Density Fluctuations as Signature of a Non–Equilibrium First Order Phase Transition

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in collaboration with

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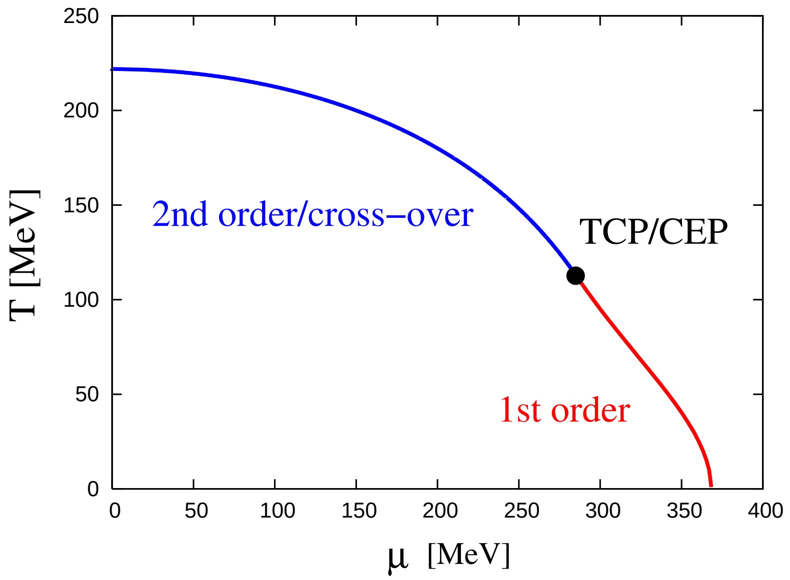
references:

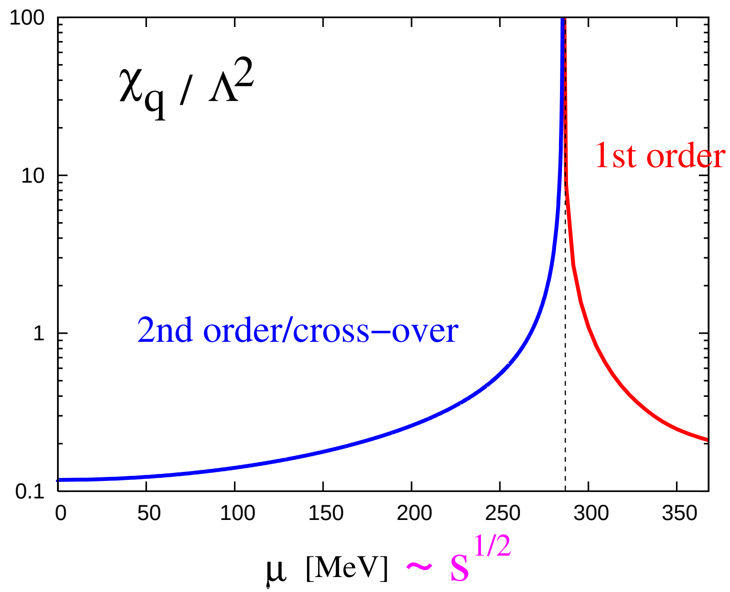
- Phys. Rev. Lett. **99**, 232301 (2007)
- Phys. Rev. D **75**, 054026 (2007)
- arXiv:0712.2761 [hep-ph], to appear in Phys. Rev. D

QCD phase structure and conserved charge fluctuations

- conserved charge fluctuations [Stephanov, Rajagopal and Shuryak (98,99)]
  baryon number, electric charge etc. : accessible in heavy-ion experiments
- net baryon number fluctuations χ_B
  lattice QCD, effective model calculations, universality arguments:

Gottlieb et al. (87); Kunihiro (91); Stephanov et al. (98); Gavai and Gupta (02);
D'Elia and Lombardo (03); Hatta and Ikeda (03); Fujii and Ohtani (04); Allton et al. (05);
Shaefer and Wambach (06); Sasaki el at. (06); Ratti et al. (06,07)

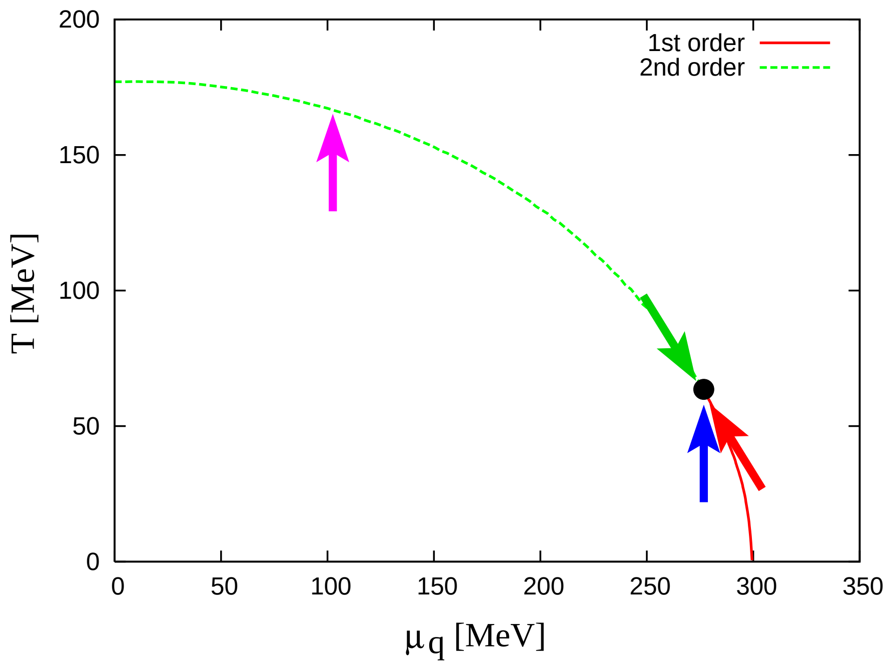


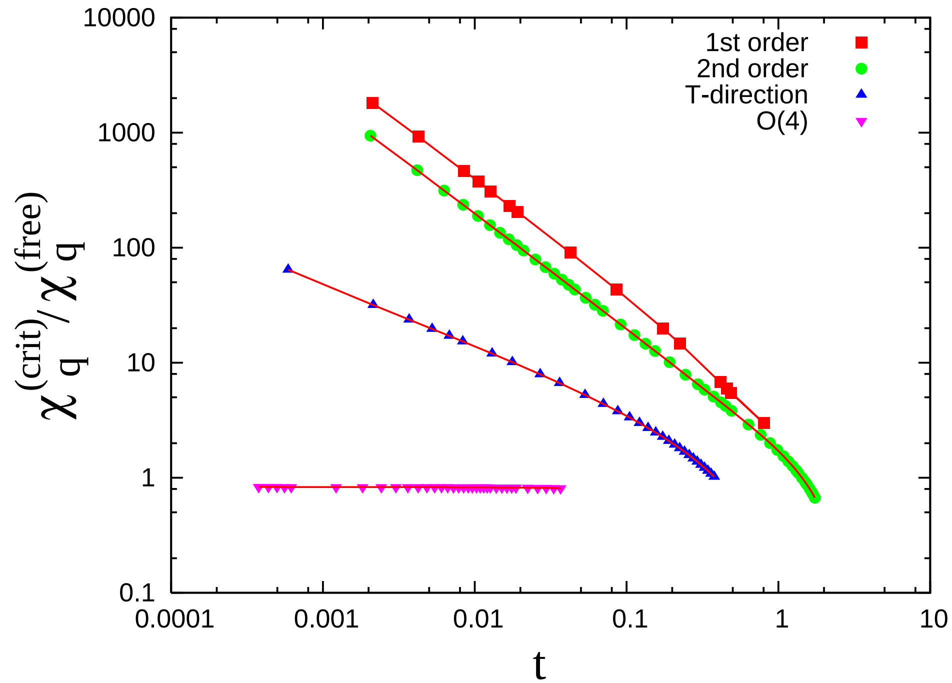


χ_B is an important observable to search for CP in QCD phase diagram

Scaling properties and critical region of χ_B

- MF critical exponents [NJL model & Landau theory: Sasaki, Friman and Redlich, PRD (07)]



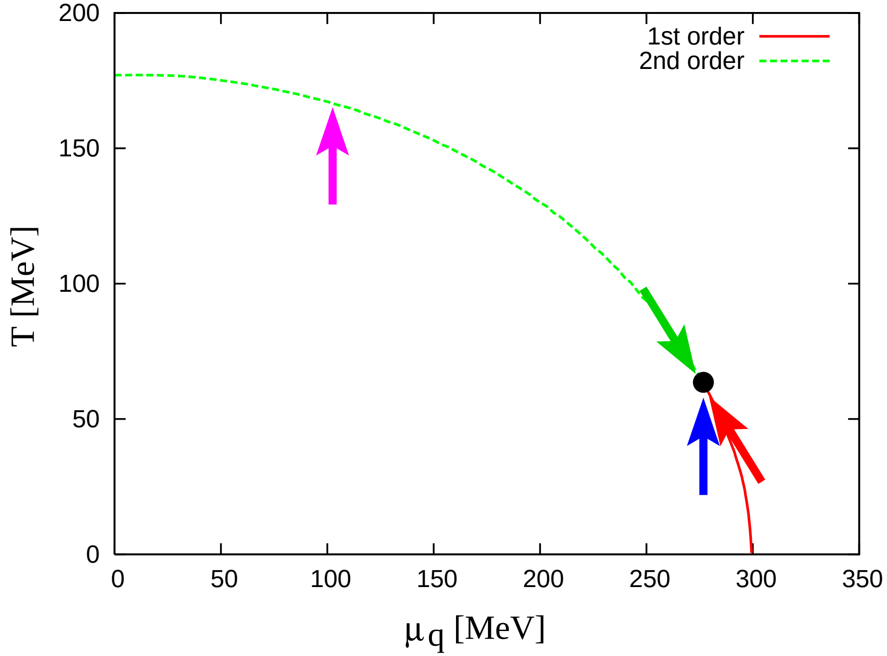


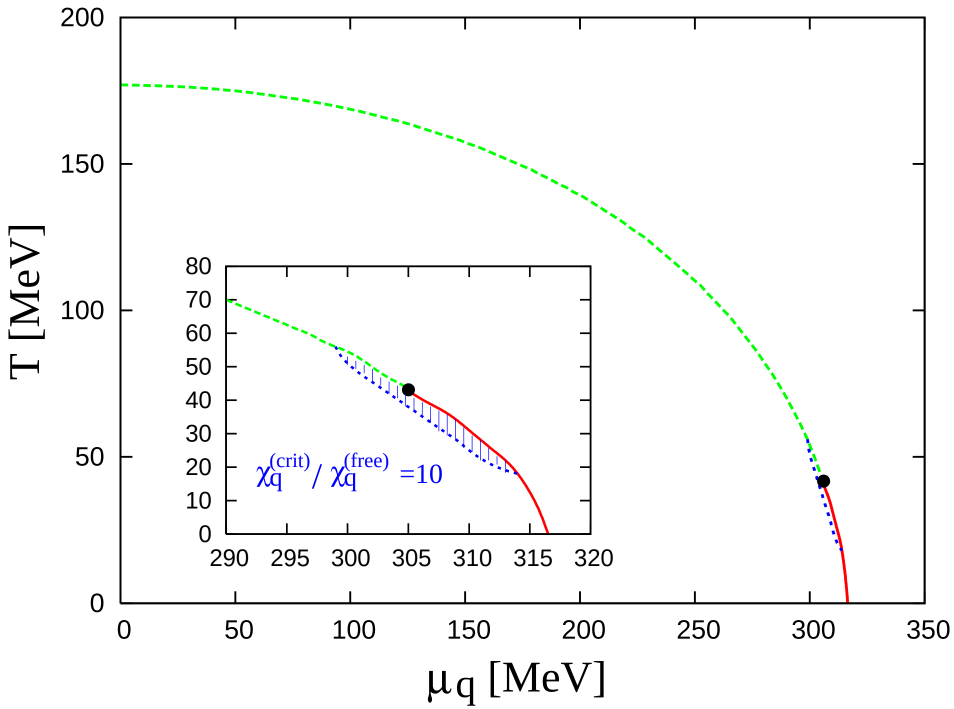
– depending on paths approaching CP [Griffiths, Wheeler (70); Hatta, Ikeda (03)]

$$\chi_q \sim \begin{cases} t^{-1} & \text{for 1st order} \\ \frac{1}{2}\,t^{-1} & \text{for 2nd order} \\ t^{-1/2} & \text{for any paths not tangential to boundary} \end{cases}$$

Scaling properties and critical region of χ_B

- MF critical region [NJL model & Landau theory: Sasaki, Friman and Redlich, PRD (07)]





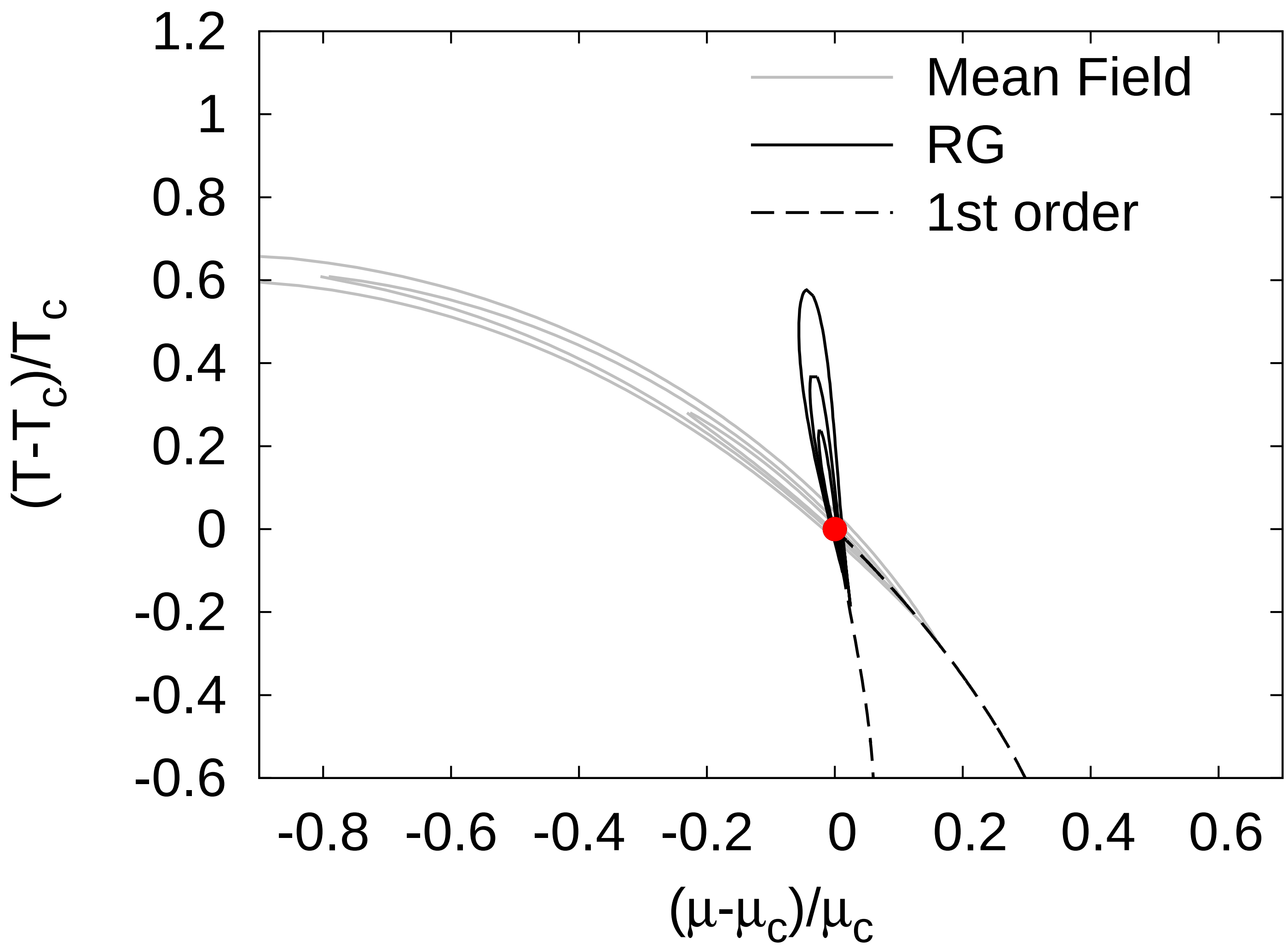
– definition of "critical region" : $\chi_q/\chi_q^{(\mathrm{free})} = 10$

– elongated due to different exponents

– narrow critical region

Scaling properties and critical region of χ_B

- MF vs. beyond MF : [RGE + quark-meson model: Schaefer and Wambach, PRD (07)]

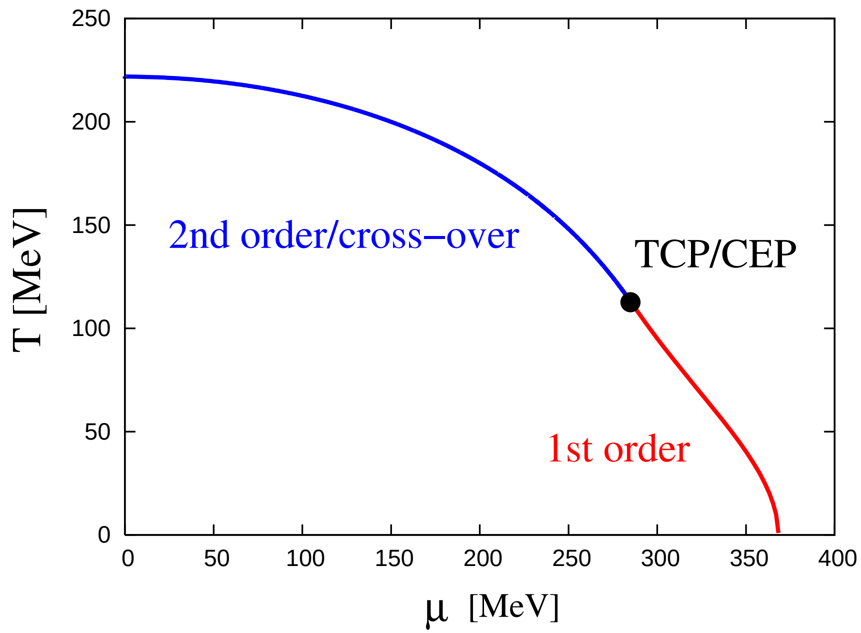


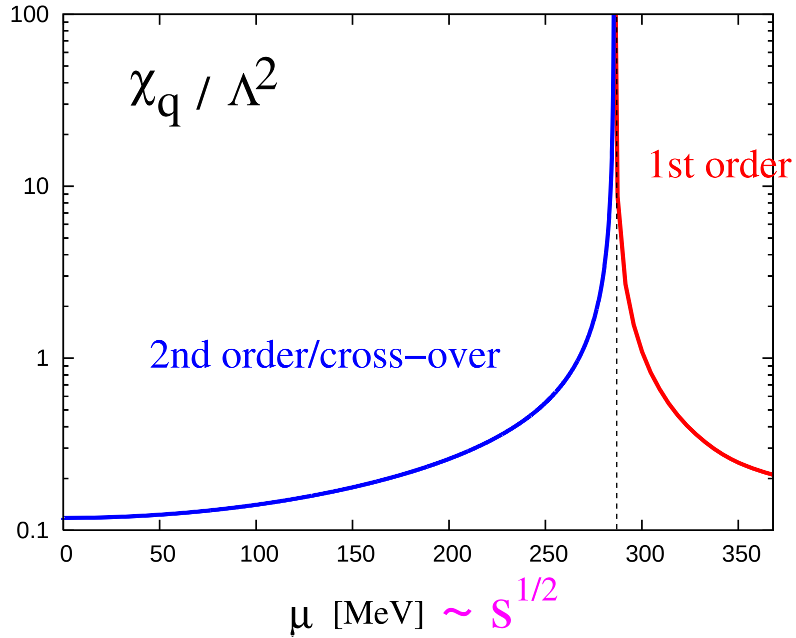
– critical region compressed with quantum fluctuations

Equilibrium vs. Non-equilibrium

- net quark number susceptibility : search for the CEP

e.g., NJL model calculation : C. S., B. Friman, K. Redlich, Phys. Rev. D, 2007





$\chi_q \to \infty$ at CEP while $\chi_q \sim$ finite for the 1st order transition in equilibrium

- heavy-ion collisions : deviation from equilibrium

spinodal decomposition for the χ/deconfinement phase transitions

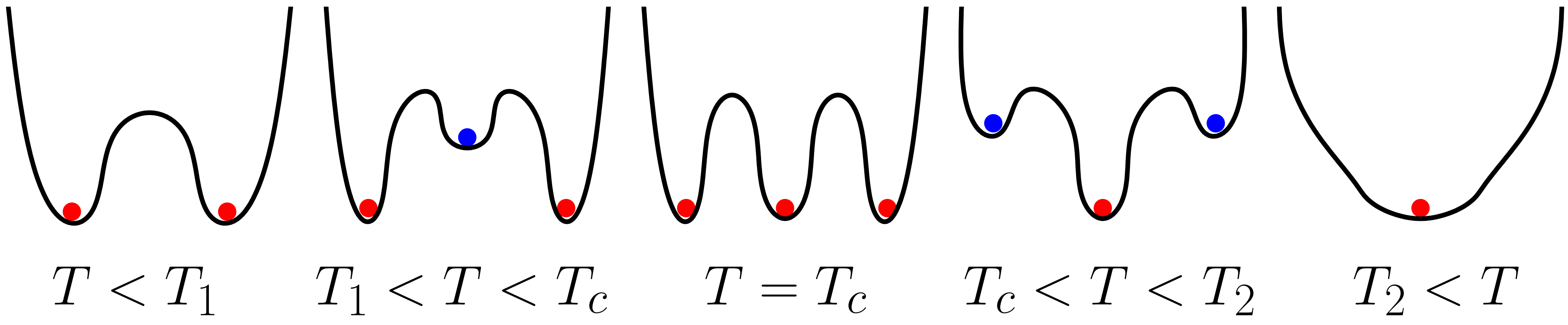
$\cdots$ instabilities, enhancement of baryon and strangeness fluctuations

Heiselberg et al. (88); Bower, Gavin (01); Chomaz et al. (04); V. Koch et al. (05)

a key question : critical behavior of charge fluctuations?

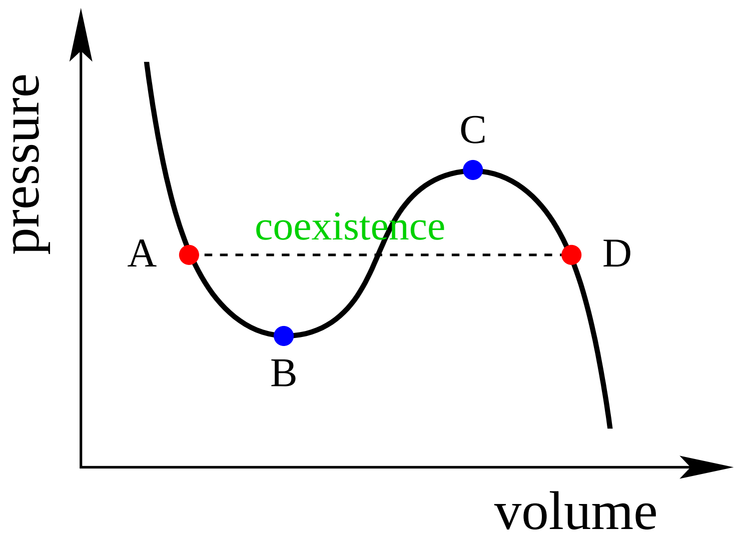
The nature of first order phase transition

- change of thermodynamic potential from broken to symmetric phase



appearance of meta-stable state in $T_1 < T < T_2$

- stability of a system



$\partial P / \partial V < 0$: stable
$\partial P / \partial V > 0$: unstable
$\partial P / \partial V = 0$: spinodals

A-B : supercooling (symmetric phase)
B-C : non-equilibrium state
C-D : superheating (broken phase)

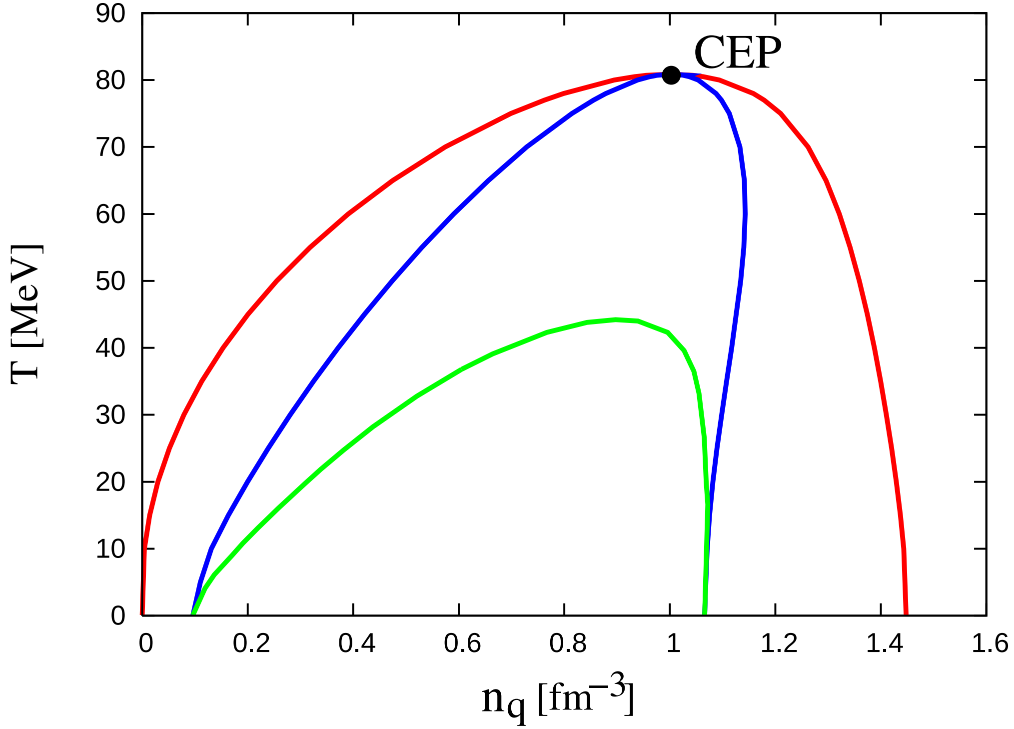
Phase diagram in the Nambu–Jona-Lasinio model

- NJL model with two flavors

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \bar{\psi}\mu\gamma_0\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\vec{\tau}\gamma_5\psi\right)^2\right]$$

$$m = 5.6\,\text{MeV}\,,\ G_S\Lambda^2 = 2.44\,,\ \Lambda = 587.9\,\text{MeV}$$

- phase diagram



critical point (CP) :
$T = 81$ MeV, $\mu = 330$ MeV

spinodal lines :

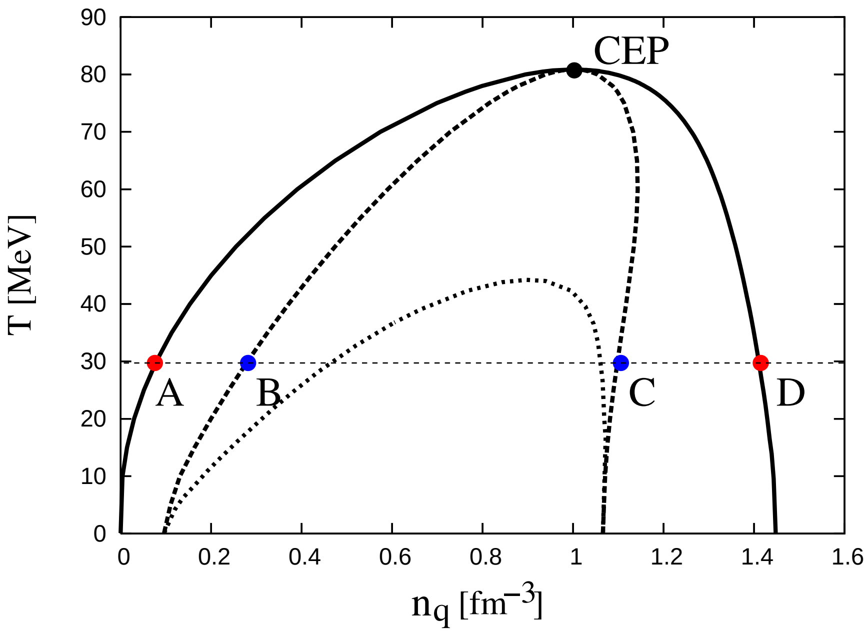
$$\left(\frac{\partial P}{\partial V}\right)_T = 0 \quad : \text{isothermal}$$

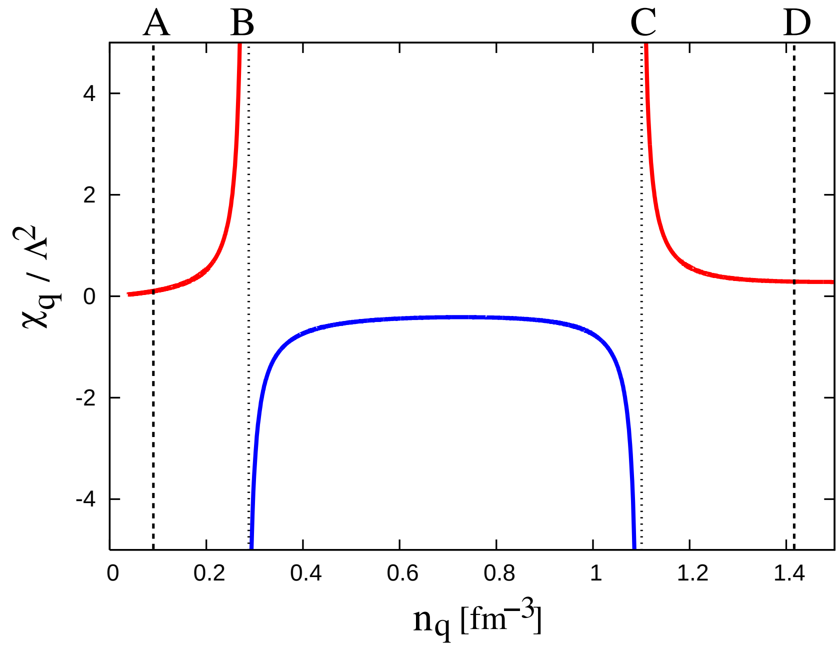
$$\left(\frac{\partial P}{\partial V}\right)_S = 0 \quad : \text{isentropic}$$

$$\left(\frac{\partial P}{\partial V}\right)_T = \left(\frac{\partial P}{\partial V}\right)_S + \frac{T}{C_V}\left[\left(\frac{\partial P}{\partial T}\right)_V\right]^2$$

Quark number susceptibility

- deviation from equilibrium, large fluctuations induced by instabilities





- at 1st order transition point (A, D) : χ_q is finite
- at isothermal spinodal point (B, C) : χ_q diverges and changes its sign
  $\frac{\partial P}{\partial V} < 0$ for stable/meta-stable state $\Rightarrow$ $\frac{\partial P}{\partial V} > 0$ for unstable state
- in unstable region (B-C) : χ_q is finite and negative

- divergence of χ_q :

$$\left(\frac{\partial P}{\partial V}\right)_T = \left(\frac{\partial P}{\partial \mu}\right)_T \Big/ \left(\frac{\partial V}{\partial \mu}\right)_T$$

$$= -\frac{n_q^2}{V}\frac{1}{\chi_q} \quad = 0 \quad \text{at any spinodals}$$

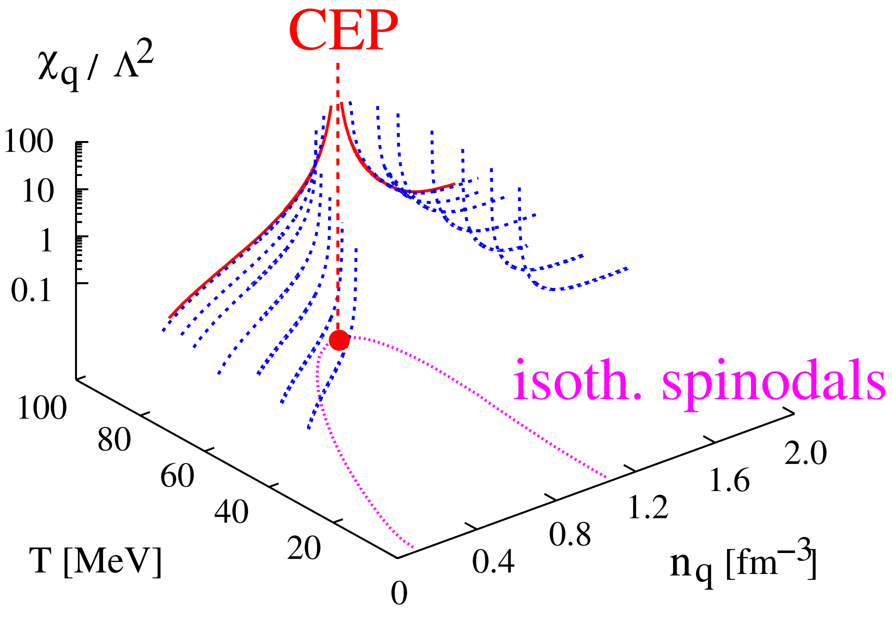
$$\Downarrow$$

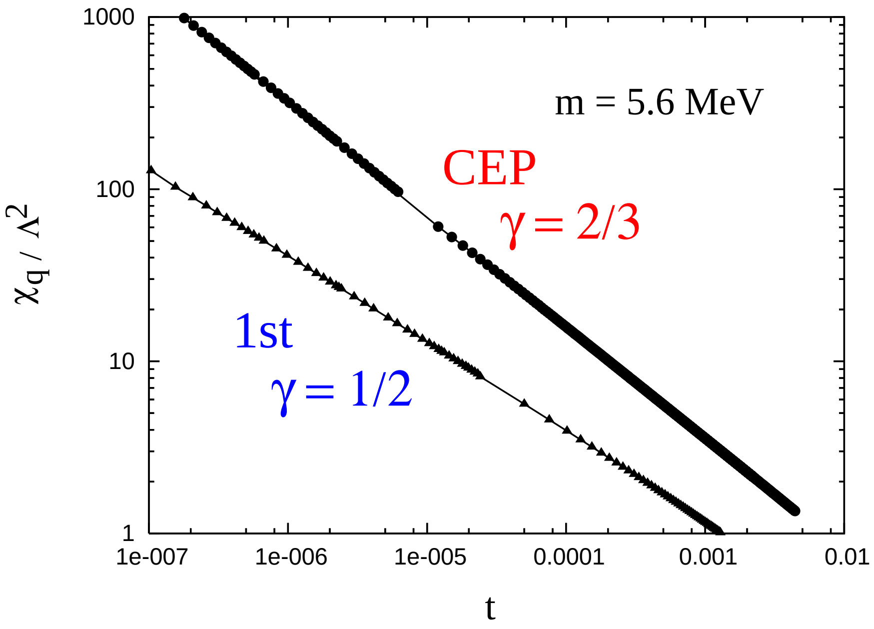
χ_q diverges and changes its sign.

- divergence in electric charge susceptibility:

$$\chi_Q = \frac{1}{36}\chi_q + \frac{1}{4}\chi_I + \frac{1}{6}\frac{\partial^2 P}{\partial \mu_q \partial \mu_I}$$

- toward the critical point





– two positive branches are approaching

– instability region shrinks

– critical exponents : $\chi_q \sim t^{-\gamma}$, $t = (\mu - \mu_c)/\mu_c$

$\gamma = 2/3$ (CP), $\gamma = 1/2$ (1st) : justified in the GL theory

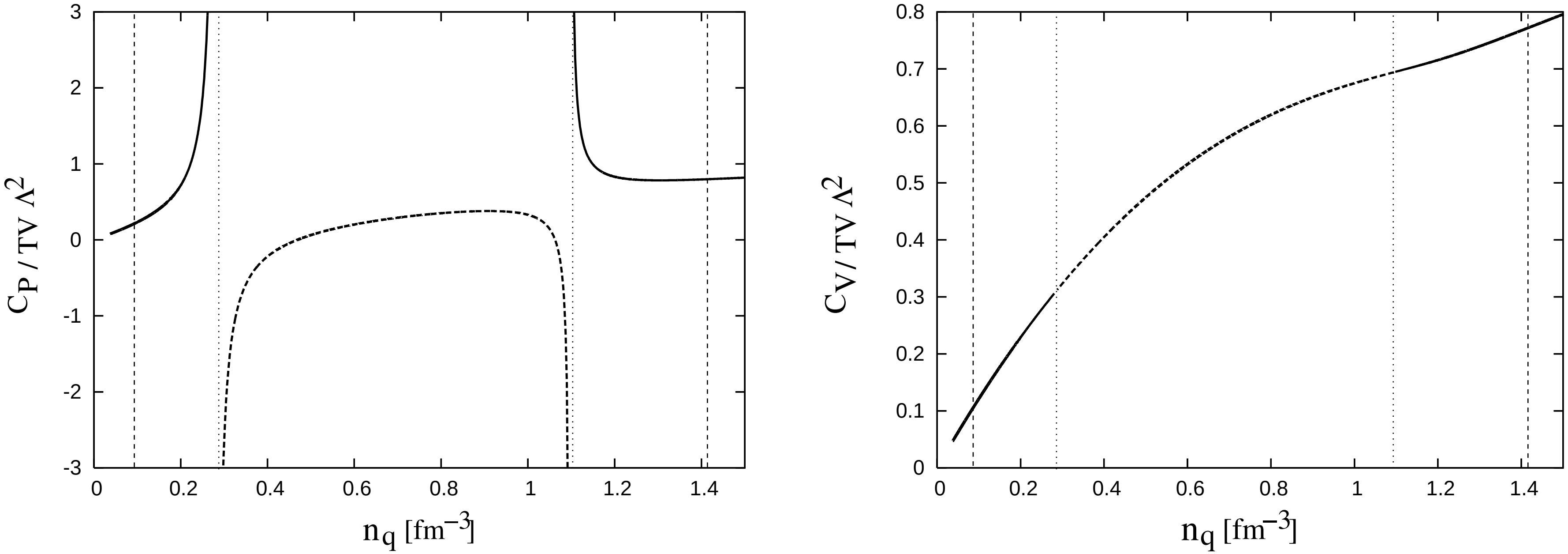
$\Rightarrow$ different universality class

[C.S., B.Friman, K.Redlich, arXiv:0712.2761 [hep-ph]]

- specific heat for constant pressure/volume

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = TV\left[\chi_{TT} - \frac{2s}{n_q}\chi_{\mu T} + \left(\frac{s}{n_q}\right)^2\chi_{\mu\mu}\right] \sim \text{divergent}$$

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V = TV\left[\chi_{TT} - \frac{\chi_{\mu T}^2}{\chi_{\mu\mu}}\right] \sim \text{finite} \qquad \left(\chi_{xy} = -\frac{\partial^2\Omega}{\partial x\partial y}\right)$$

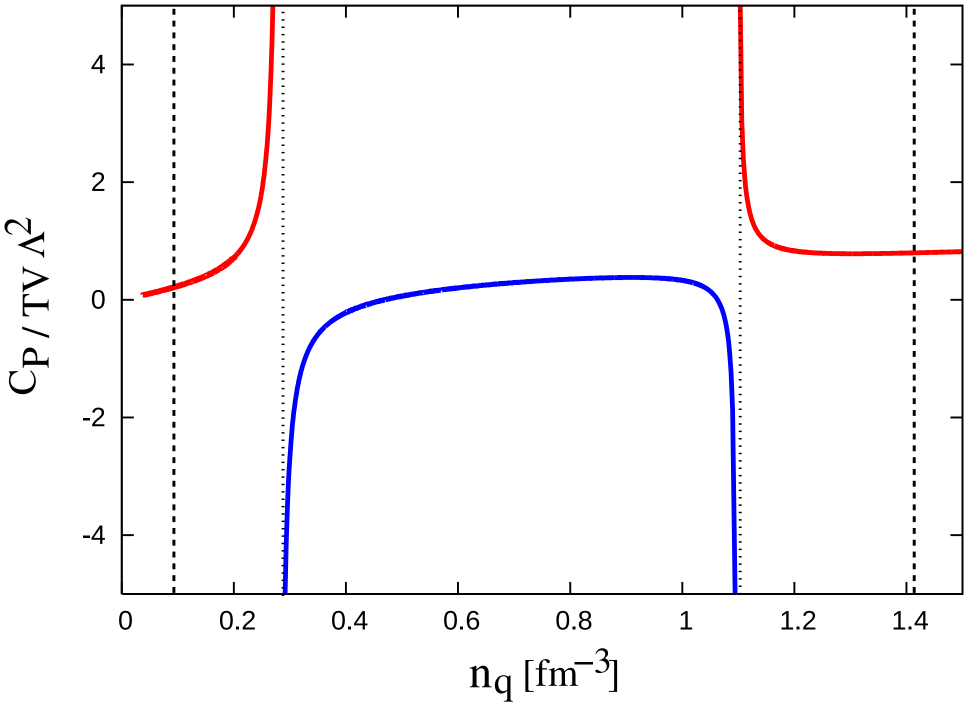


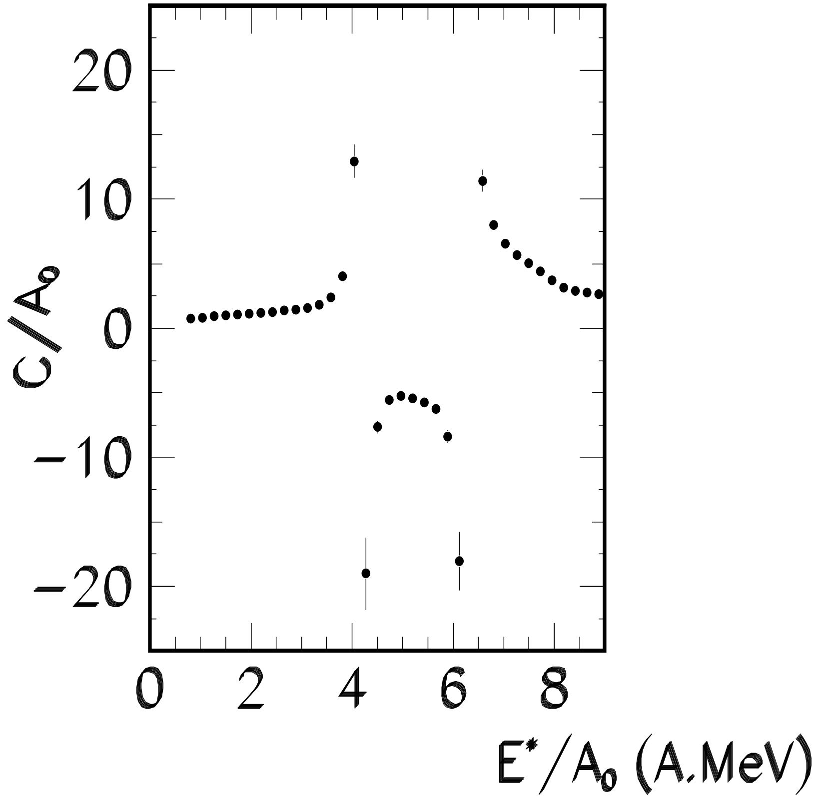
– NOTE : C_V is finite (MF) but divergent (beyond MF).

– specific heat can be a signal of the 1st order transition.

Experimental evidence

- low-energy nuclear collisions





$$C_P = \frac{C^2_{\text{can.exp.}}}{C_{\text{can.exp.}} - \sigma^2/T^2}$$

M. D'Agostino *et al.*, Phys. Lett. B **473**, 219 (2000)

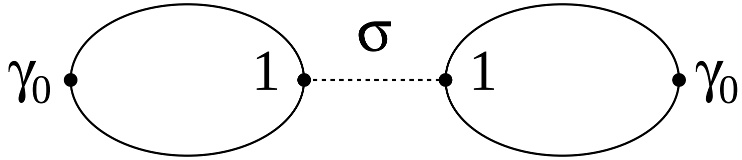
negative heat capacity : anomalously large fluctuations

$\Rightarrow$ an evidence of the 1st order liquid-gas phase transition

Relation of χ_B to chiral susceptibility

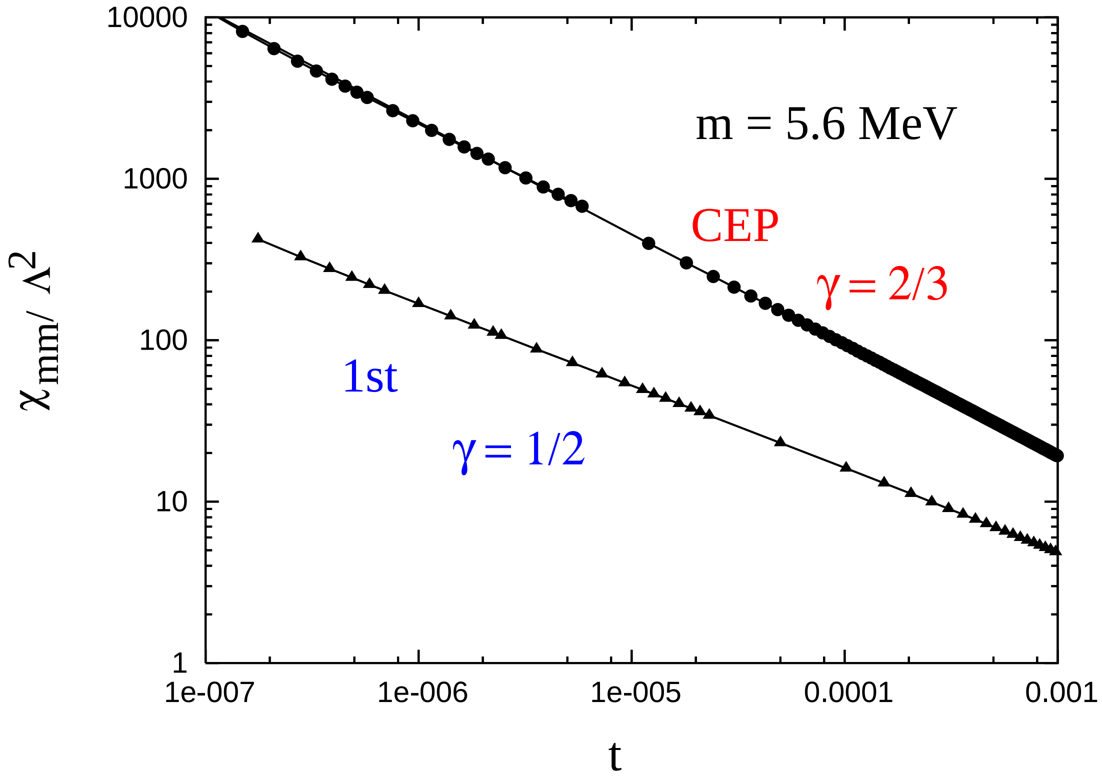
- singular part : scalar mode exchange

[Kunihiro (91); Hatsuda and Kunihiro (94)]



$$\chi_q = \chi_q^{(\text{regular})} + \underline{\chi_q}^{(\text{singular})}$$

$$\sim M^2 \chi_{\text{ch}} \sim M^2 / m_\sigma^2$$



– at CP/1st-order transitions $(M \neq 0)$: $\chi_q \sim \chi_{\text{ch}} \sim t^{-\gamma_{\text{ch}}}$

Summary and conclusions

- The quark number susceptibility diverges if spinodal phase separation occurs. (independently of any models)
  cf. finite in the equilibrium transition
- a signal not only for the CP but also for the 1st order phase transitions
  $\Rightarrow$ large fluctuations will be seen in a wider range of the phase diagram.
- J-PARC, FAIR/GSI energies : the 1st order phase transition
  large fluctuations of baryon number will be expected.