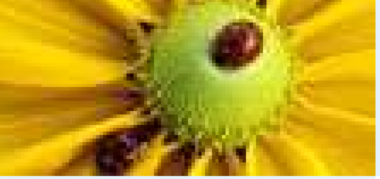
Parton picture for a strongly–coupled plasma

Edmond Iancu
IPhT Saclay & CNRS

Collaboration with Yoshitaka Hatta and Al Mueller

(arXiv:0710.2148 and 0710.5297 [hep-th])



Hard probes for sQGP

- QGP just above the phase transition: $T \simeq 2 \div 5\,T_c$
  - deconfined
  - nearly conformal: $(\mathcal{E} - 3P)/\mathcal{E} \lesssim 20\%$
  - strongly coupled: $\lambda \equiv g^2 N_c \approx 3 \div 6$

    (RHIC data: v_2, low viscosity, early thermalization)
- a cousin of the strongly-coupled $\mathcal{N} = 4$ SYM plasma

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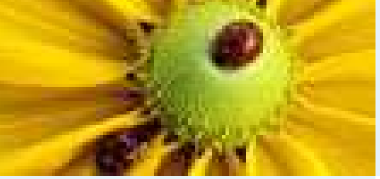
- AdS/CFT –based studies of medium properties ...
  - thermodynamics, hydrodynamics, transport coefficients
- ... and also of 'hard probes' : $\omega,\ q\ \gg\ T$
  - energy loss, momentum broadening, meson dissociation
- How does a strongly–coupled plasma look, when probed on short distances and at high energies ?



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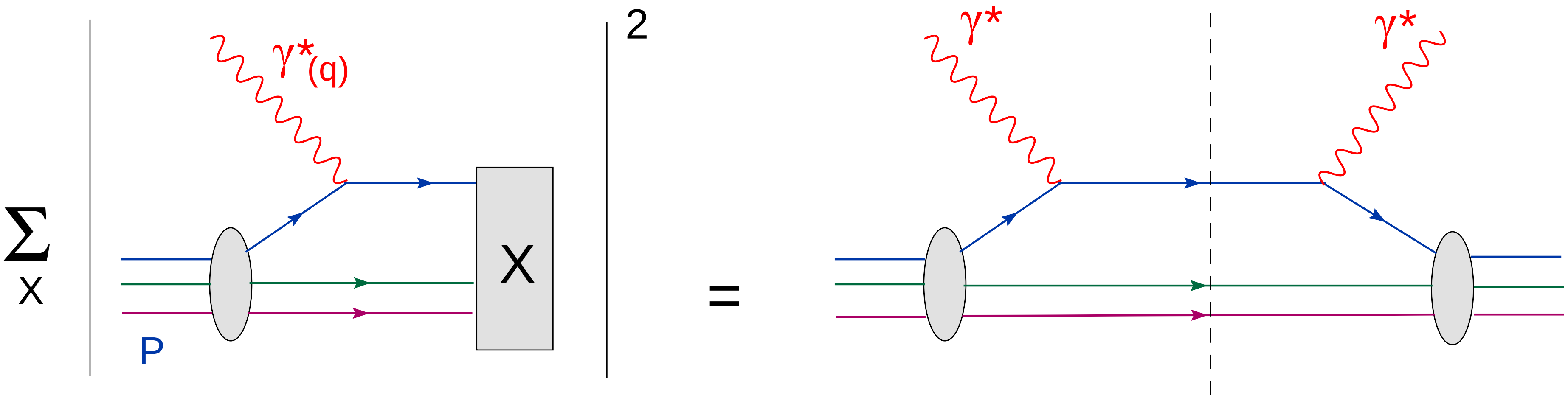
- DIS: the best suited (Gedanken) experiment to measure this

The proton structure functions



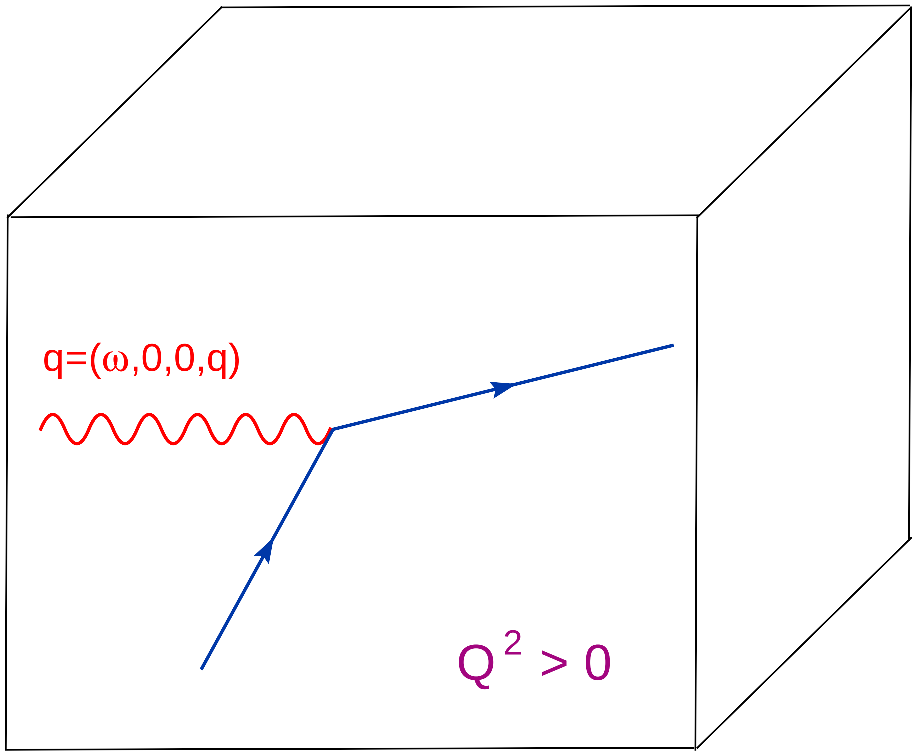
$$F_{1,2}(x,Q^2) \sim \mathrm{Im}\int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\,\langle P|\,[J_\mu(x),J_\nu(0)]\,|P\rangle$$

- Virtual photon absorbed by a quark excitation with
  - transverse size $\Delta x_\perp \sim 1/Q$, where $Q^2 \equiv -q^\mu q_\mu \geq 0$
  - longitudinal momentum $k_z = xP$, where $x \equiv \frac{Q^2}{2P\cdot q} \simeq \frac{Q^2}{s}$
- $F_2(x,Q^2)$: the quark distribution in the proton

DIS off a $\mathcal{N} = 4$ SYM plasma

- An Abelian $\mathcal{R}$–current : $J_\mu(q) \propto \mathrm{e}^{-i\omega t + iqz}$ with $Q^2 = q^2 - \omega^2$

$$\Pi_{\mu\nu}(q) \equiv \int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, i\theta(x_0)\, \langle\, [J_\mu(x), J_\nu(0)]\, \rangle_T$$



- $Q^2 > 0 \Longrightarrow$ DIS : $F_{1,2}(x, Q^2) \sim \mathrm{Im}\, \Pi_{\mu\nu}(q)$

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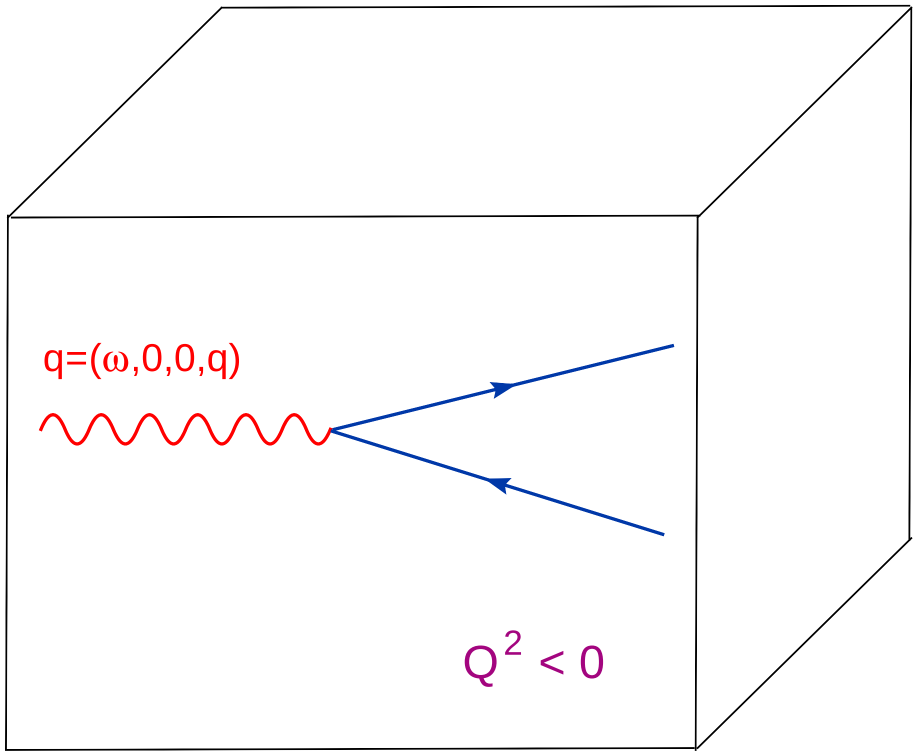
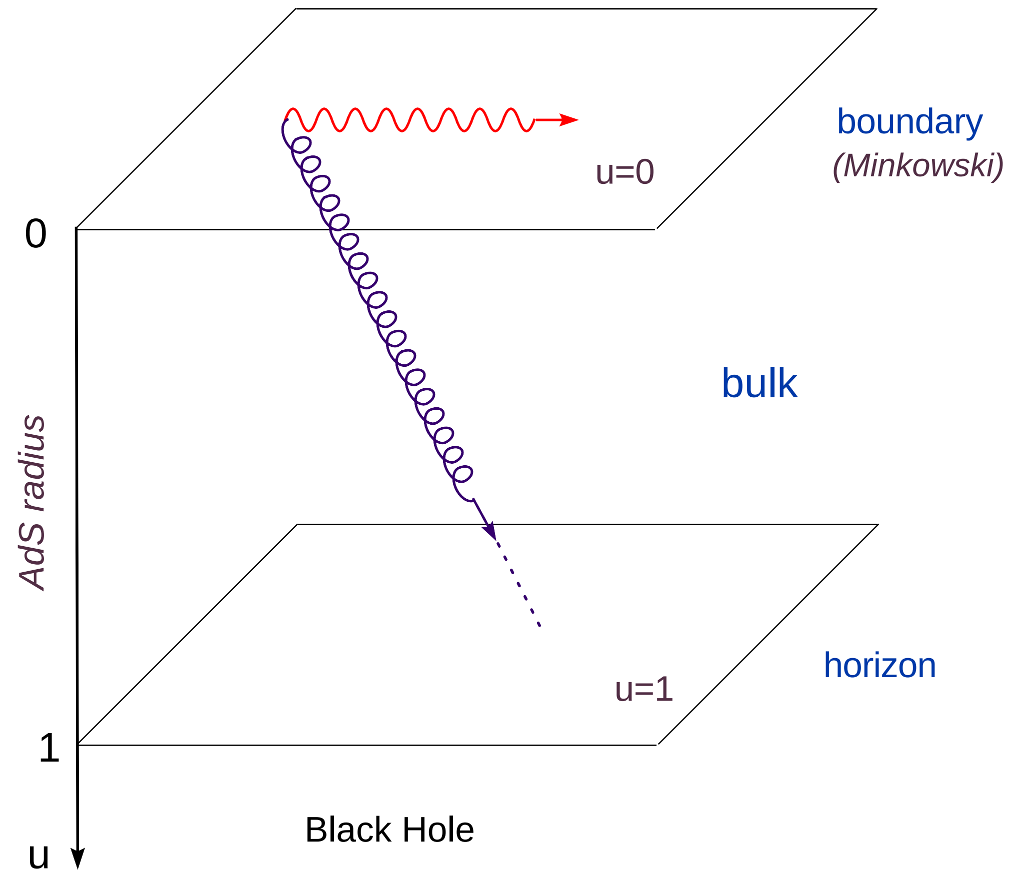


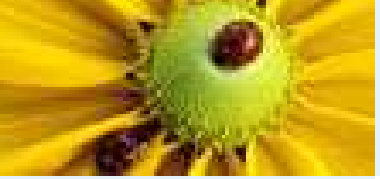
- $Q^2 < 0 \Longrightarrow$ Jet physics : Energy loss, jet quenching ...

The string dual: AdS_5 **Black Hole**

- Strong coupling limit $\lambda \to \infty$ $(N_c \to \infty)$: supergravity
- Metric perturbation in AdS_5 : $A_\mu(t, \boldsymbol{x}, u) \;=\; \mathrm{e}^{-i\omega t + iqz} A_\mu(u)$

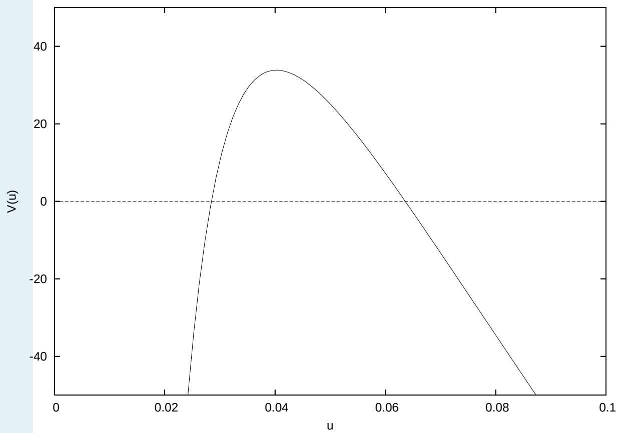


- Inelastic scattering $(\mathrm{Im}\,\Pi_{\mu\nu} \neq 0)$ $\Longleftrightarrow$ Absorbtion by the BH

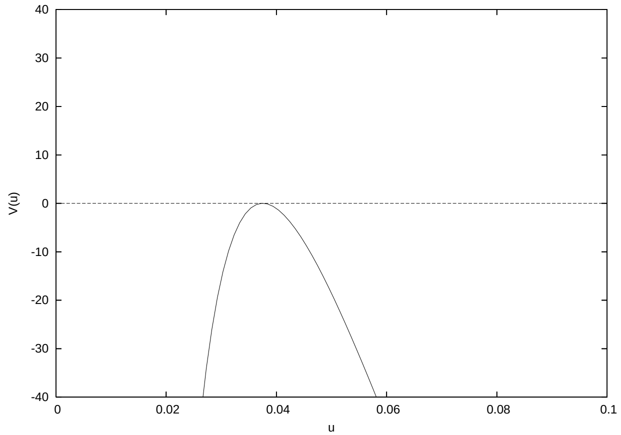


The gravitational wave equation

- Einstein equations linearized around AdS_5
- Effective Schroedinger equation: $\psi'' - V\psi = 0$



$\omega T^2 \ll Q^3$



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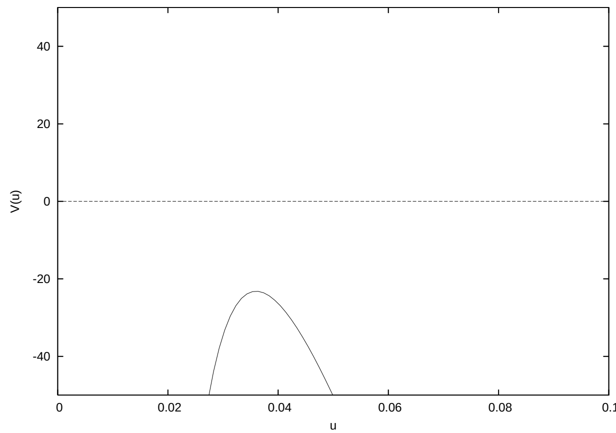
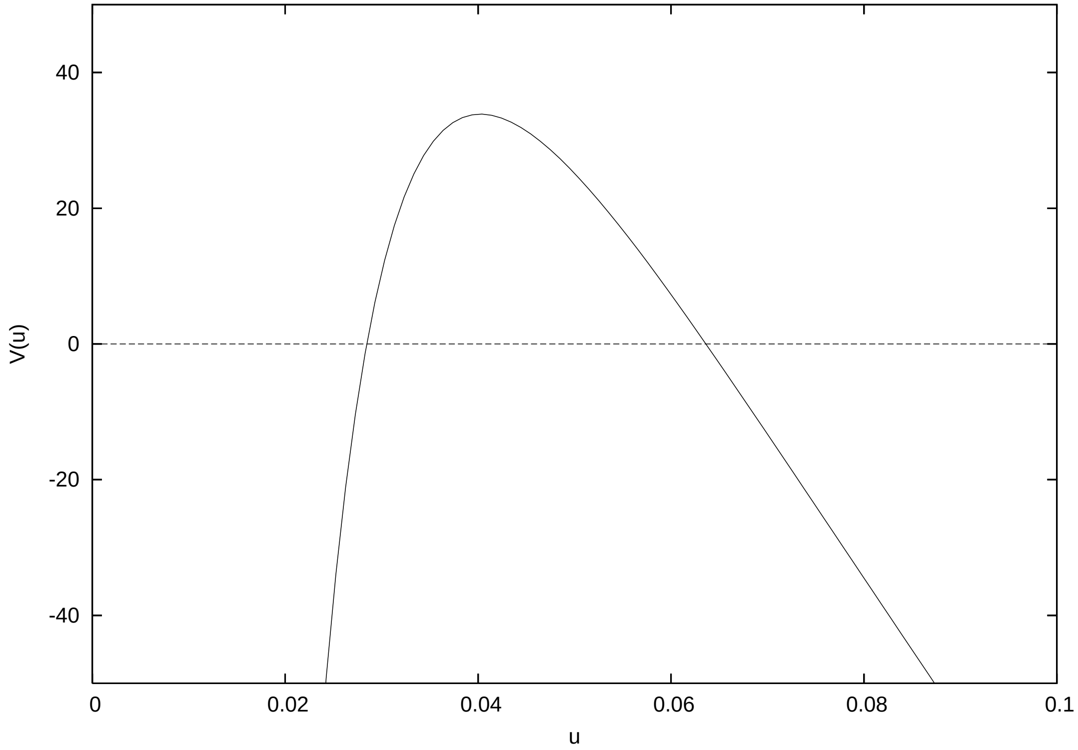


$\omega T^2 \gg Q^3$

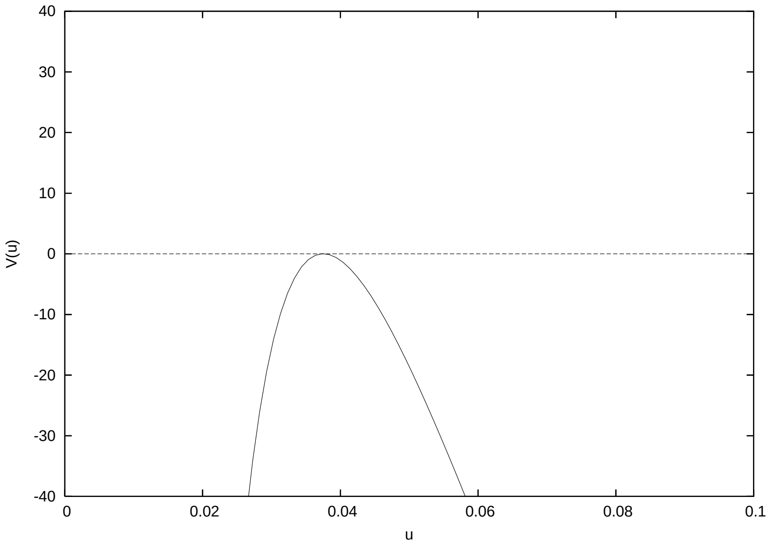
  - Repulsive barrier due to the virtuality Q^2
  - Attractive potential due to the black hole
- Gravitational interactions are proportional to the energy
- Change of regime when increasing ω and/or T at fixed Q^2

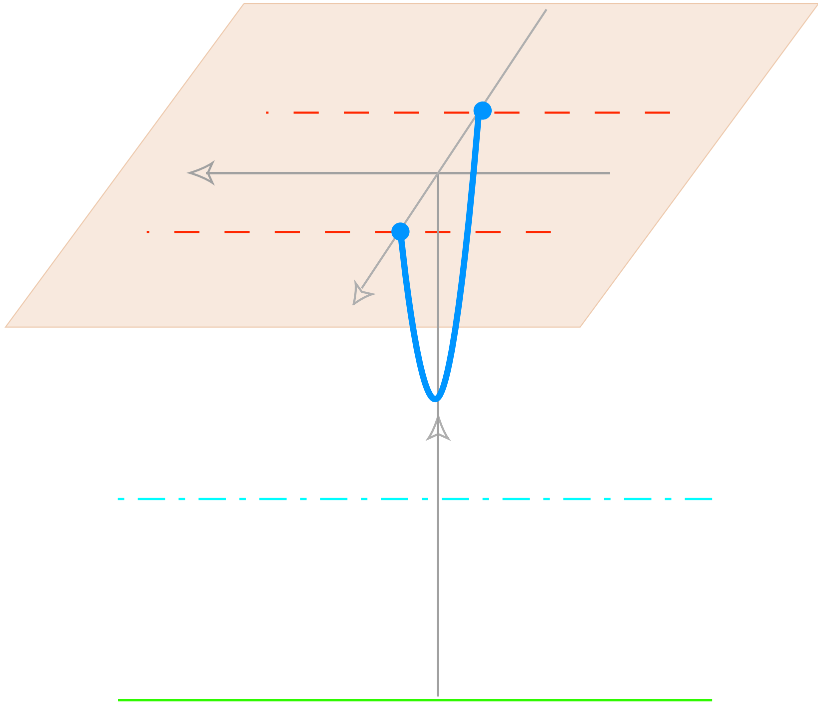
Low energy (or low temperature) : $\omega T^2 \ll Q^3$



- Potential barrier: energy–momentum conservation
- Vacuum–like dynamics: a space–like current cannot decay
- Very small imaginary part due to double tunnel effect

Saturation momentum : $\omega T^2 \sim Q^3$





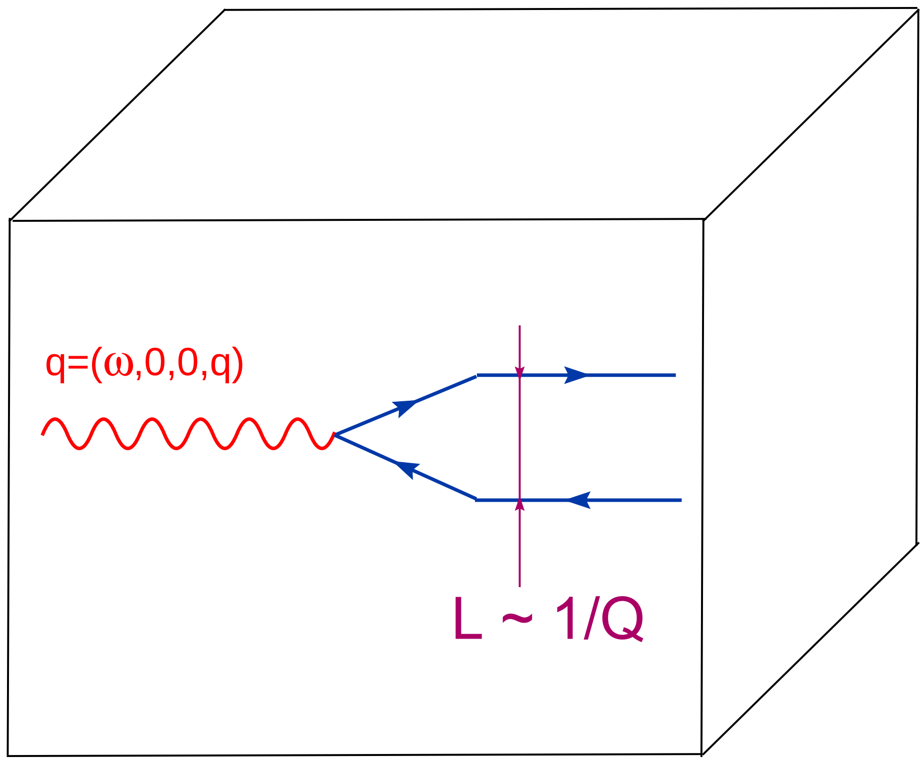
- The barrier disappears when $Q \sim Q_s \equiv (\omega T^2)^{1/3}$
- The same as the meson screening length

$$L \sim \frac{1}{Q} \quad \& \quad \gamma \sim \frac{\omega}{Q} \quad \Longrightarrow \quad L_s \sim \frac{1}{\sqrt{\gamma} T} = \frac{(1-v^2)^{1/4}}{T}$$

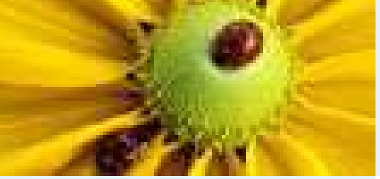
[Liu, Rajagopal, Wiedemann; Chernicoff, Garcia, Guijosa; 2006]

From current to meson

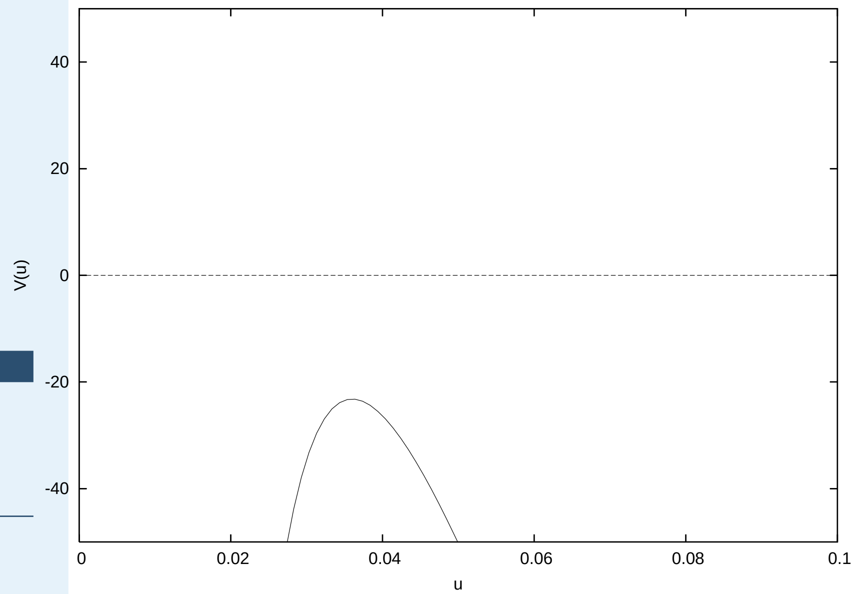
- At very high energy ($\omega \sim q \gg Q$) the current is like a bunch of light–like partons with transverse size $L \sim 1/Q$

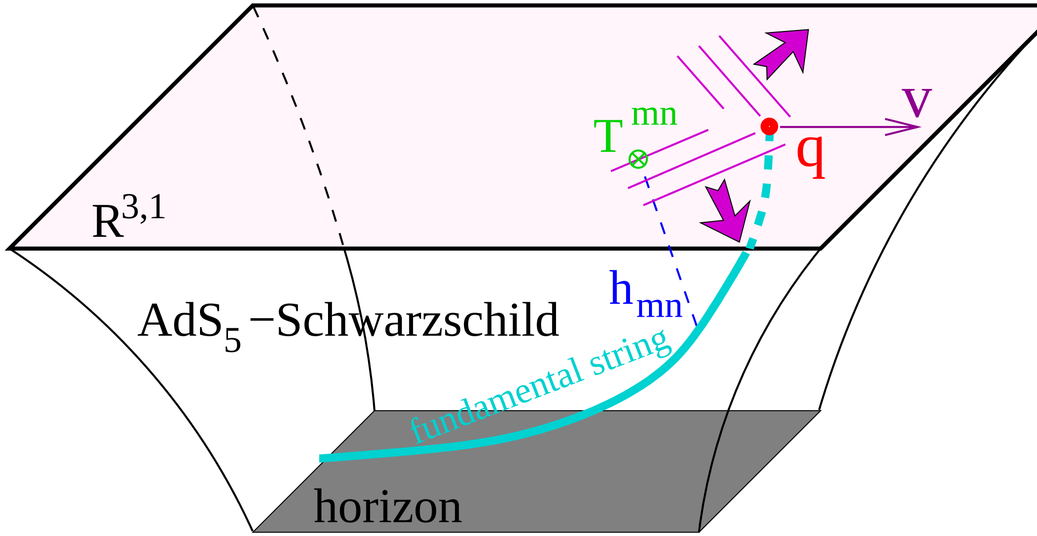


- The current dynamics can be studied also after the ‘meson’ breaking $\Longrightarrow$ a bunch of partons falling in the black hole



High energy (or high temperature) : $\omega T^2 \gg Q^3$





- The wave falls into the BH $\Longrightarrow$ large imaginary part
- No reflected wave $\Longrightarrow$ Total absorption (unitarity limit)
- The line of stationary phase :

  same as the 'trailing string solution' for a heavy quark

  (here, with $v \simeq 1$) [Herzog, Yaffe, et al; Gubser et al, 2006]

Partonic interpretation

- ‘Partonic’ variables: Q^2 and $x \equiv \frac{Q^2}{2\omega T}$

$$Q_s \simeq (\omega T^2)^{1/3} \iff x_s \simeq \frac{T}{Q}$$

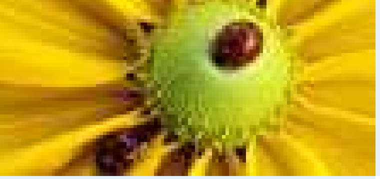
- Low energy ($x \gg x_s$) : $F_2 \simeq 0 \Longrightarrow$ No large–x partons
- High energy ($x \lesssim x_s$) : $F_2(x, Q^2) \sim xN_c^2Q^2$

Interpretation : $$\frac{1}{x}F_2(x, Q^2) \sim \int^{Q^2} \mathrm{d}^2k_\perp \frac{\mathrm{d}n}{\mathrm{d}^2x_\perp \mathrm{d}^2k_\perp}$$

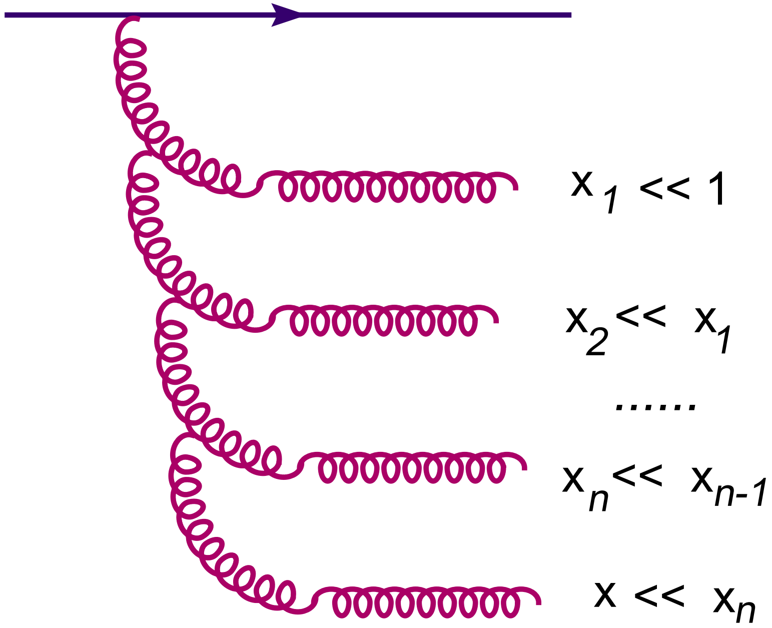
$$\Longrightarrow \frac{1}{N_c^2}\frac{\mathrm{d}n}{\mathrm{d}^2x_\perp \mathrm{d}^2k_\perp} \sim \mathcal{O}(1)$$

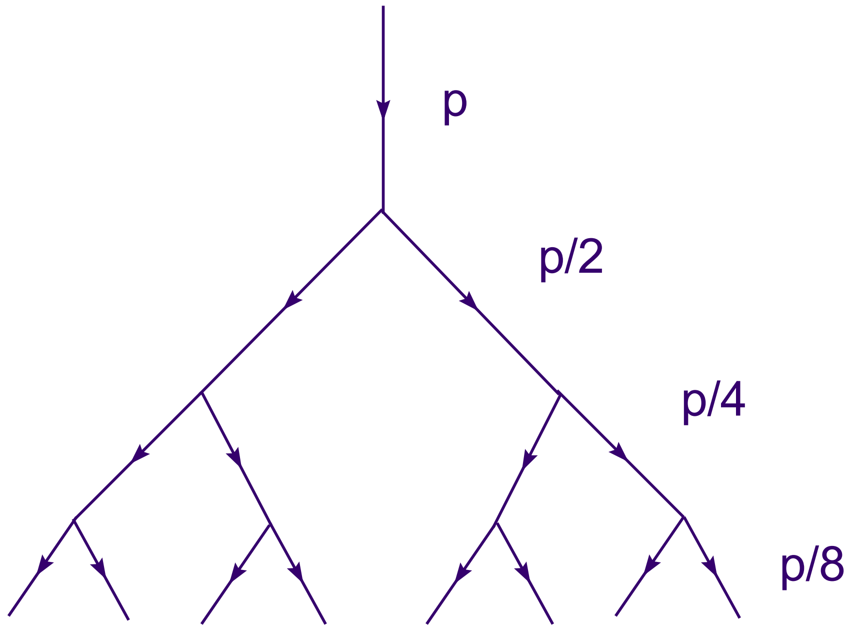
$\Longrightarrow$ one parton of each color per unit cell in phase–space

$\Longrightarrow$ parton saturation (occupation numbers of $\mathcal{O}(1)$)

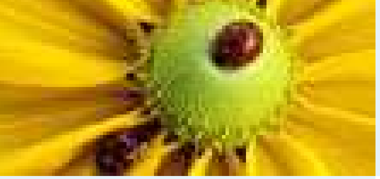


Parton branching: weak vs. strong coupling

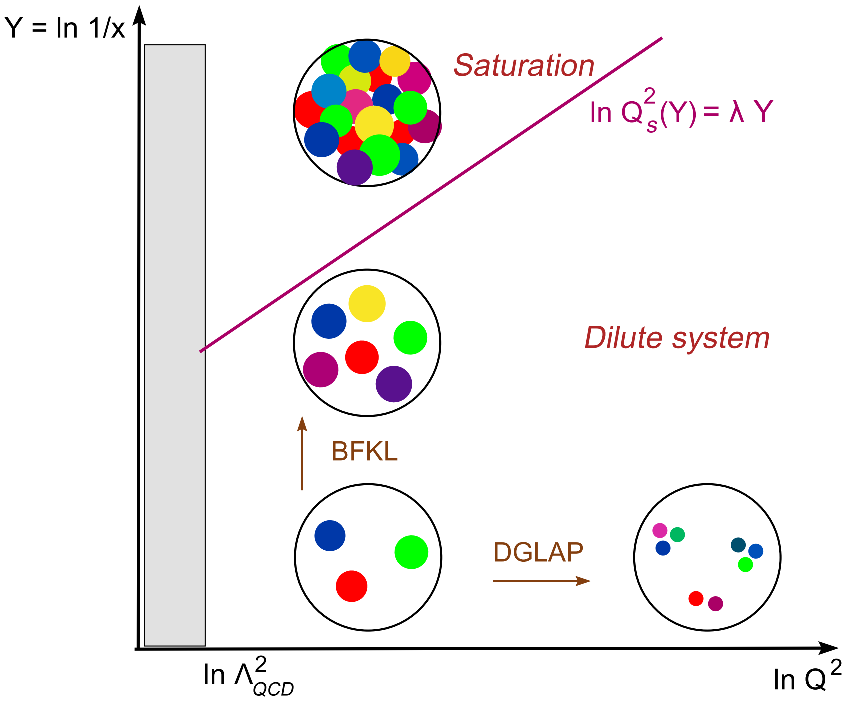


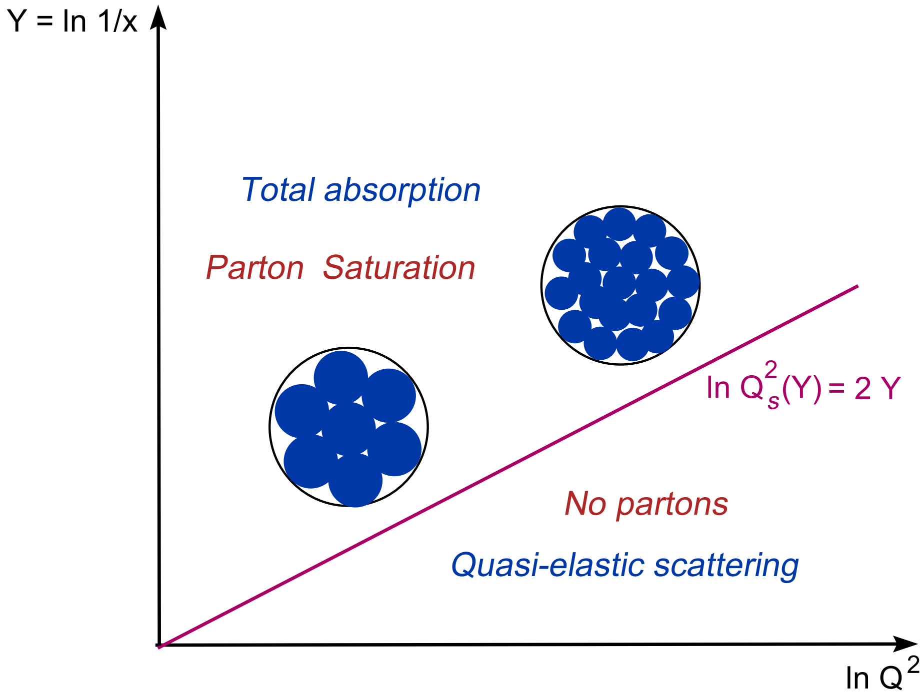


- Weak coupling: very little energy loss per branching
  - energy is carried by the few remaining partons at large x
- Strong coupling: energy is democratically divided
  - all partons fall down at very small $x \lesssim x_s \equiv T/Q$
  - the energy is carried by the partons with $x \simeq x_s$

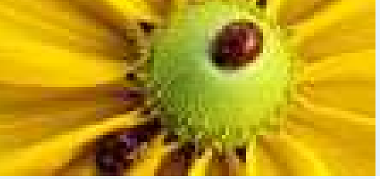


Saturation line: weak vs. strong coupling





- Saturation exponent : $Q_s^2(x) \propto 1/x^{\lambda} \equiv e^{\lambda Y}$
  - weak coupling (pQCD): $\lambda \approx 1.23\, g^2 N_c$ (BFKL Pomeron)
  - strong coupling (plasma): $\lambda = 2$ (graviton)



The string dual of the $\mathcal{N}=4$ SYM plasma

- The $AdS_5 \times S^5$ black hole :

$$\mathrm{d}s^2 = \frac{r^2}{R^2}(-f(r)\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2) + \frac{R^2}{r^2 f(r)}\mathrm{d}r^2 + R^2\mathrm{d}\Omega_5^2$$

  where $f(r) = 1 - \frac{r_0^4}{r^4}$ and the horizon $r_0 = \pi R^2 T$

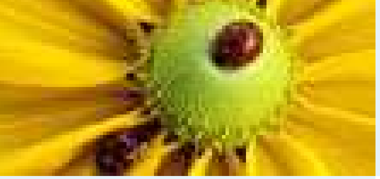
- Strong coupling limit $\lambda \to \infty$ ($N_c \to \infty$) : supergravity
- The black hole entropy: $S_{\rm BH} = A/4G$, with $A =$ horizon area

$$\Longrightarrow \mathcal{S} \equiv \frac{S_{\rm BH}}{V_{3D}} = \frac{\pi^2}{2}N_c^2T^3 = \frac{3}{4}\mathcal{S}_0$$

- For generic values of $\lambda = g^2 N_c$:

  $\mathcal{S} = f(\lambda)\, N_c^2 T^3$ where f interpolates between 1 and 3/4

  $\Longrightarrow$ Same number of d.o.f. at either weak or strong coupling!



The wave equations

- Classical EOM (Maxwell eqs. in the AdS_5 BH metric) :

$$\partial_m\left(\sqrt{-g}g^{mn}F_{np}\right)\,=\,0\,,\qquad F_{mn}=\partial_m A_n-\partial_n A_m$$

- Boundary conditions at $r\to\infty$

$$A_\mu(t,\boldsymbol{x},r\to\infty)\,=\,\mathrm{e}^{-i\omega t+iqz}\,A_\mu\,,\qquad A_r(t,\boldsymbol{x},r\to\infty)\,=\,0$$

- Boundary conditions at the BH horizon $r=r_0$

No wave returning from the horizon $\Longrightarrow$ purely outgoing wave

N.B. The origin of the imaginary part !

- The classical action $\Longrightarrow$ the current–current correlator:

$$R_{\mu\nu}(q)\,=\,\frac{\partial^2 S_{cl}}{\partial A_\mu \partial A_\nu}$$

- The classical action involves just $(A_\mu\partial_r A_\mu)|_{r\to\infty}$