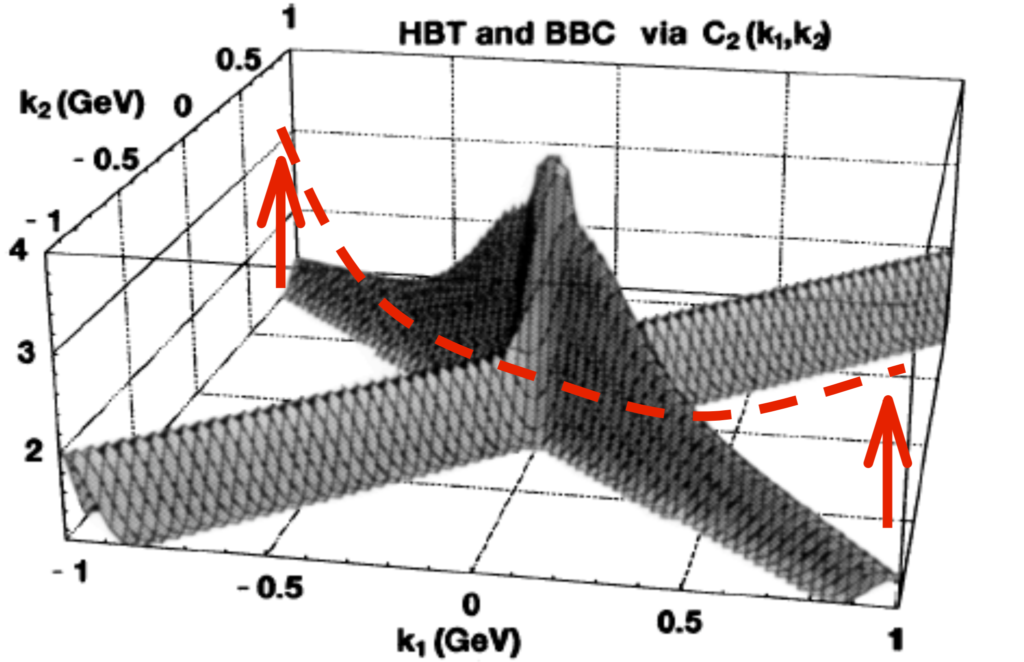

FIG. 2. Schematic illustration of the new kind of correlations for mass-shifted π^0 pairs, assuming $T = 140$ MeV, $G_c \sim \exp[-q_{12}^2 R_G^2/2]$, $G_s \sim \exp[-2K_{12}^2 R_G^2]$, with $R_G = 2$ fm. The fall of the BBC for increasing values of $|\mathbf{k}|$ is controlled here by a momentum-dependent effective mass, $m_\pi^* = m_\pi[1 + \exp(-\mathbf{k}^2/\Lambda_s^2)]$, with $\Lambda_s = 325$ MeV in the sudden approximation. Without the Λ_s cutoff, the BBC would increase indefinitely as $|\mathbf{k}| \to \infty$.

$C_s(K_{12}, q_{12})$ vs. 2^*K (vs q) slices