Initial two-particle correlations in nucleus-nucleus collisions

François Gelis

CERN and CEA/Saclay

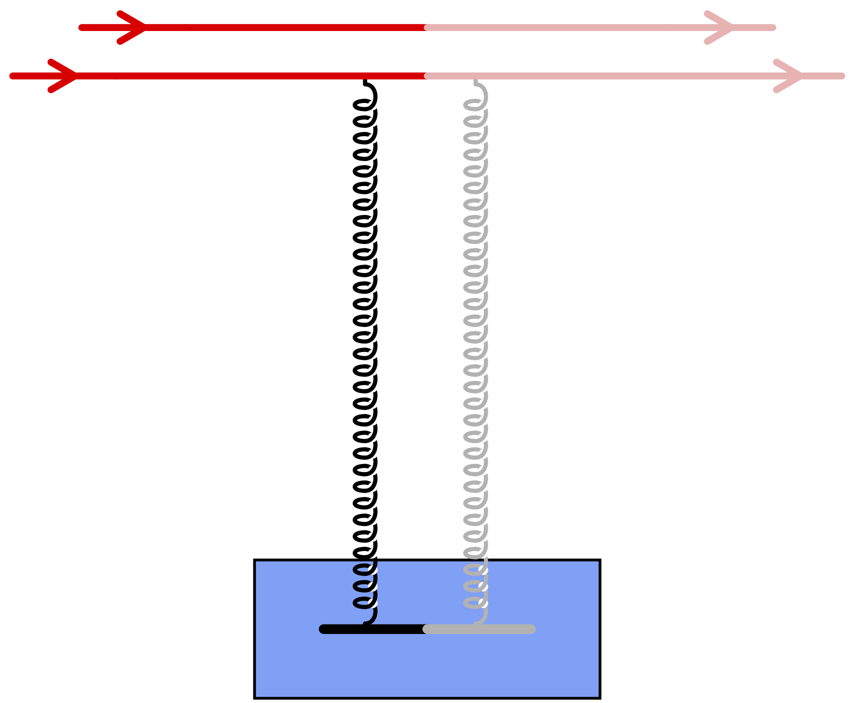
Outline

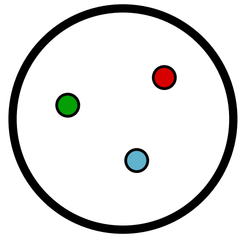
- Saturation and Color Glass Condensate
- Single gluon spectrum
- Two gluon spectrum
- Possible link to the ridge

(work with T. Lappi and R. Venugopalan)

Gluon saturation

Parton saturation



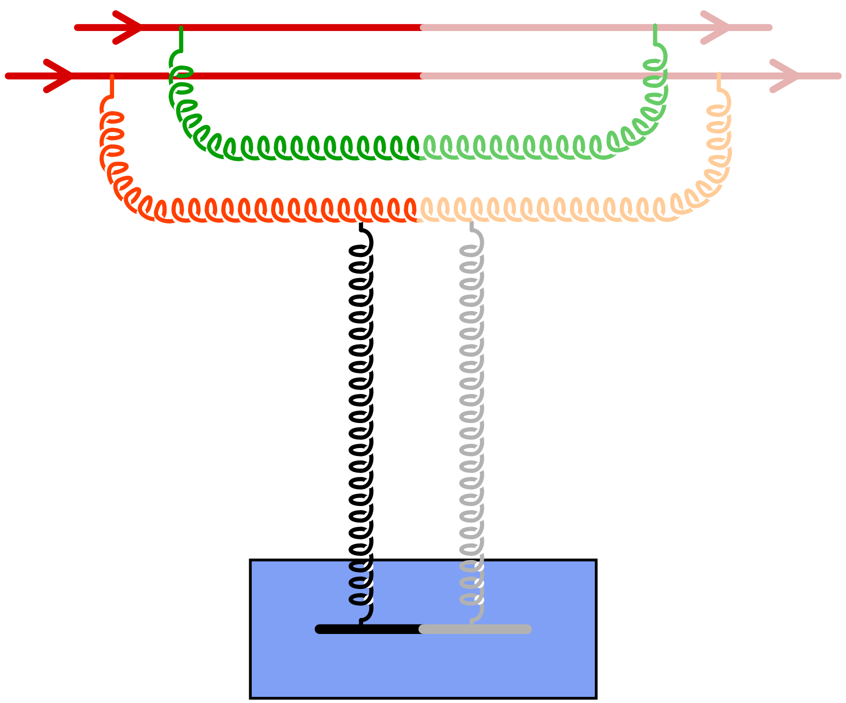


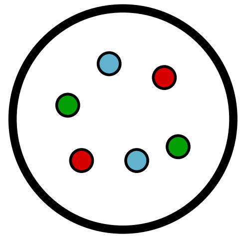
▷ assume that the projectile is big, e.g. a nucleus, and has many valence quarks (only two are represented)

▷ on the contrary, consider a small probe, with few partons

▷ at low energy, only valence quarks are present in the hadron wave function

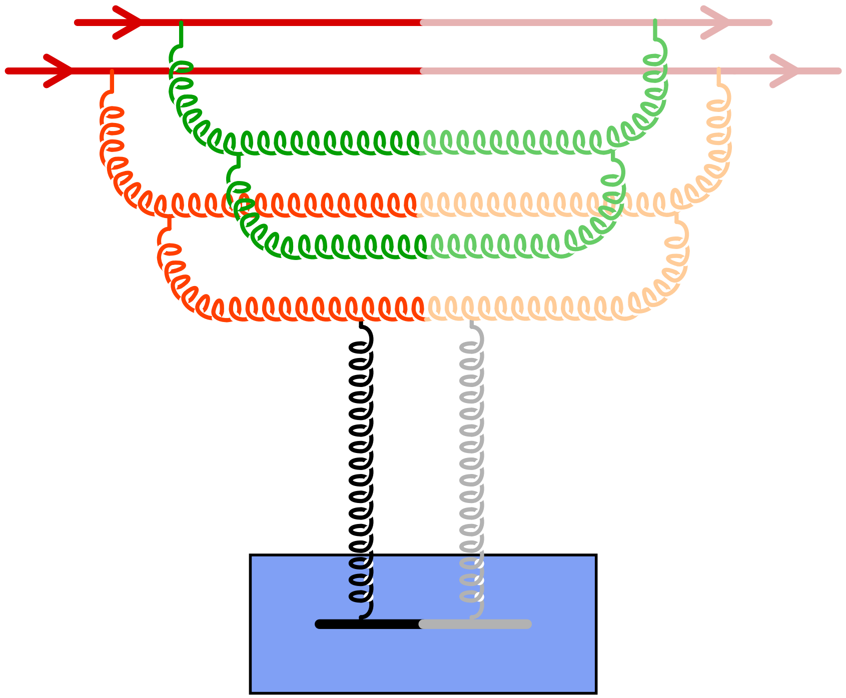
Parton saturation

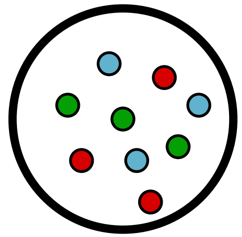




- ▷ when energy increases, new partons are emitted
- ▷ the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon
- ▷ at small-x (i.e. high energy), these logs need to be resummed

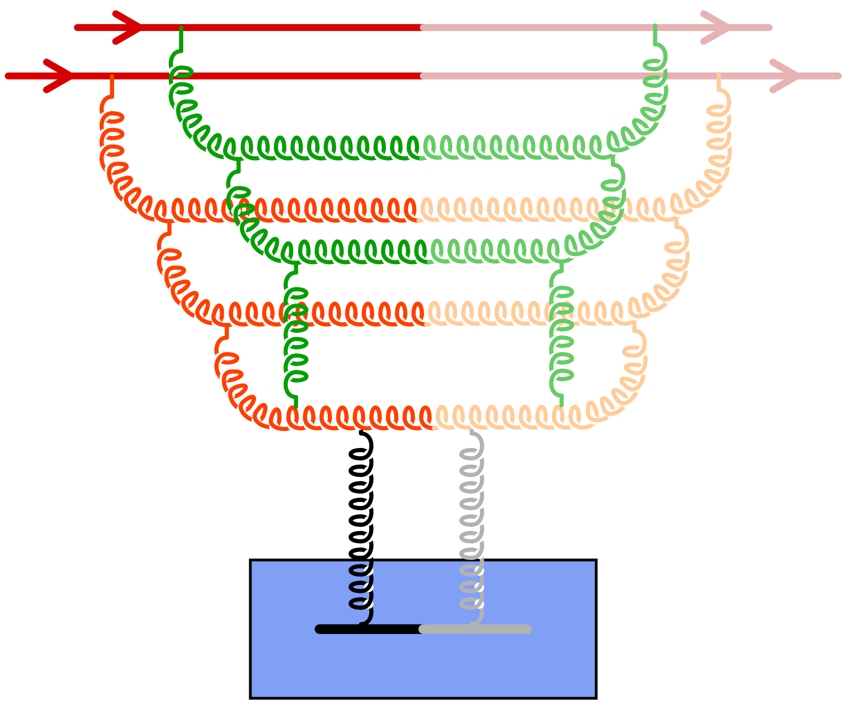
Parton saturation

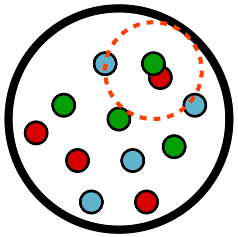




▷ as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

Parton saturation





- ▷ eventually, the partons start overlapping in phase-space
- ▷ parton recombination becomes favorable
- ▷ after this point, the evolution is non-linear:

the number of partons created at a given step depends non-linearly on the number of partons present previously

Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

- Number of gluons per unit area:

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

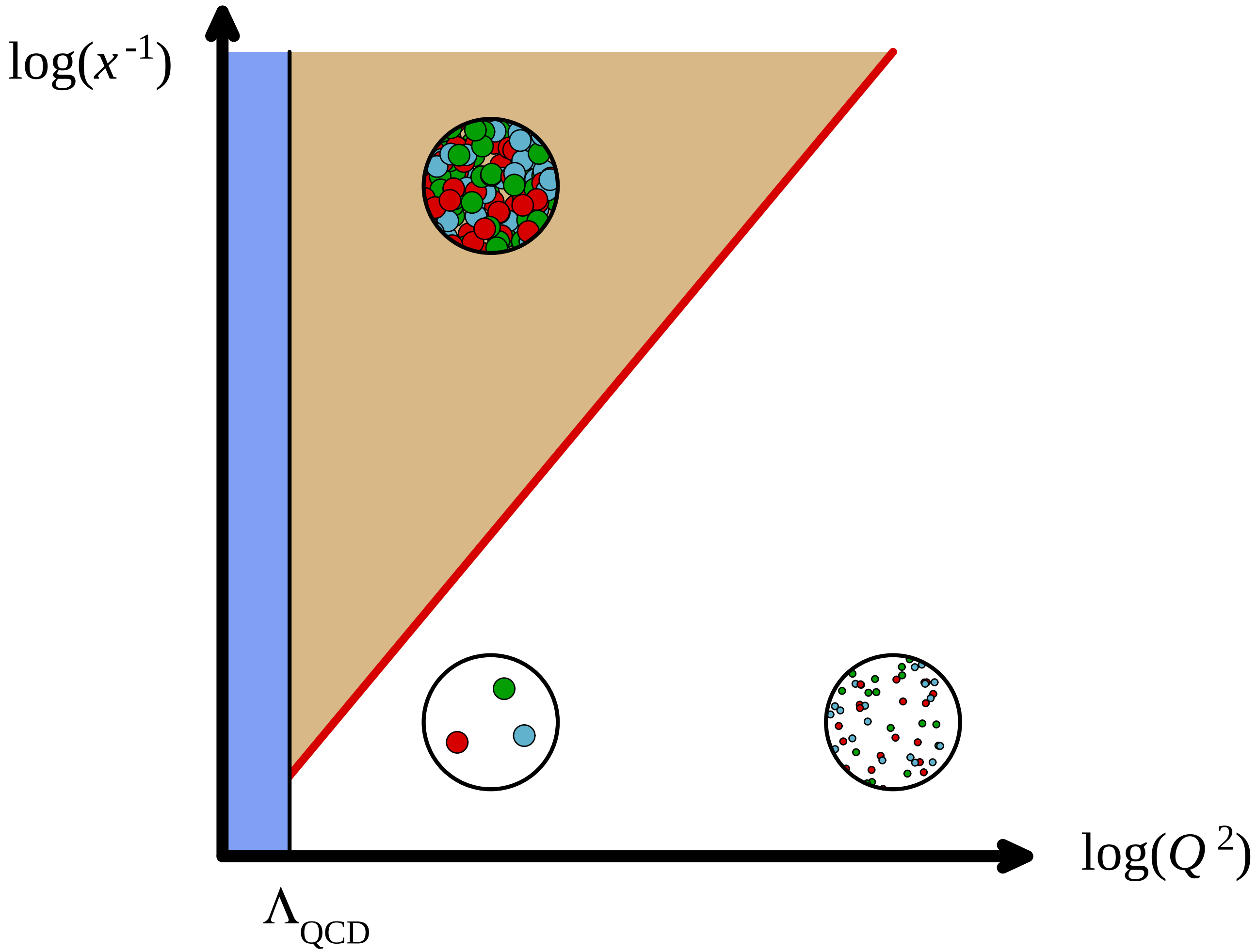
- Recombination cross-section:

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

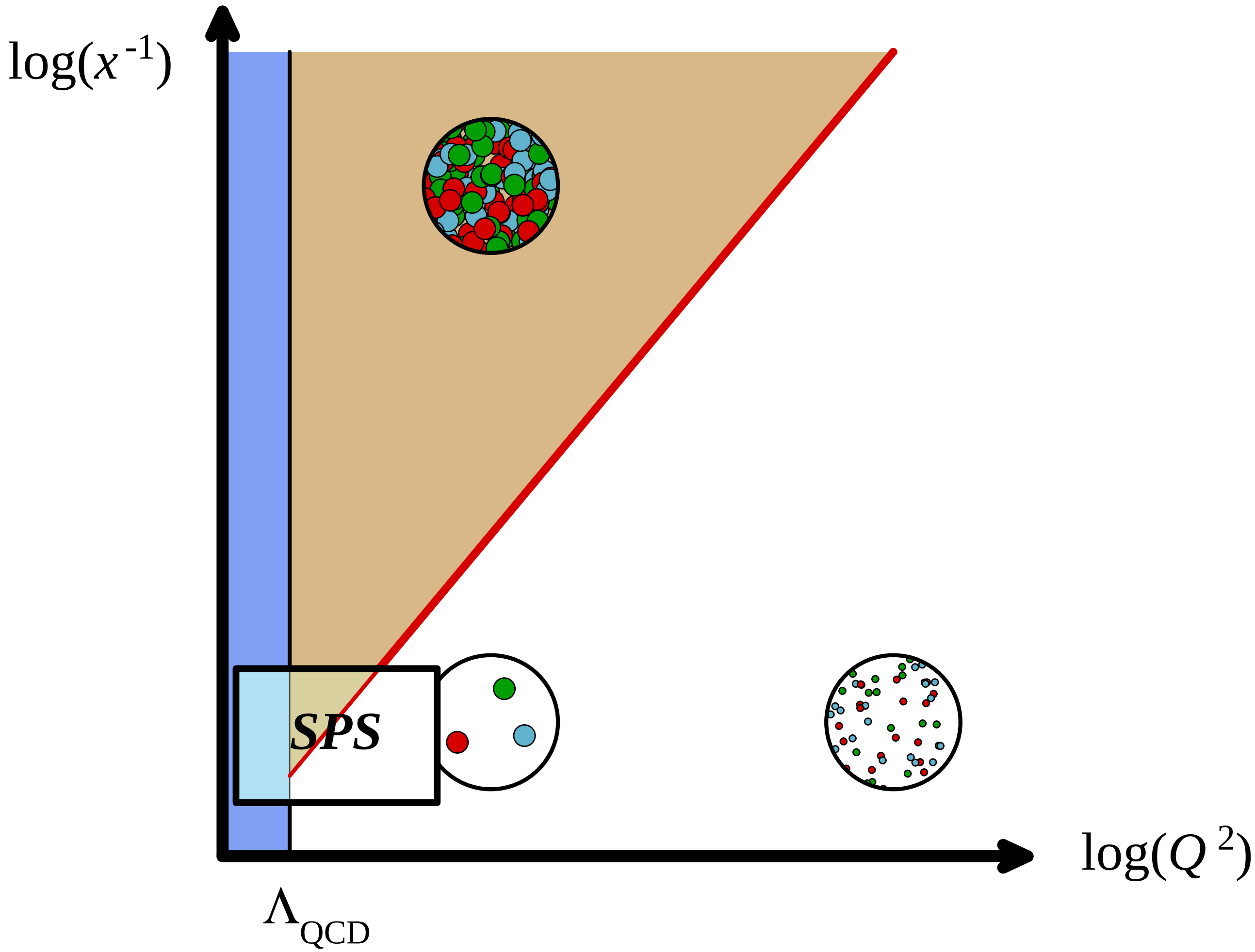
- Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with:

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

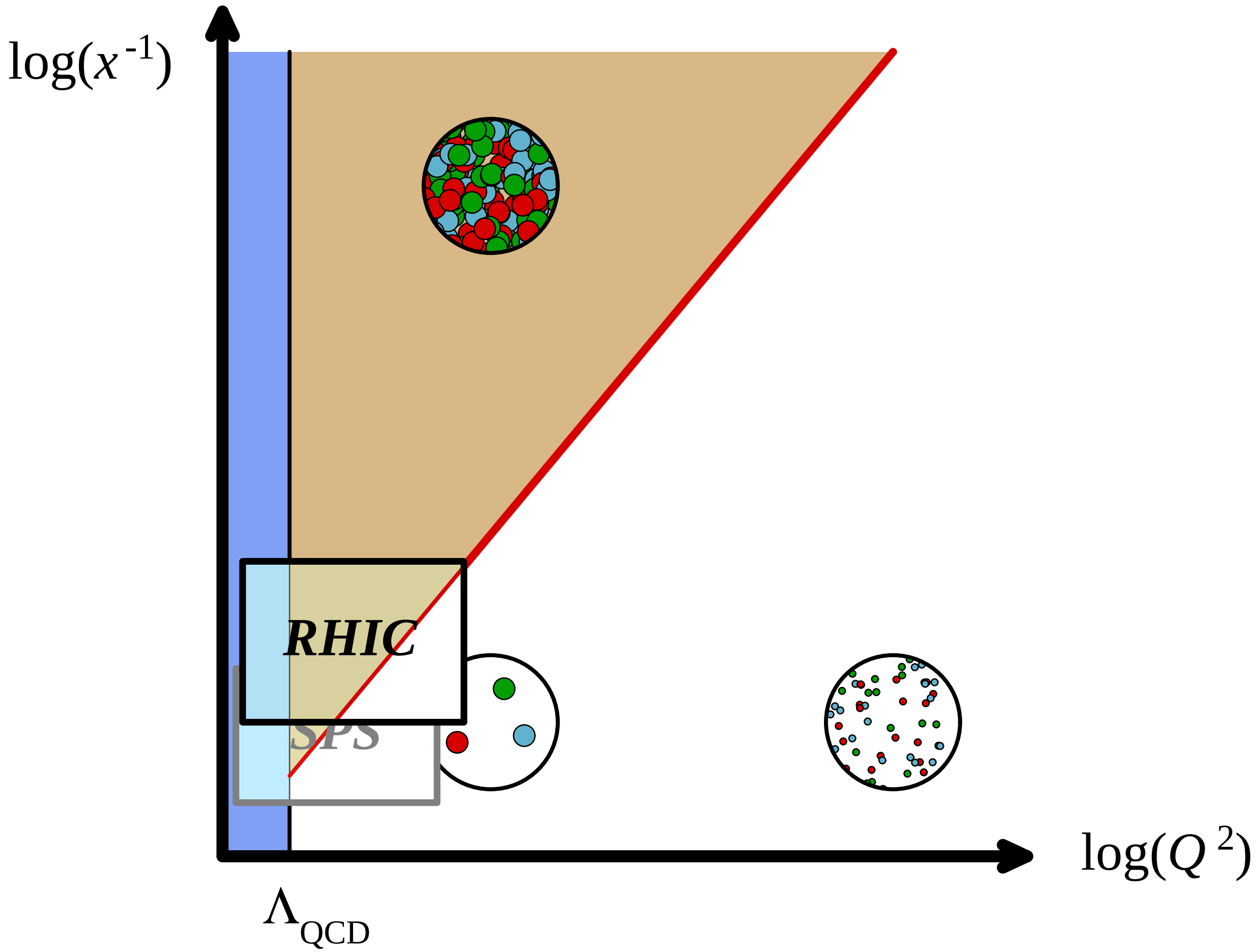
Saturation domain



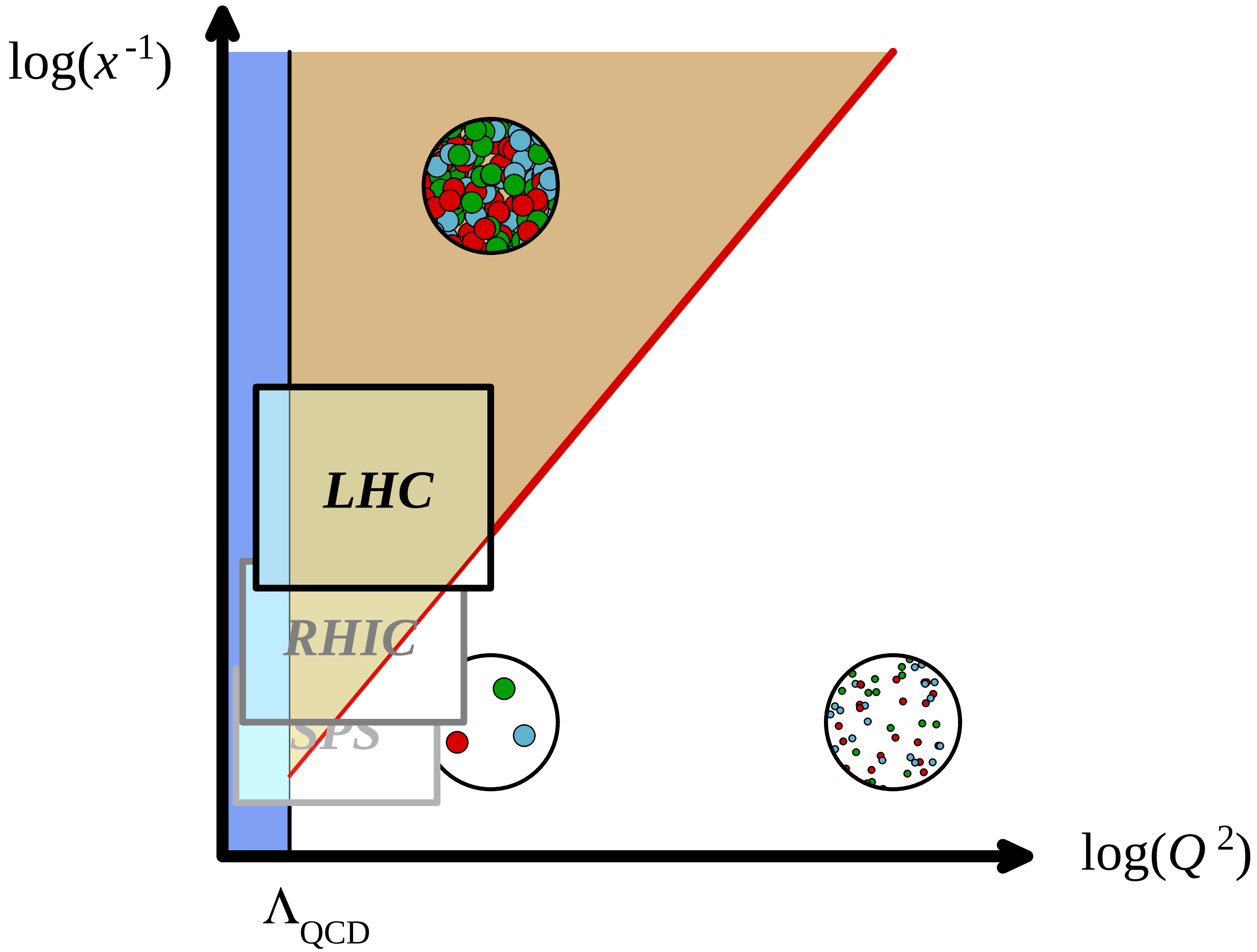
Saturation domain



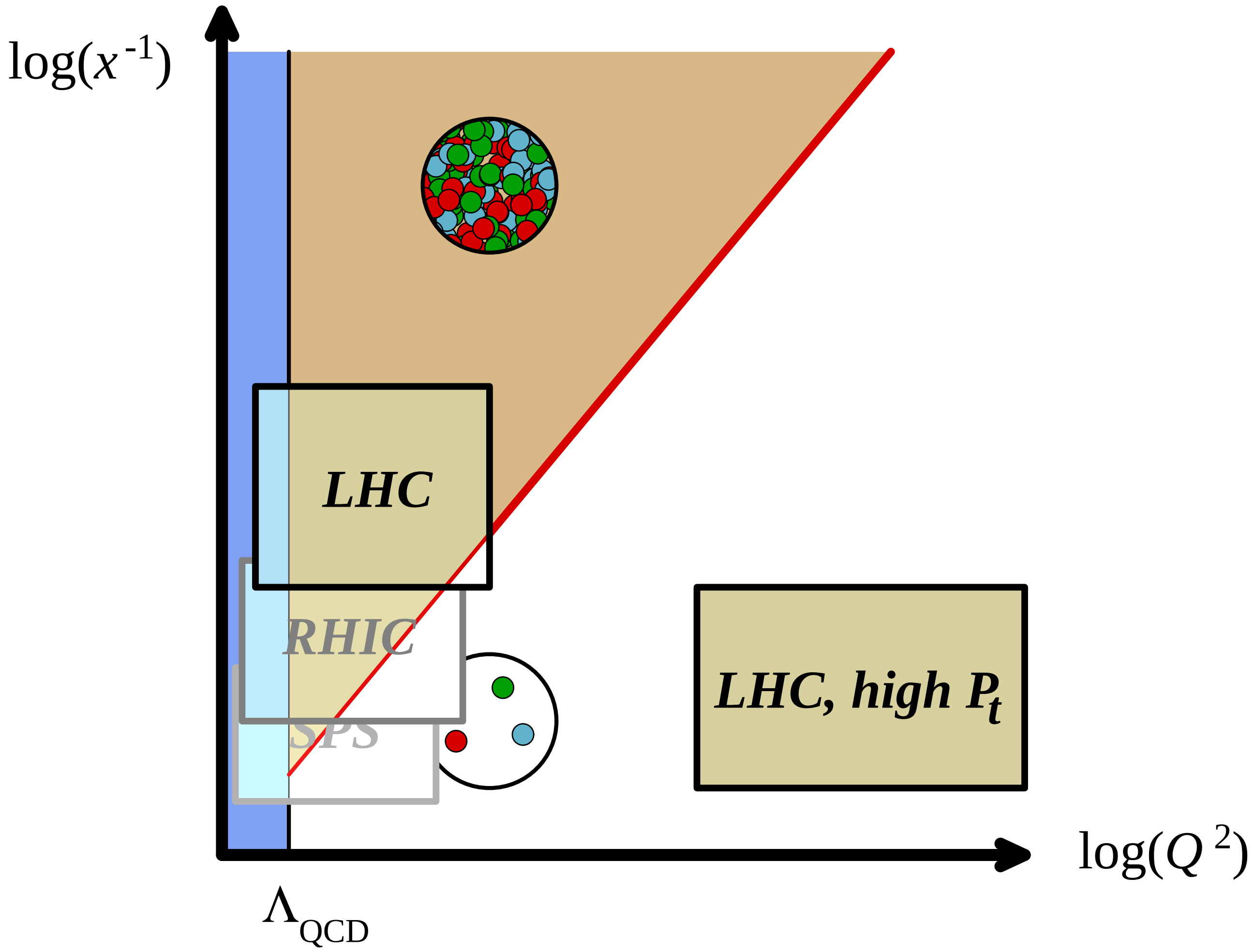
Saturation domain



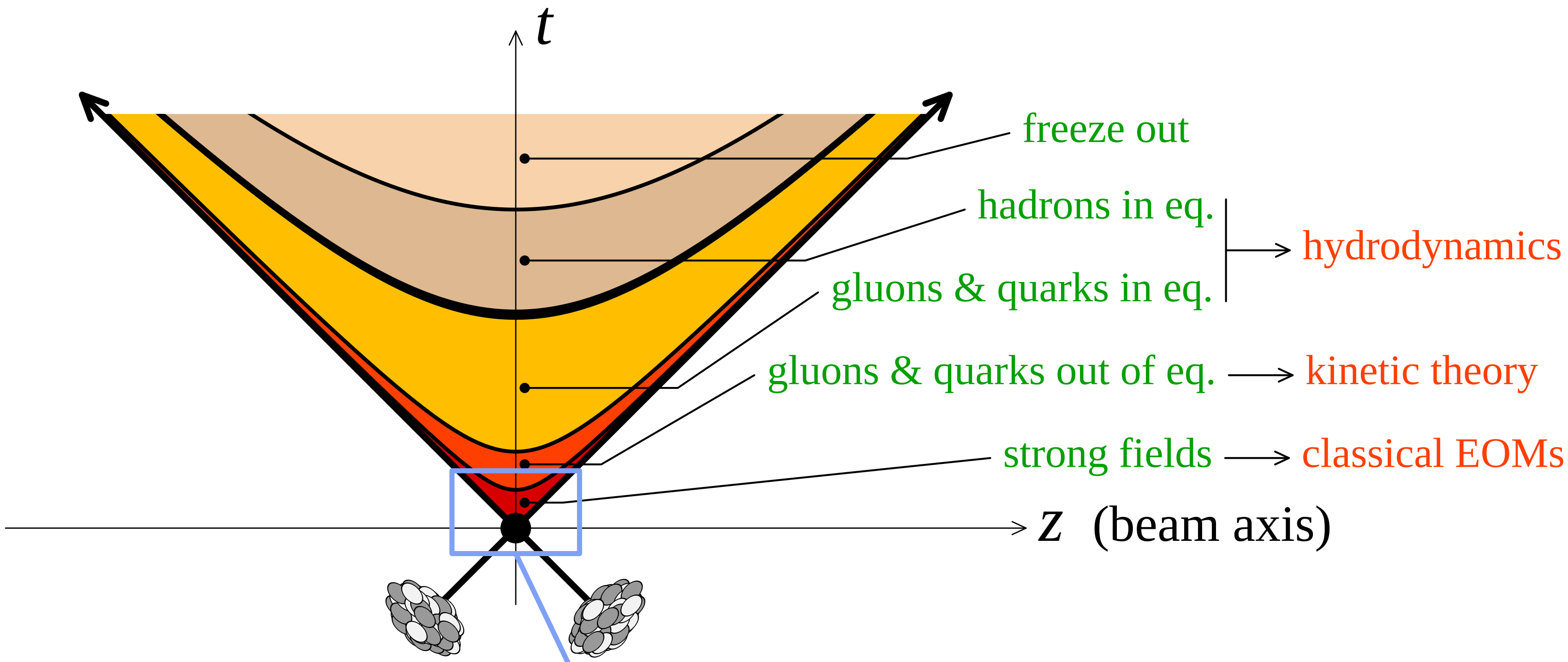
Saturation domain



Saturation domain



Heavy Ion Collisions



- calculate the initial production of semi-hard particles
- provide initial conditions for hydrodynamics

CGC degrees of freedom

- The fast partons (large x) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J_a^\mu = \delta^{\mu+}\delta(x^-)\rho_a(\vec{\boldsymbol{x}}_\perp) \qquad (x^- \equiv (t-z)/\sqrt{2})$$

- Slow partons (small x) cannot be considered static over the time-scales of the collision process ▷ they must be treated as the usual gauge fields

  Since they are radiated by the fast partons, they must be coupled to the current J_a^μ by a term : $A_\mu J^\mu$

- The color sources ρ_a are random, and described by a distribution functional $W_Y[\rho]$, with Y the rapidity that separates “soft” and “hard”

CGC evolution

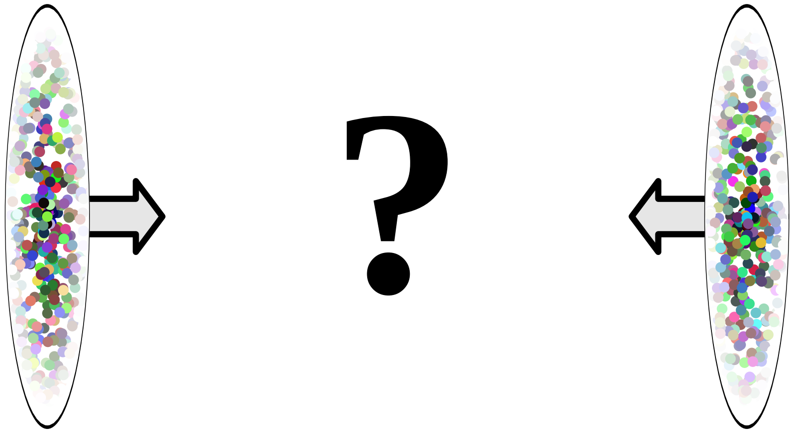
- Evolution equation (JIMWLK) :

$$\frac{\partial W_Y[\rho]}{\partial Y} = \mathcal{H}[\rho]\ \ W_Y[\rho]$$

$$\mathcal{H}[\rho] = \int_{\vec{\boldsymbol{x}}_\perp} \sigma(\vec{\boldsymbol{x}}_\perp) \frac{\delta}{\delta\rho(\vec{\boldsymbol{x}}_\perp)} + \frac{1}{2} \int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp} \chi(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp) \frac{\delta^2}{\delta\rho(\vec{\boldsymbol{x}}_\perp)\delta\rho(\vec{\boldsymbol{y}}_\perp)}$$

- σ and χ are non-linear functionals of ρ
- This evolution equation resums the powers of $\alpha_s \ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- This equation simplifies into the BFKL equation when the color density ρ is small (one can expand σ and χ in ρ)

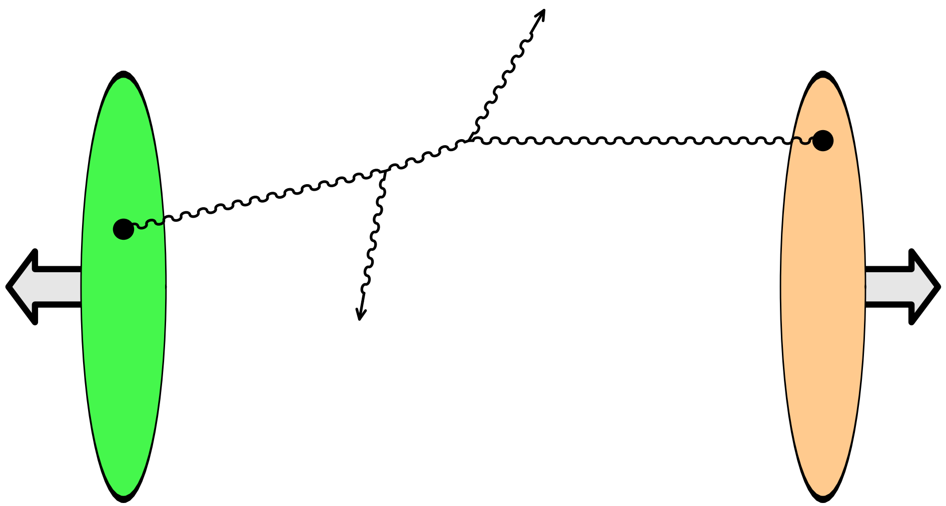
CGC and Nucleus-Nucleus collisions



$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} + \underbrace{(J_1^\mu + J_2^\mu)}_{J^\mu} A_\mu$$

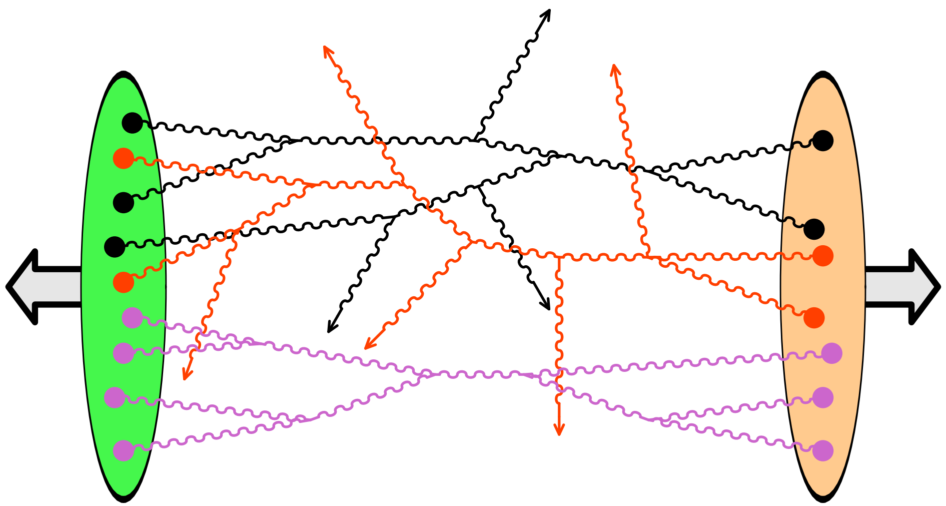
- Given the sources $\rho_{1,2}$ in each projectile, how do we calculate observables? Is there some kind of perturbative expansion?
- Loop corrections and factorization?

Initial particle production



- Dilute regime : one parton in each projectile interact

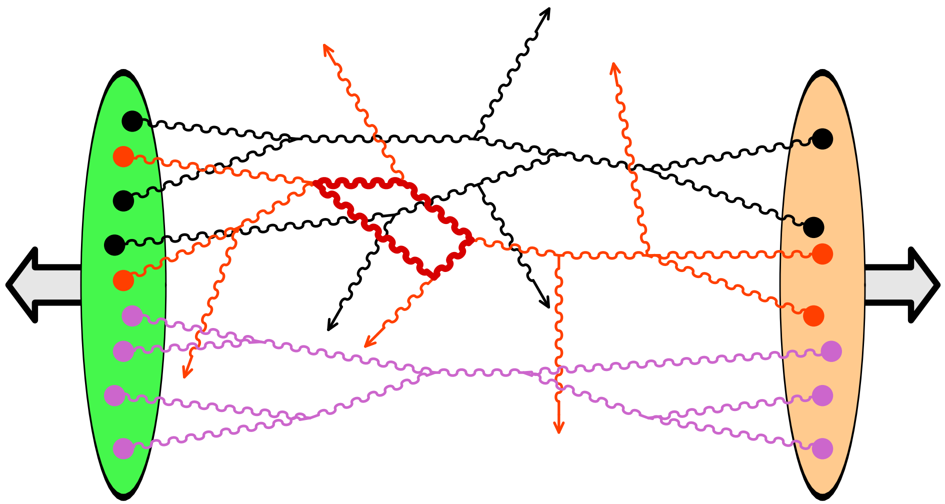
Initial particle production



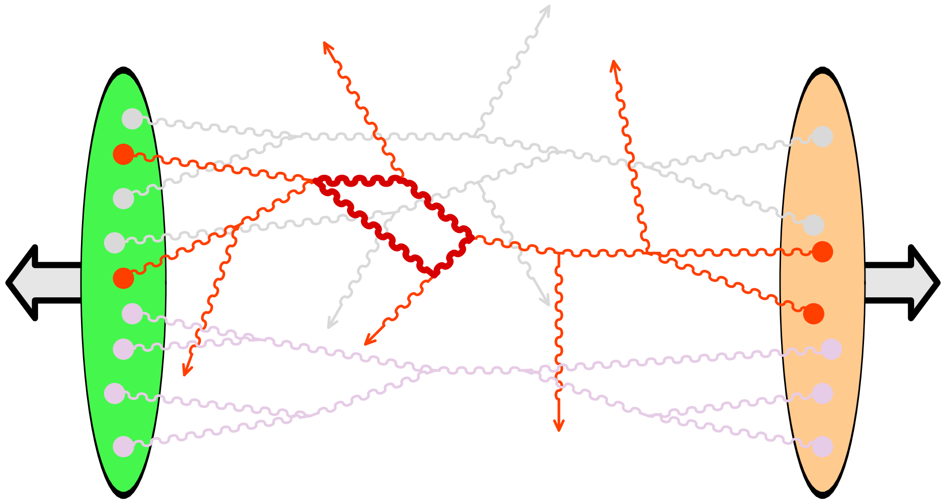
- Dilute regime : one parton in each projectile interact
- Dense regime : multiparton processes become crucial

  (+ pileup of many partonic scatterings in each AA collision)

Power counting



Power counting



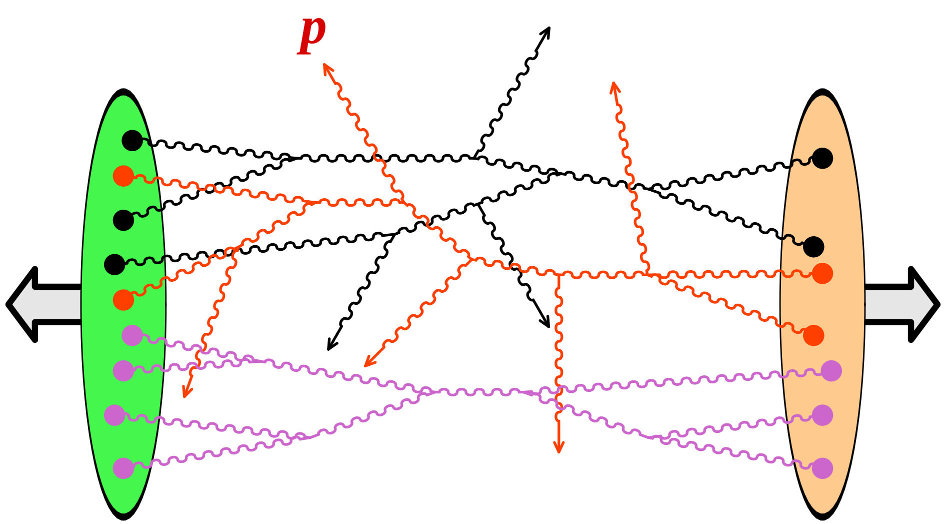
- In the saturated regime, the sources are of order $1/g$ (because $\langle\rho\rho\rangle \sim$ occupation number $\sim 1/\alpha_s$)
- The order of each connected diagram is given by :

$$\frac{1}{g^2} \quad g^{\#\ \text{produced gluons}} \quad g^{2(\#\ \text{loops})}$$

- The total order of a graph is the product of the orders of its disconnected subdiagrams

Single gluon spectrum

Single gluon spectrum at LO



- Leading Order = tree diagrams only
- Tag one gluon of momentum $\vec{\boldsymbol{p}}$
- Integrate out the phase-space of all the other gluons

$$\frac{dN}{d^3\vec{\boldsymbol{p}}} \sim \sum_{n=0}^{\infty} \frac{1}{n!} \int \left[d^3\vec{\boldsymbol{p}}_1 \cdots d^3\vec{\boldsymbol{p}}_n\right] \; \left|\left\langle \vec{\boldsymbol{p}} \;\; \vec{\boldsymbol{p}}_1 \cdots \vec{\boldsymbol{p}}_n \middle| 0 \right\rangle\right|^2$$

Single gluon spectrum at LO

- LO results for the single gluon spectrum :
  - Disconnected graphs cancel in the inclusive spectrum
  - At LO, the single gluon spectrum can be expressed in terms of classical solutions of the field equation of motion
  - These classical fields must obey boundary conditions

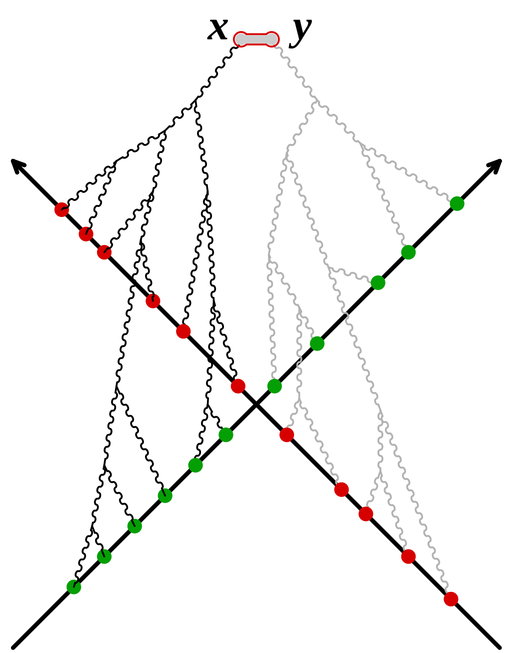
$$\frac{dN}{d^3\vec{\boldsymbol{p}}} \sim \lim_{t\to+\infty} \int d^3\vec{\boldsymbol{x}} d^3\vec{\boldsymbol{y}}\; e^{i\vec{\boldsymbol{p}}\cdot(\vec{\boldsymbol{x}}-\vec{\boldsymbol{y}})} \;\cdots \mathcal{A}^\mu(t,\vec{\boldsymbol{x}})\;\mathcal{A}^\nu(t,\vec{\boldsymbol{y}})$$

$$\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J^\nu$$

$$\lim_{t\to-\infty} \mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0$$

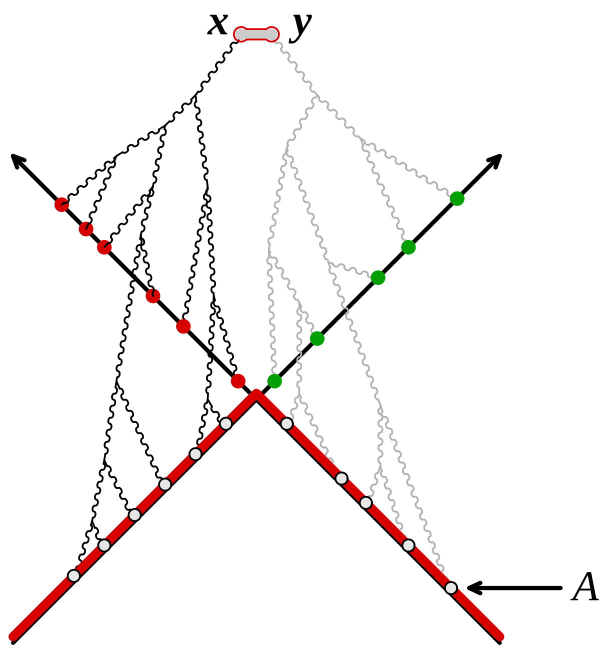
Single gluon spectrum at LO

- Retarded classical fields are sums of tree diagrams :



Single gluon spectrum at LO

- Retarded classical fields are sums of tree diagrams :

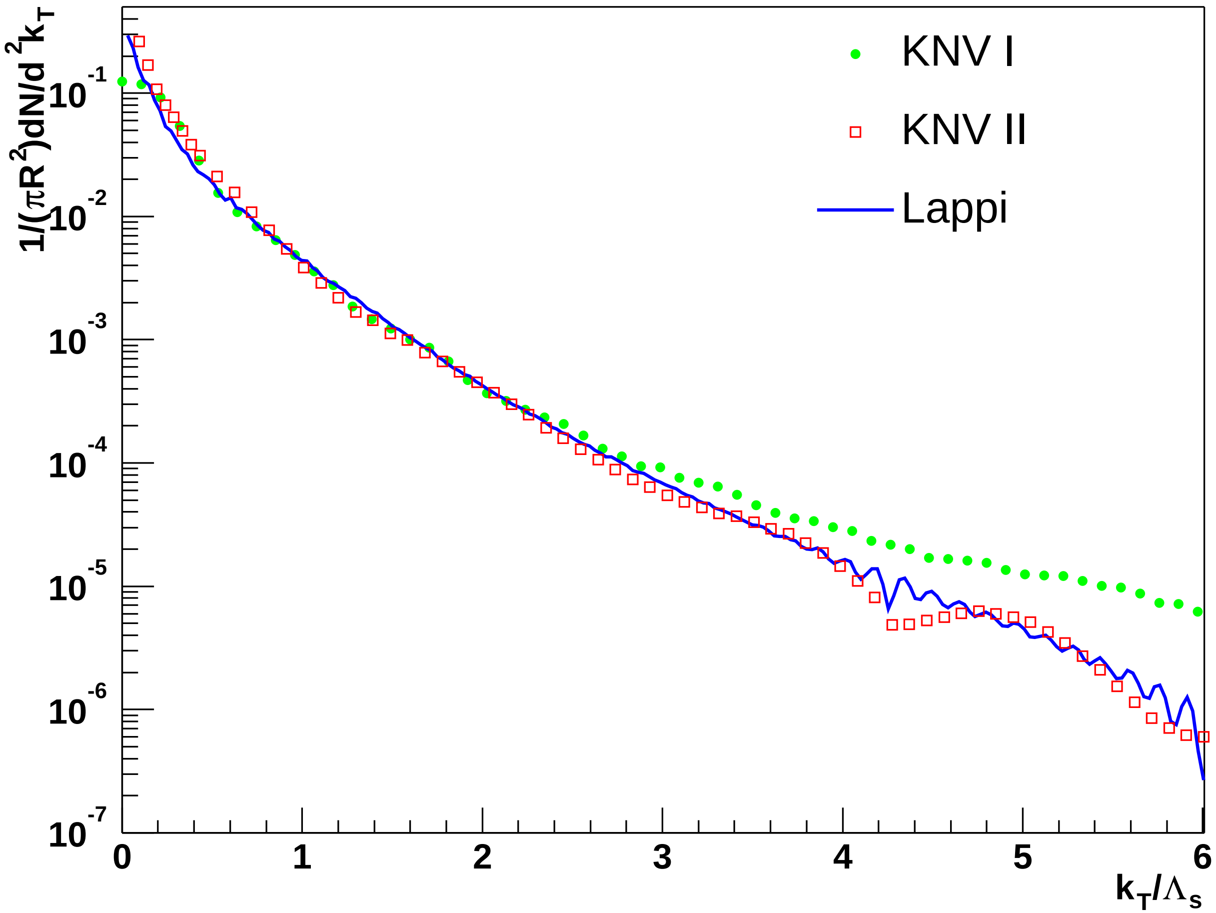


- Note : the gluon spectrum can be seen as a functional of the value of the classical field just above the backward light-cone :

$$\frac{dN}{d^3\vec{\boldsymbol{p}}} = \mathcal{F}[\mathcal{A}]$$

Single gluon spectrum at LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

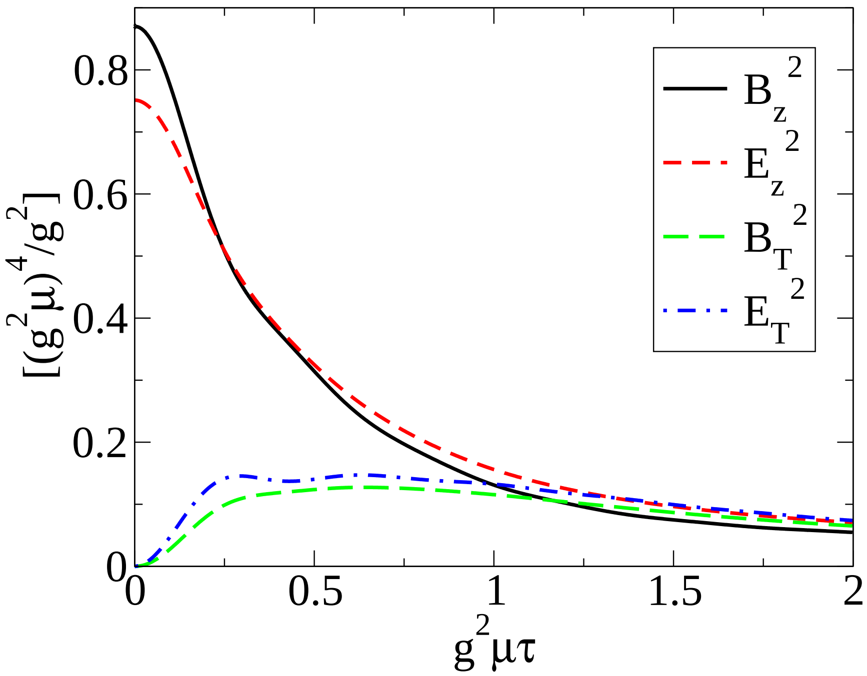


- Important softening at small $k_\perp$ compared to pQCD (saturation)

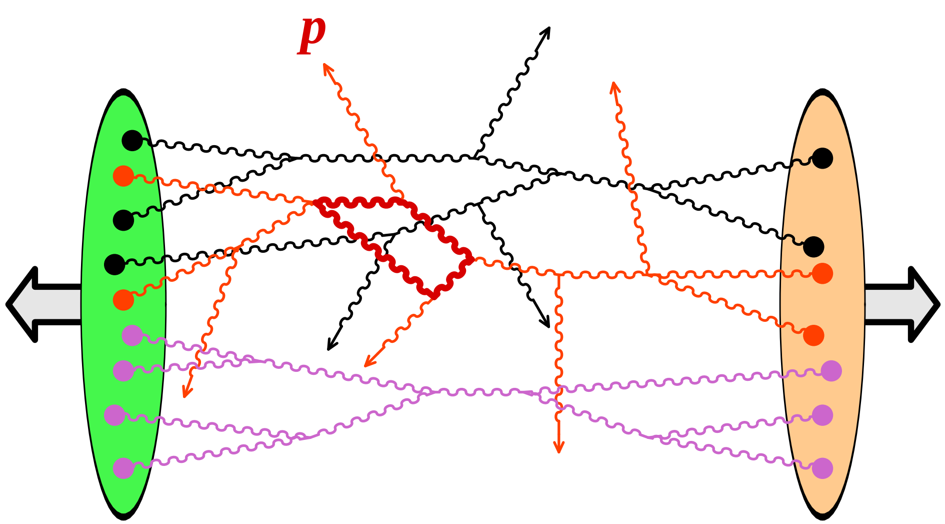
Initial fields

Lappi, McLerran (2006)

- Before the collision, the chromo-$\vec{E}$ and $\vec{B}$ fields are localized in two sheets transverse to the beam axis
- Immediately after the collision, the chromo-$\vec{E}$ and $\vec{B}$ fields have become longitudinal :



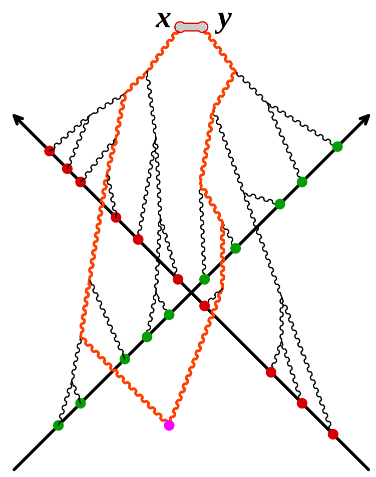
Single gluon spectrum at NLO

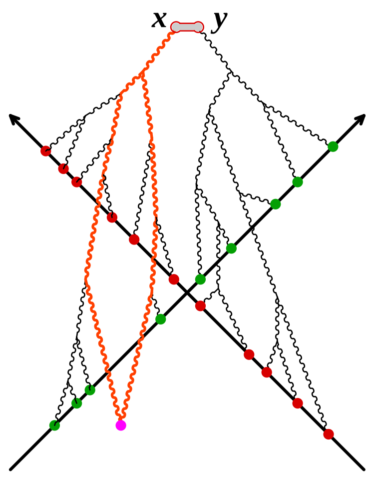


- Next to Leading Order = 1-loop diagrams
- Connected diagrams only
- Expressible in terms of classical fields, and small fluctuations about the classical field, both with retarded boundary conditions

Single gluon spectrum at NLO

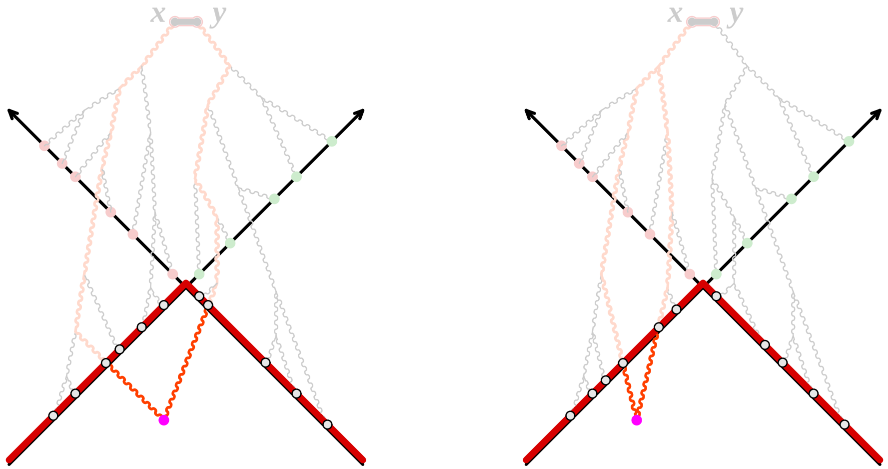
- 1-loop graphs contributing to the gluon spectrum at NLO :





Single gluon spectrum at NLO

- 1-loop graphs contributing to the gluon spectrum at NLO :



- Their contribution can be written as a perturbation of the initial fields on the light-cone

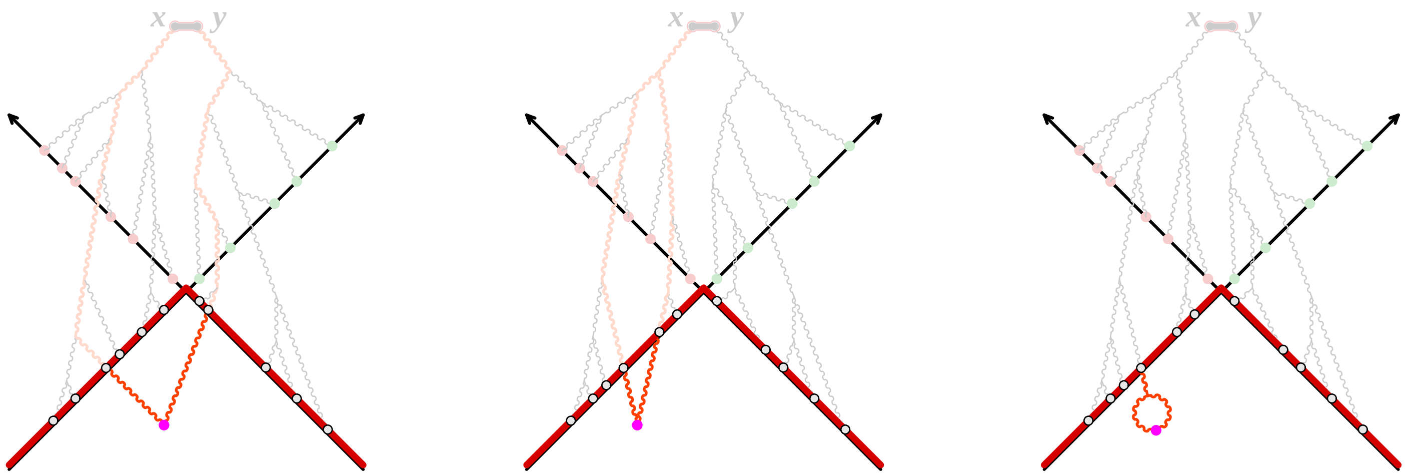
$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm NLO} \sim \left[\frac{1}{2}\int_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in {\rm LC}} \boldsymbol{\Sigma}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\,\mathbb{T}_{\boldsymbol{v}}\right]\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

  ▷ $\mathbb{T}_{\boldsymbol{u}}$ is the generator of shifts of the initial field at the point $\vec{\boldsymbol{u}}$:

$$\mathcal{F}[\mathcal{A}+a] \equiv \left[\exp\int_{\vec{\boldsymbol{u}}\in {\rm LC}} a(\vec{\boldsymbol{u}})\,\mathbb{T}_{\boldsymbol{u}}\right]\mathcal{F}[\mathcal{A}]$$

Single gluon spectrum at NLO

- 1-loop graphs contributing to the gluon spectrum at NLO :



- The loop correction can also be below the light-cone :

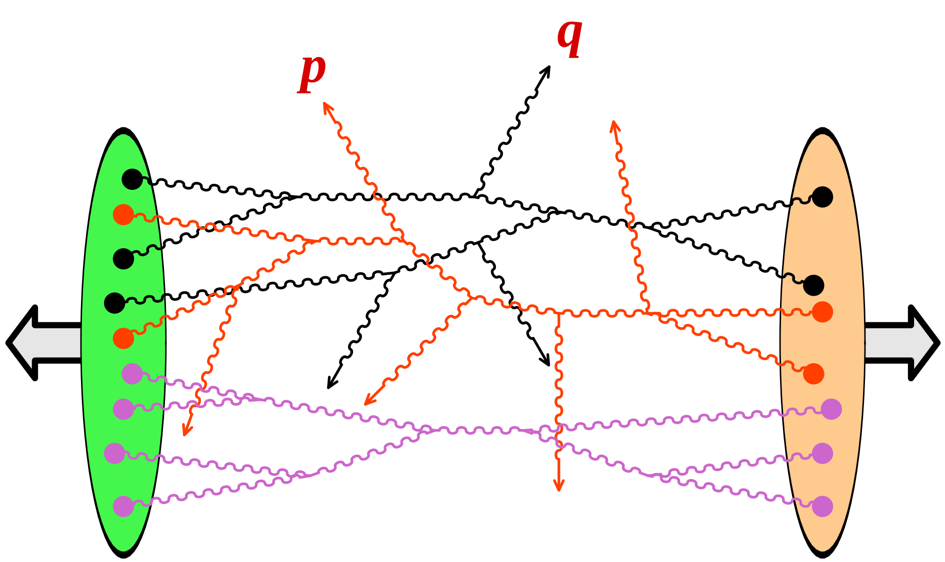
$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm NLO} \sim \Big[\frac{1}{2}\int\limits_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in{\rm LC}} \boldsymbol{\Sigma}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}} + \int\limits_{\vec{\boldsymbol{u}}\in{\rm LC}} \boldsymbol{\beta}(\vec{\boldsymbol{u}})\,\mathbb{T}_{\boldsymbol{u}}\Big]\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

- At leading log, one gets the JIMWLK Hamiltonian $\mathcal{H}[\rho]$, and one can prove the following factorization theorem

$$\left\langle\frac{dN}{d^3\vec{\boldsymbol{p}}}\right\rangle_{\rm LLog} = \int [D\rho_1\, D\rho_2]\; W_{Y_{\rm beam}-y}[\rho_1]\; W_{Y_{\rm beam}+y}[\rho_2]\;\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

Two gluon spectrum

Two gluon spectrum at LO



- Leading Order = tree diagrams only
- Tag two gluons of momenta $\vec{\boldsymbol{p}}$ and $\vec{\boldsymbol{q}}$
- Integrate out the phase-space of all the other gluons

$$\frac{d^2N}{d^3\vec{\boldsymbol{p}}d^3\vec{\boldsymbol{q}}} \sim \sum_{n=0}^{\infty} \frac{1}{n!} \int \left[d^3\vec{\boldsymbol{p}}_1 \cdots d^3\vec{\boldsymbol{p}}_n\right] \left|\left\langle \vec{\boldsymbol{p}}\, \vec{\boldsymbol{q}}\ \vec{\boldsymbol{p}}_1 \cdots \vec{\boldsymbol{p}}_n \middle| 0\right\rangle\right|^2$$

Two gluon spectrum at LO

- LO results for the double gluon spectrum :
  - Disconnected graphs cancel in the quantity

$$\frac{d^2N}{d^3\vec{\boldsymbol{p}}d^3\vec{\boldsymbol{q}}} - \frac{dN}{d^3\vec{\boldsymbol{p}}}\frac{dN}{d^3\vec{\boldsymbol{q}}}$$

  - At LO, this quantity is made of tree diagrams, whose sum can be expressed in terms of the retarded classical field $\mathcal{A}^\mu(x)$ and of small fluctuations $\eta^\mu_{\pm\boldsymbol{k}}(x)$ above the classical field, also with retarded boundary conditions :

$$\big[\mathcal{D}_\mu,\big[\mathcal{D}^\mu,\eta^\nu_{\pm\boldsymbol{k}}\big]\big] - \big[\mathcal{D}_\mu,\big[\mathcal{D}^\nu,\eta^\mu_{\pm\boldsymbol{k}}\big]\big] - ig\big[\eta^\mu_{\pm\boldsymbol{k}},\mathcal{F}_\mu{}^\nu\big] = 0$$

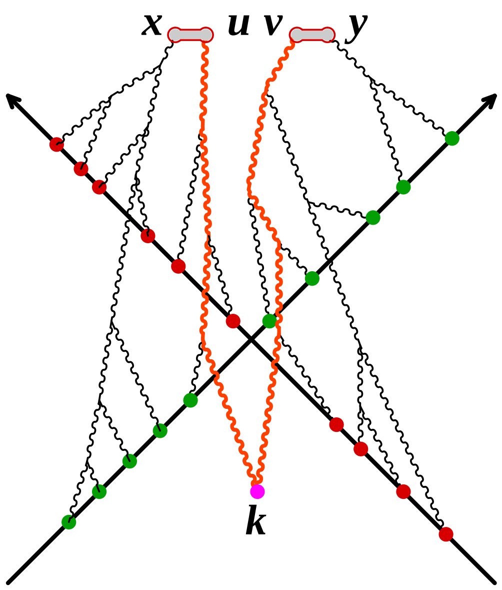
$$\lim_{t\to-\infty}\eta^\mu_{\pm\boldsymbol{k}}(t,\vec{\boldsymbol{x}}) = \epsilon^\mu(\boldsymbol{k})\,e^{\pm ik\cdot x}$$

Two gluon spectrum at LO

- The structure of the 2-gluon correlation at LO is :

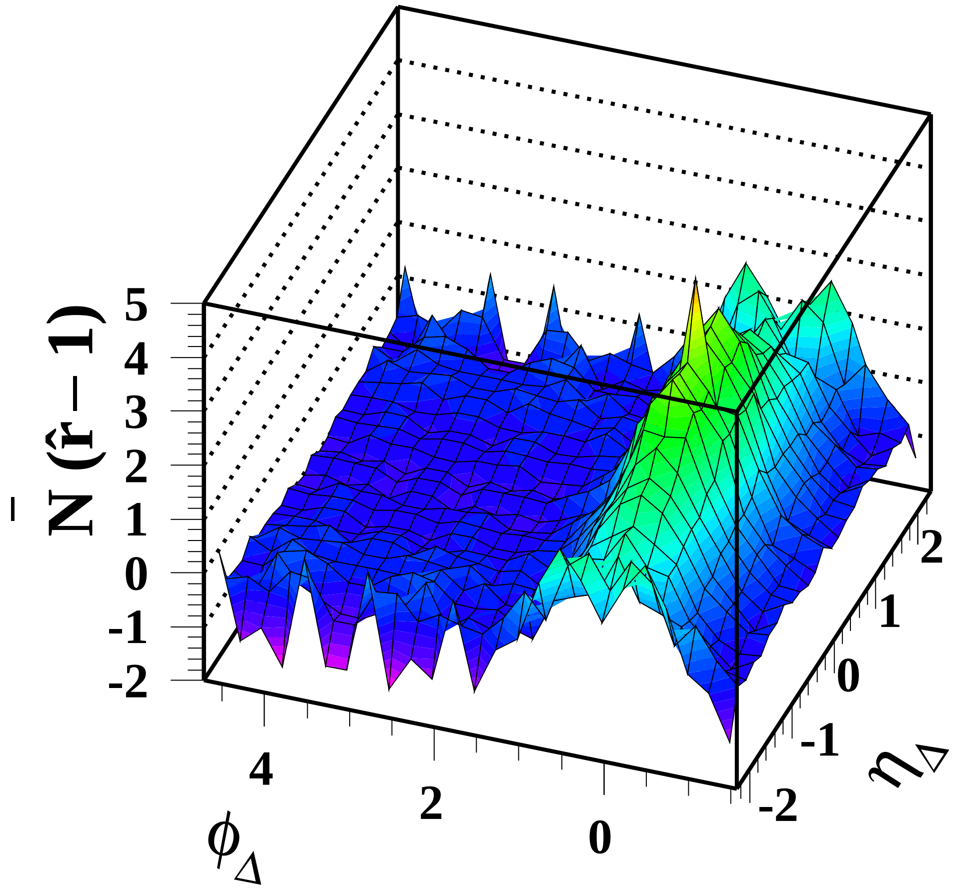
$$\frac{d^2N}{d^3\vec{p}\,d^3\vec{q}} - \frac{dN}{d^3\vec{p}}\,\frac{dN}{d^3\vec{q}} \underset{t\to+\infty}{\sim} \int_{\vec{k}} \int_{\vec{x},\vec{y},\vec{u},\vec{v}} e^{i\vec{p}\cdot(\vec{x}-\vec{u})}\, e^{-i\vec{q}\cdot(\vec{y}-\vec{v})} \cdots$$
$$\cdots \mathcal{A}^{\mu}(t,\vec{x})\mathcal{A}^{\nu}(t,\vec{y})\eta^{\rho}_{-\boldsymbol{k}}(t,\vec{u})\eta^{\sigma}_{+\boldsymbol{k}}(t,\vec{v})$$

- Diagrammatically, this corresponds to graphs such as :



Possible explanation of the ridge

- 2-hadron correlation in central AA collisions (STAR, 2006)



(a)

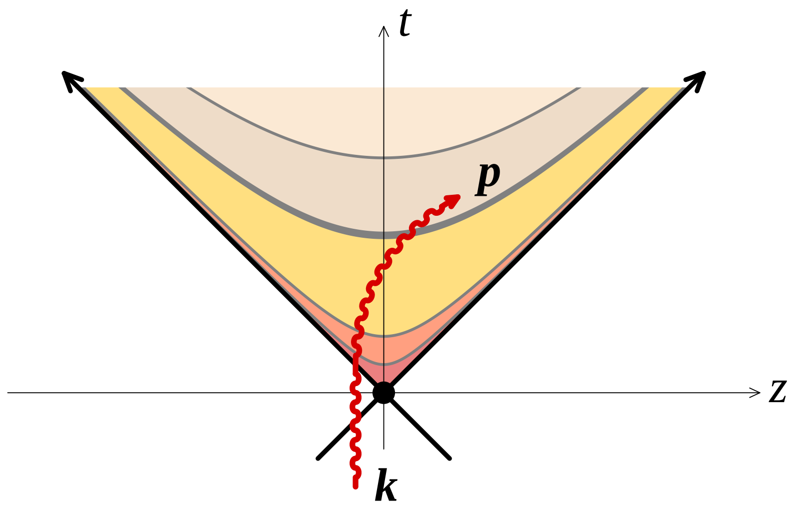
Possible explanation of the ridge

- In the vacuum, the Fourier modes of the fluctuations $\eta^\mu_{\pm\boldsymbol{k}}(x)$ are either zero or delta functions :

$$\int_{\vec{\boldsymbol{x}}} e^{\mp i\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{x}}} \cdots \eta^\mu_{\pm\boldsymbol{k}}(x) = (2\pi)^3 2E_{\boldsymbol{p}}\delta(\vec{\boldsymbol{p}}-\vec{\boldsymbol{k}})$$

▷ gluon correlations can only be local in $(\vec{\boldsymbol{p}},\vec{\boldsymbol{q}})$ in the dilute regime

- In AA collisions, these fluctuations propagate on top of a classical color field (solution of the Yang-Mills equations)
  - In the fluctuations Fourier modes, $\vec{\boldsymbol{k}}$ is the initial momentum of a colored particle moving on top of this electric field, and $\vec{\boldsymbol{p}}$ its final momentum



Possible explanation of the ridge

- In the vacuum, the Fourier modes of the fluctuations $\eta^{\mu}_{\pm\boldsymbol{k}}(x)$ are either zero or delta functions :

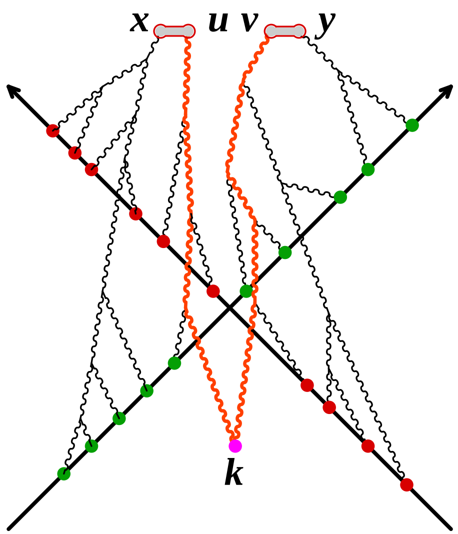
$$\int_{\vec{\boldsymbol{x}}} e^{\mp i\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{x}}} \cdots \eta^{\mu}_{\pm\boldsymbol{k}}(x) = (2\pi)^3 2E_{\boldsymbol{p}}\delta(\vec{\boldsymbol{p}} - \vec{\boldsymbol{k}})$$

  ▷ gluon correlations can only be local in $(\vec{\boldsymbol{p}}, \vec{\boldsymbol{q}})$ in the dilute regime

- In AA collisions, these fluctuations propagate on top of a classical color field (solution of the Yang-Mills equations)
  - In the fluctuations Fourier modes, $\vec{\boldsymbol{k}}$ is the initial momentum of a colored particle moving on top of this electric field, and $\vec{\boldsymbol{p}}$ its final momentum
  - If the background field has a strong electric field in the longitunal direction (and small transverse components), these Fourier modes have support for k_z quite different from p_z, while $\vec{\boldsymbol{p}}_\perp \approx \vec{\boldsymbol{k}}_\perp$
  - When inserted into the formula for the 2-gluon correlations, the correlation is elongated in the z direction, and remains narrow in the transverse direction

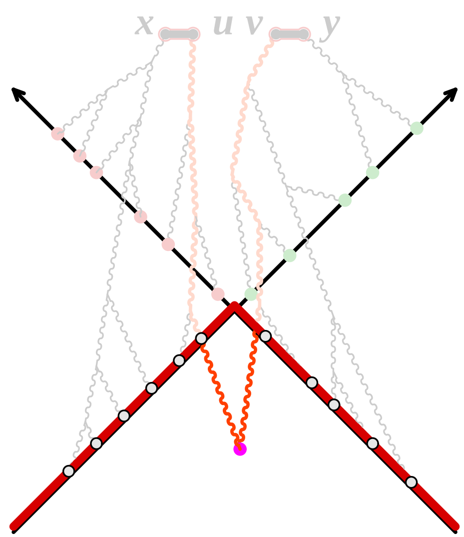
NLO corrections and factorization

- Many terms to evaluate at 1-loop (work in progress with T. Lappi and R. Venugopalan)
- Recall the structure of the tree-level terms :



NLO corrections and factorization

- Many terms to evaluate at 1-loop (work in progress with T. Lappi and R. Venugopalan)
- Recall the structure of the tree-level terms :



- They can be rewritten as a perturbation of the initial conditions in the product of two 1-particle spectra :

$$\frac{d^2N}{d^3\vec{\boldsymbol{p}}d^3\vec{\boldsymbol{q}}} - \frac{dN}{d^3\vec{\boldsymbol{p}}}\,\frac{dN}{d^3\vec{\boldsymbol{q}}}\Big|_{\rm LO} = \frac{1}{2}\int_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in{\rm LC}} \boldsymbol{\Sigma}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\;\left[\mathbb{T}_{\boldsymbol{u}}\frac{dN}{d^3\vec{\boldsymbol{p}}}\right]_{\rm LO}\times\left[\mathbb{T}_{\boldsymbol{v}}\frac{dN}{d^3\vec{\boldsymbol{q}}}\right]_{\rm LO}$$

NLO corrections and factorization

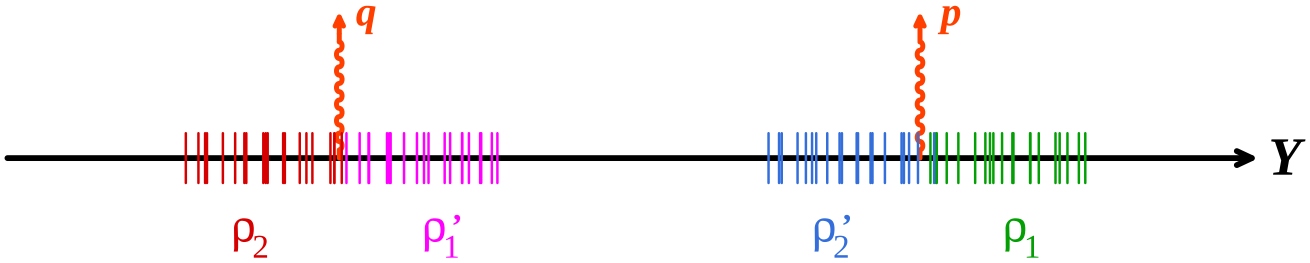
- Based on this Leading Order expression, one can conjecture the following factorization formula for the Leading Log expression of the 2-gluon correlation :

$$\left\langle \frac{d^2N}{d^3\vec{\boldsymbol{p}}\,d^3\vec{\boldsymbol{q}}} - \frac{dN}{d^3\vec{\boldsymbol{p}}}\,\frac{dN}{d^3\vec{\boldsymbol{q}}} \right\rangle_{\rm LLog} = \int [D\rho_1\, D\rho_2]\; W_{Y_{\rm beam}-y_{\boldsymbol{p}}}[\rho_1]\; W_{Y_{\rm beam}+y_{\boldsymbol{q}}}[\rho_2]$$

$$\times \frac{1}{2}\int [D\rho_1'\, D\rho_2'] \int\limits_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in{\rm LC}} \boldsymbol{\Sigma}_{y_{\boldsymbol{p}}-y_{\boldsymbol{q}}}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}|\rho_1',\rho_2')$$

$$\times \left[\mathbb{T}_{\boldsymbol{u}}\frac{dN}{d^3\vec{\boldsymbol{p}}}(1,2')\right]_{\rm LO} \times \left[\mathbb{T}_{\boldsymbol{v}}\frac{dN}{d^3\vec{\boldsymbol{q}}}(1',2)\right]_{\rm LO}$$

  - Interpretation :



NLO corrections and factorization

- Based on this Leading Order expression, one can conjecture the following factorization formula for the Leading Log expression of the 2-gluon correlation :

$$\left\langle \frac{d^2N}{d^3\vec{\boldsymbol{p}}d^3\vec{\boldsymbol{q}}} - \frac{dN}{d^3\vec{\boldsymbol{p}}}\,\frac{dN}{d^3\vec{\boldsymbol{q}}} \right\rangle_{\rm LLog} = \int [D\rho_1\, D\rho_2]\; W_{Y_{\rm beam}-y_{\boldsymbol{p}}}[\rho_1]\; W_{Y_{\rm beam}+y_{\boldsymbol{q}}}[\rho_2]$$

$$\times \frac{1}{2}\int [D\rho_1'\, D\rho_2'] \int_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in{\rm LC}} \boldsymbol{\Sigma}_{y_{\boldsymbol{p}}-y_{\boldsymbol{q}}}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}|\rho_1',\rho_2')$$

$$\times \left[\mathbb{T}_{\boldsymbol{u}}\frac{dN}{d^3\vec{\boldsymbol{p}}}(1,2')\right]_{\rm LO} \times \left[\mathbb{T}_{\boldsymbol{v}}\frac{dN}{d^3\vec{\boldsymbol{q}}}(1',2)\right]_{\rm LO}$$

  - The function $\boldsymbol{\Sigma}_{y_{\boldsymbol{p}}-y_{\boldsymbol{q}}}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}|\rho_1',\rho_2')$ resums the large logs that arise when the rapidity between the two gluons is large :

    Evol. equation : $\partial_Y\,\boldsymbol{\Sigma}_Y = ???$

    Init. condition : $\boldsymbol{\Sigma}_{Y=0}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}|\rho_1',\rho_2') = \boldsymbol{\Sigma}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\delta(\rho_1-\rho_1')\delta(\rho_2-\rho_2')$

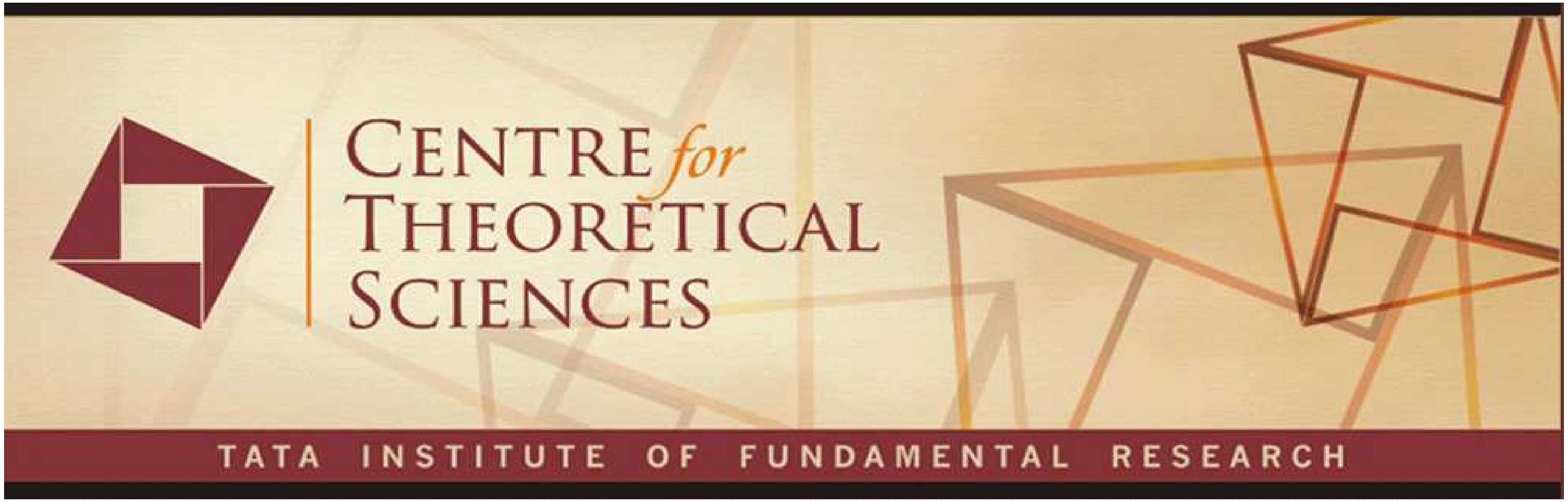
Summary

Summary

- The single gluon spectrum at LO involves only retarded classical fields
- The double inclusive spectrum at LO involves classical fields and small fluctuations, both with retarded boundary conditions
- Because the classical $\vec{E}$ field is longitudinal shortly after the collision, deformation of the 2-gluon correlation function in the η direction, that may cause the ridge
- Factorization :
  - Works (as expected) for the single inclusive gluon spectrum
  - More complicated for the 2-gluon spectrum when there is a large rapidity interval between the two gluons (the 1-loop corrections to the 2-gluon spectrum must be fully evaluated to assess that)
- Note : with a complete knowledge of the 1- and 2-gluon initial spectra, one could in principle build a CGC-based event generator for AA collisions, that has the correct correlations up to 2 particles (but not beyond that)

Upcoming workshop

Gluon saturation
Single gluon spectrum
Two gluon spectrum
Summary



Initial Conditions in Heavy-Ion Collisions

QCD at high parton densities

- Place : International Center, Dona Paula, Goa, India
- Dates : September 1-22, 2008
- School : September 8-12, 2008
- Organizers : R. Gavai, FG, S. Gupta, R. Venugopalan
- Webpage : http://theory.tifr.res.in/ qcdinit/