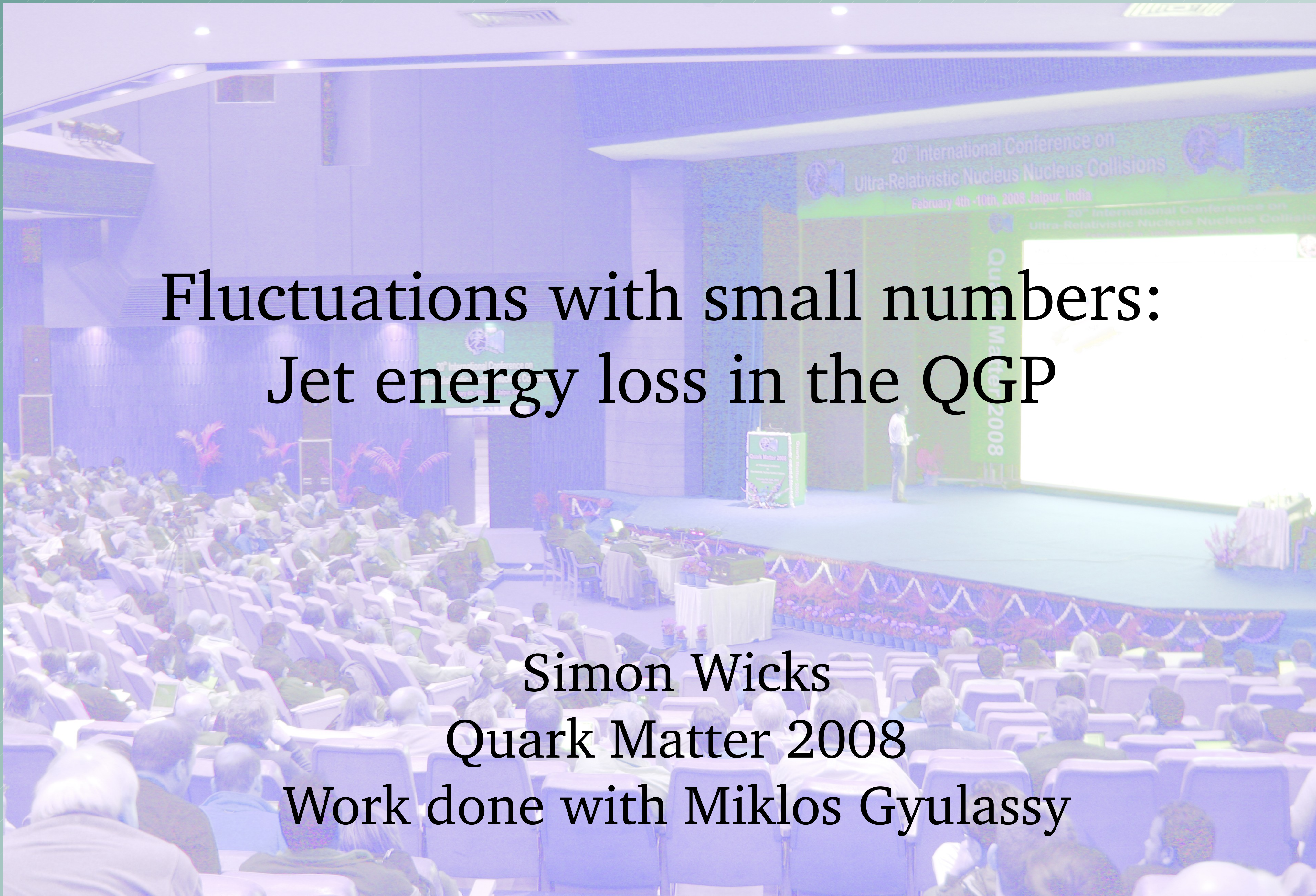
Fluctuations with small numbers: Jet energy loss in the QGP



Simon Wicks
Quark Matter 2008
Work done with Miklos Gyulassy

Questions:

Do we have a calibrated probe?

What are our major sources of theoretical uncertainty?

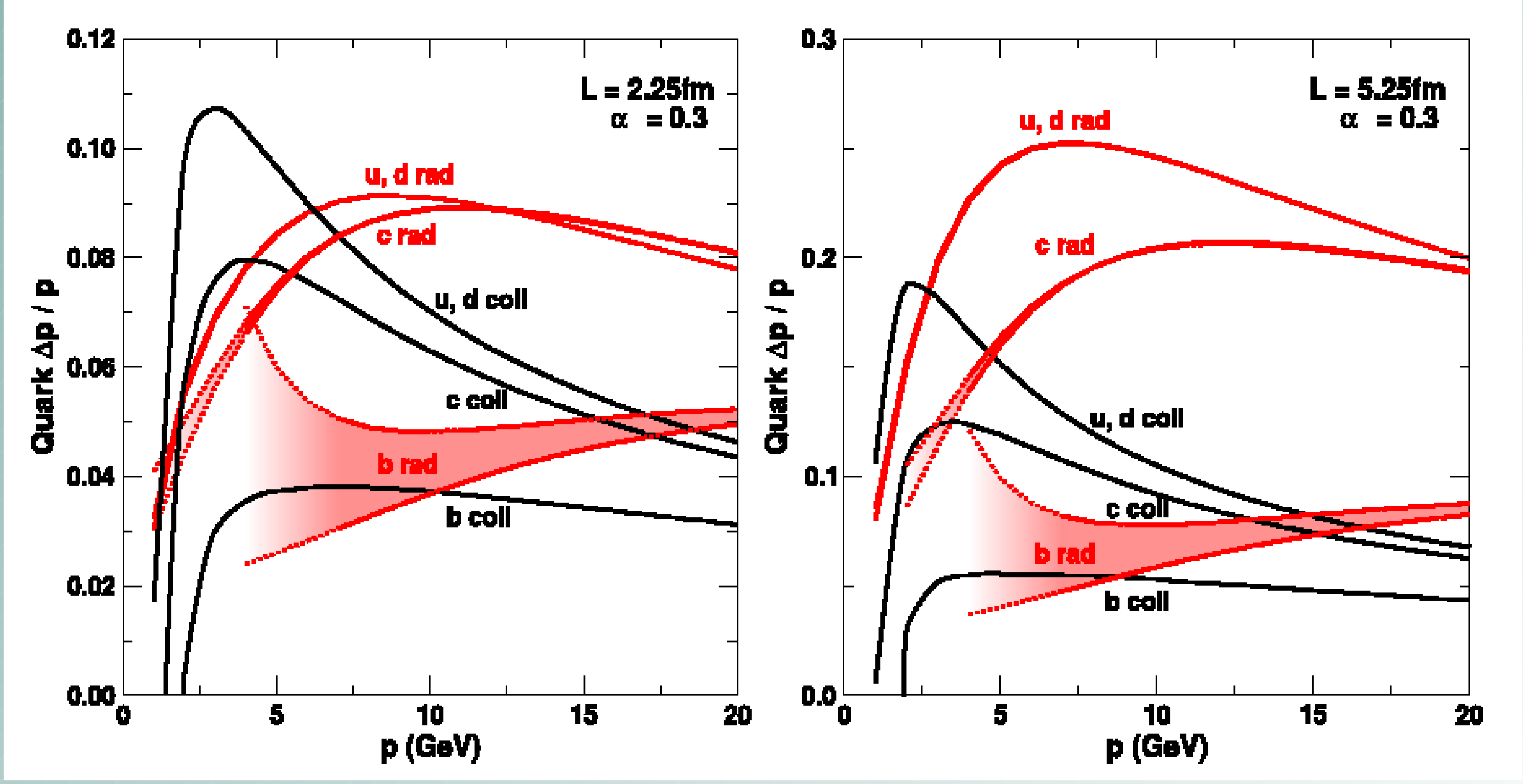
Can we use brute force numerical work to check our simple, analytical formulae?

Uncertainty?

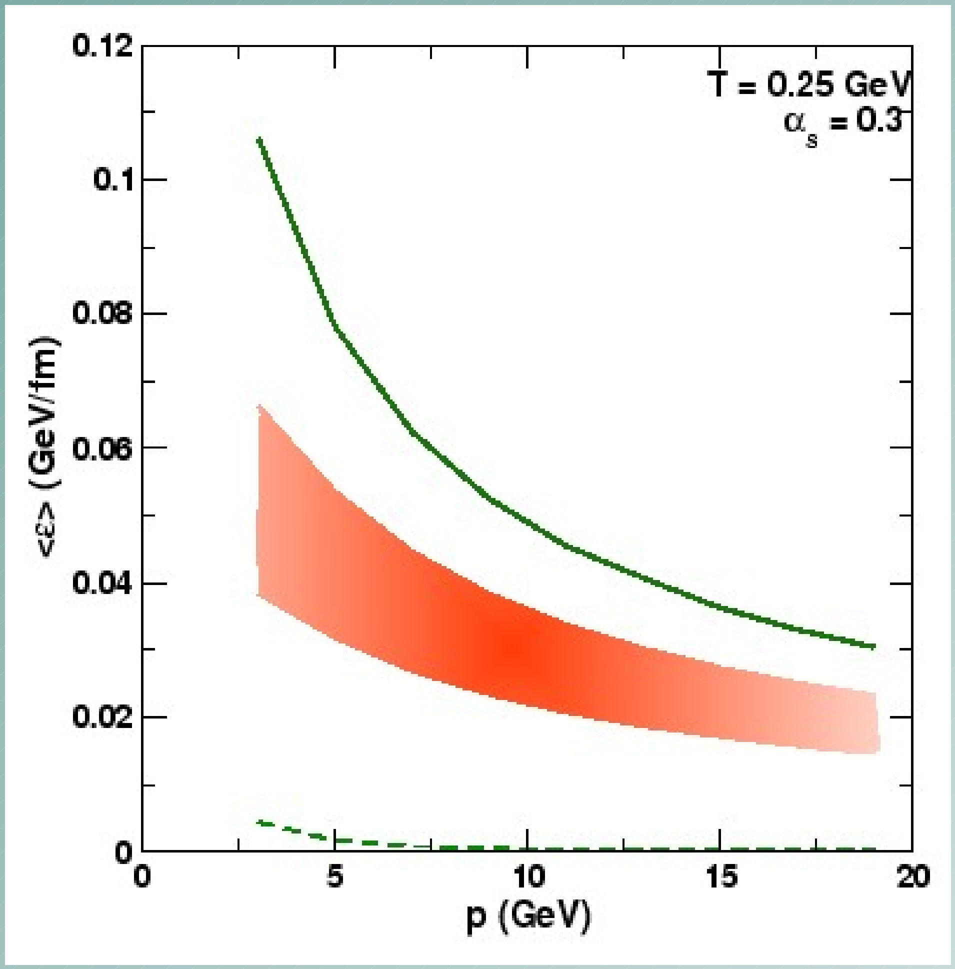
1. Mechanisms of energy loss
2. Multiple event convolution
   - Collisional
   - Radiative
     - Orders in opacity
       - Brute force numerics

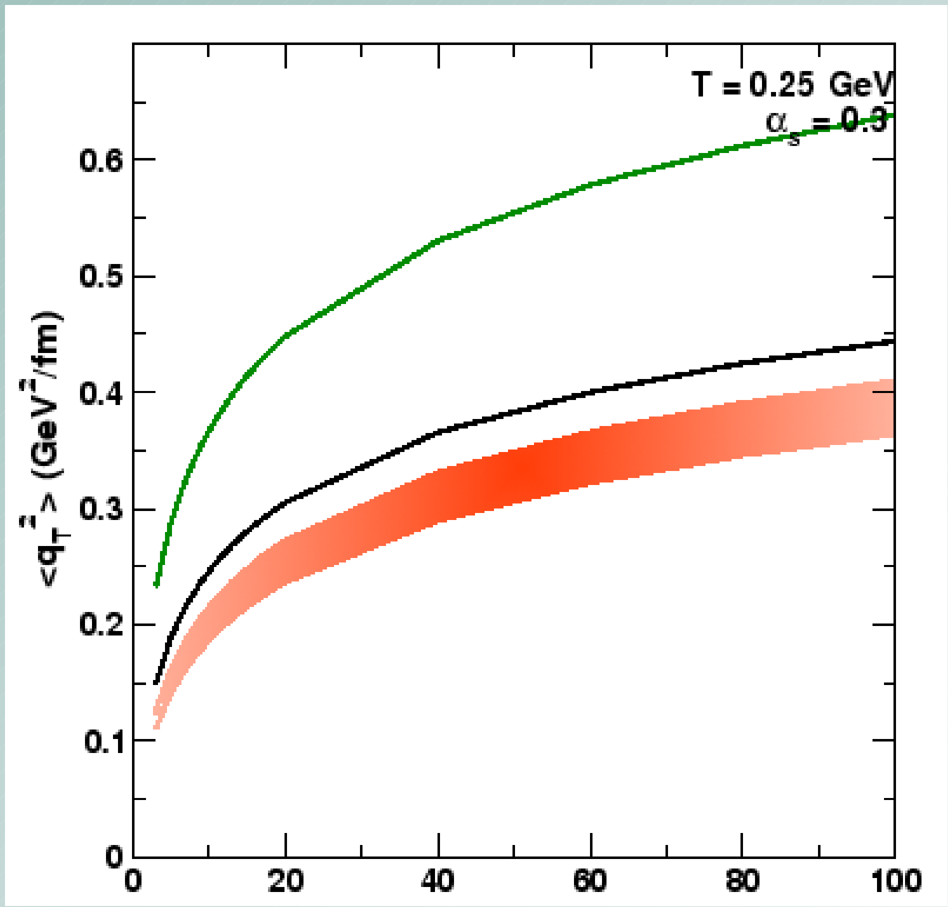
Mechanisms of energy loss

- Radiative
- Collisional



Soft collision approximations





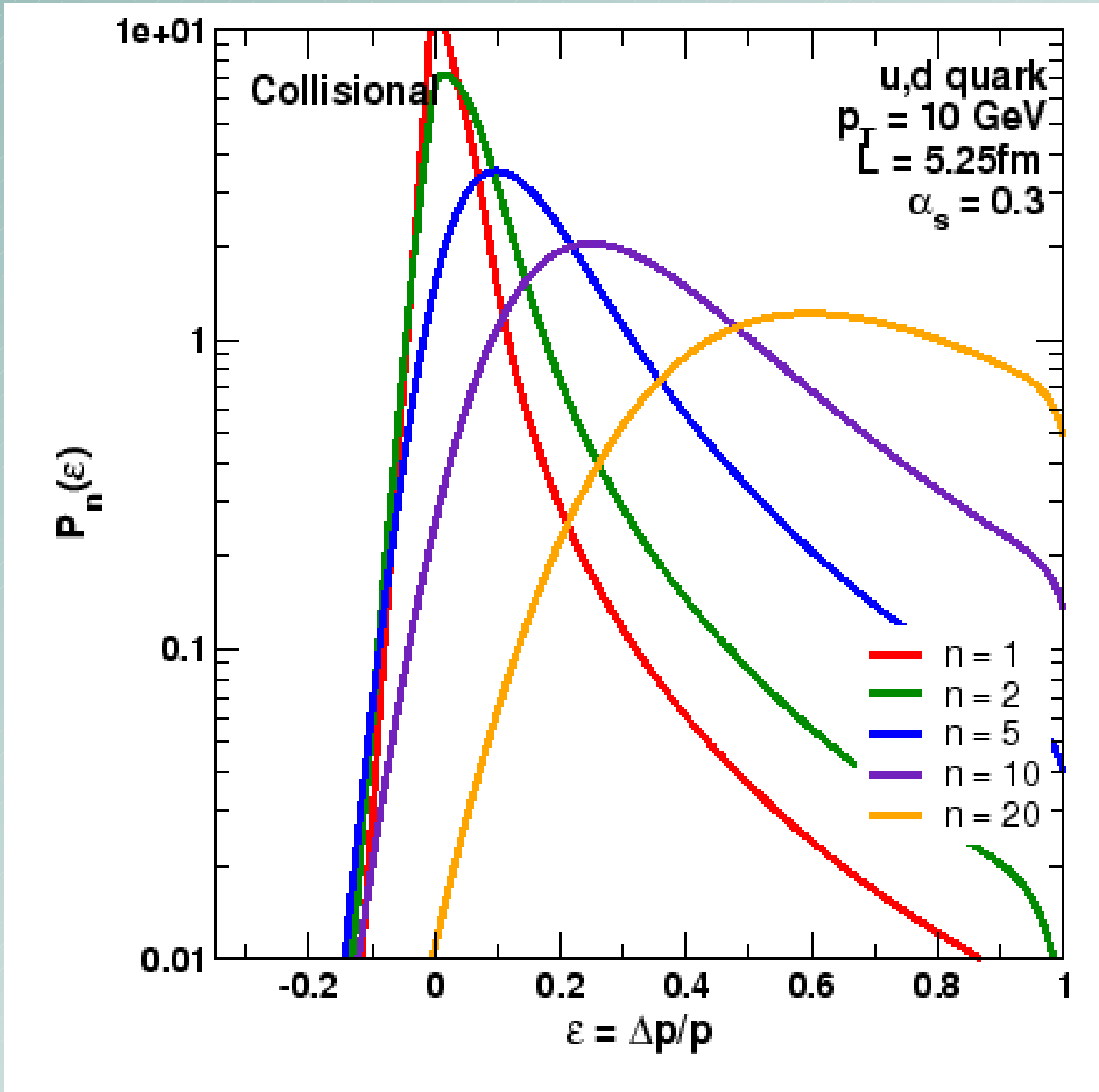
Systematic assumption that all important q << mu overestimates collisional energy loss and $<q_T^2>$ by a factor ~ 1.5->2

High q tails are important (at least for the average)

Multiple collisions
Collisional energy loss

- Take into account the finite, small number of collisions of a typical jet

  -> Fluctuations with small numbers



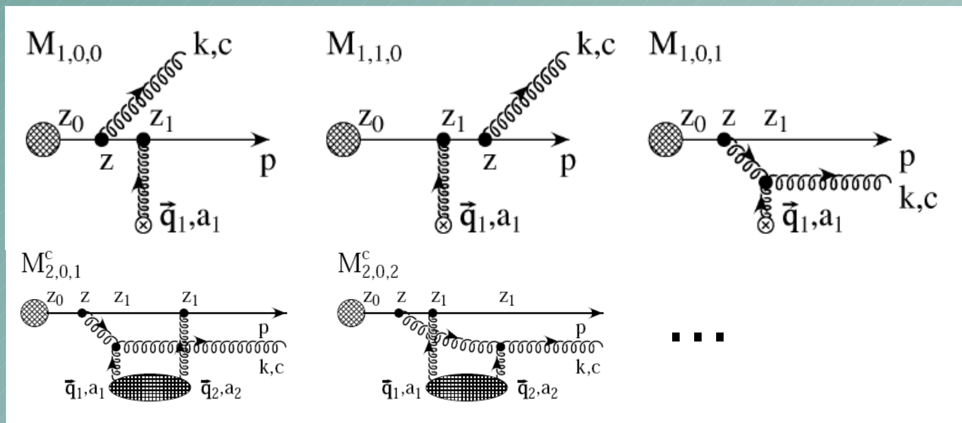
Multiple collisions
Radiative energy loss

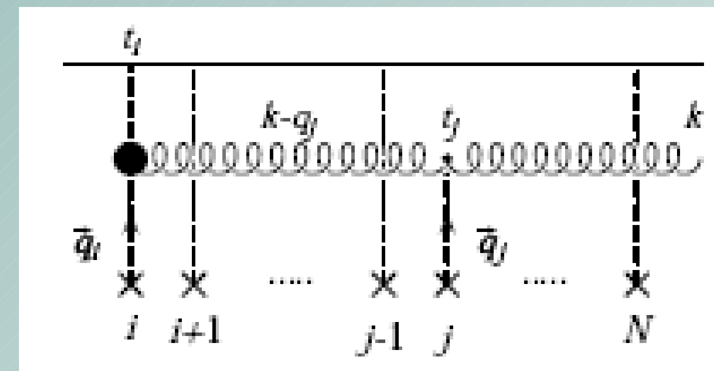
- Orders in opacity
  - Again, fluctuations with small numbers

Orders in opacity

- Opacity expansion
  - Short distances -> radiation dominated by 'creation radiation', and small induced component which interferes with it
  - Long distances, short formation time gluon -> radiation dominated by incoherent radiative emission
  - Long distances, long formation time gluon -> radiation dominated by induced radiation, interference between multiple scattering centres

Thin or thick plasma?





Thin

n =1

Thick

n -> infinity

(A)SW Quenching Weights

Higher Twist

'GLV'

BDMPS

AMY

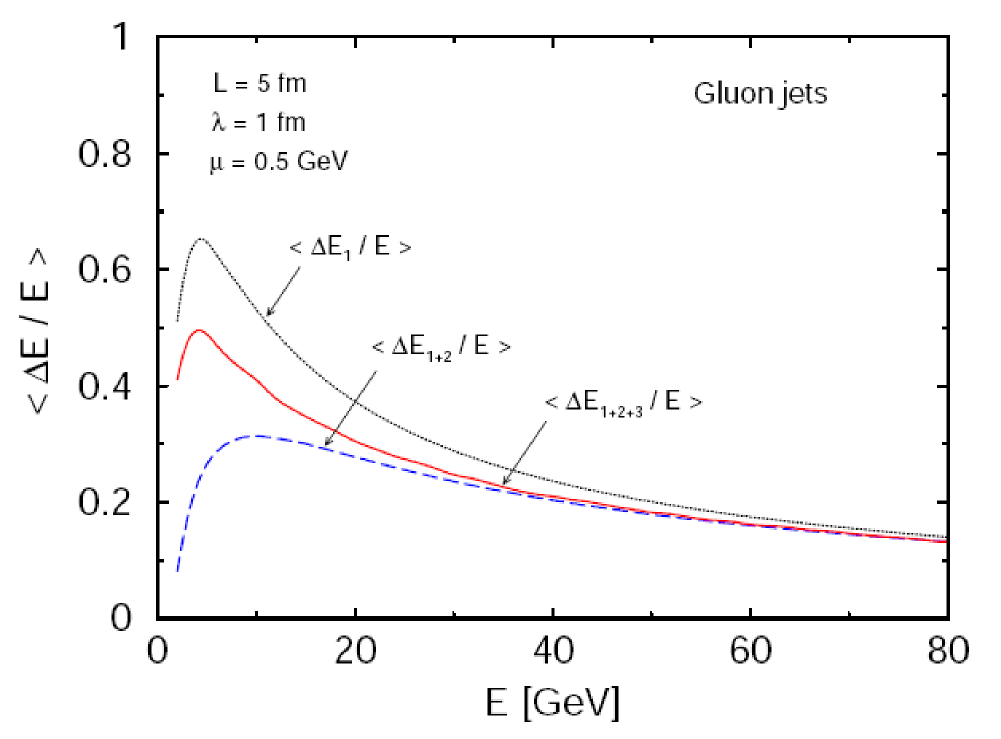
Disclaimer: the arrows correspond to (my interpretation of) the main numerical implementations

Does it matter?

- For
  - Average ΔE?
  - dN/dx (ie R_{AA})?
    - For interpreting our extracted parameter?
  - $dN/dxdk_T$
    - Multi-particle correlations
    - Jet shapes

GLV recursion formula

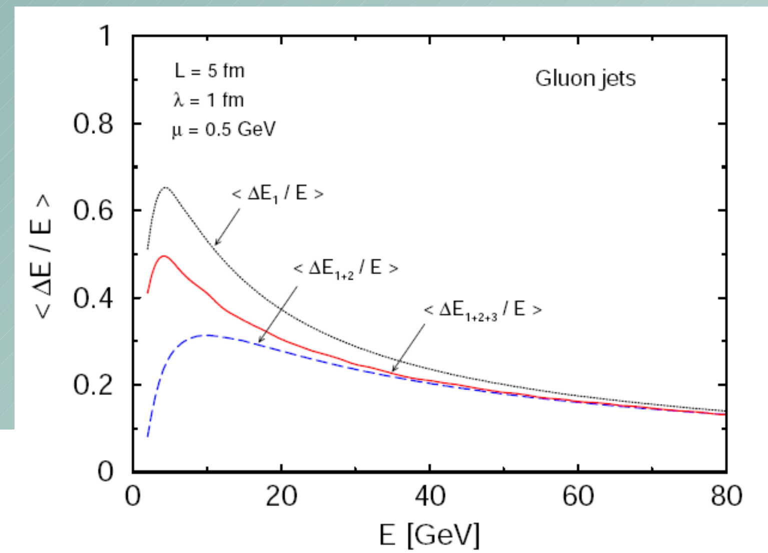
- GLV numerical implentation(s):
  - 1^{st} order in opacity
- GLV formula
  - Sum up to arbitrary order in opacity (within certain approximations)
  - Numerically expensive
    - 3^{rd} order ~ 1^{st} order => use 1^{st} order result

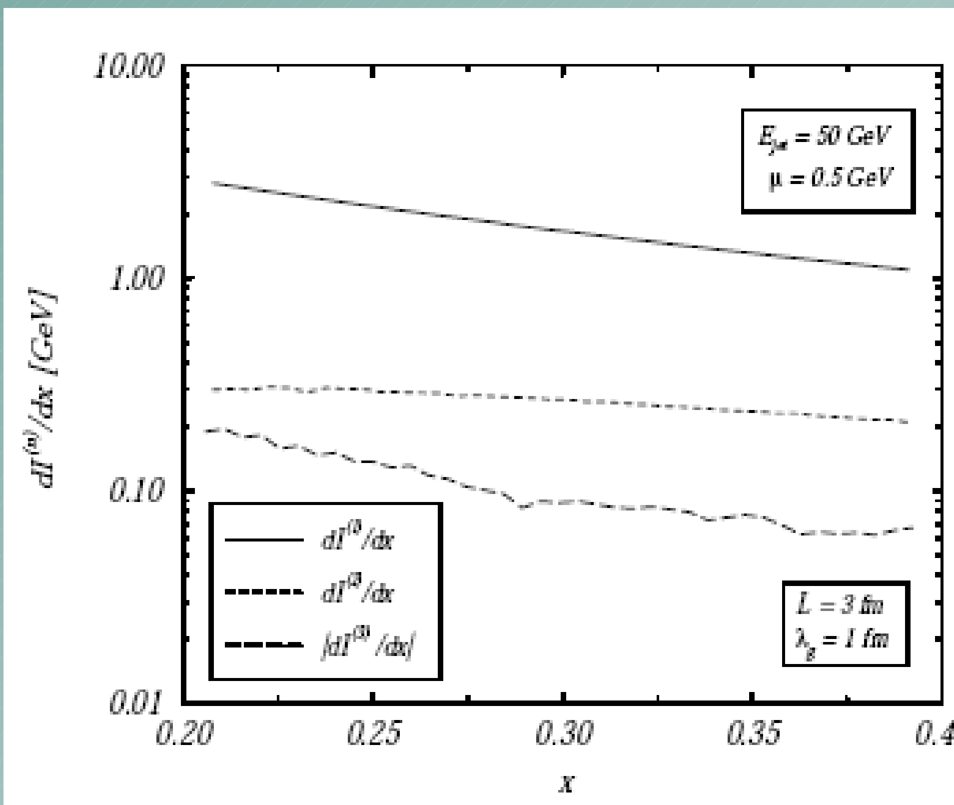


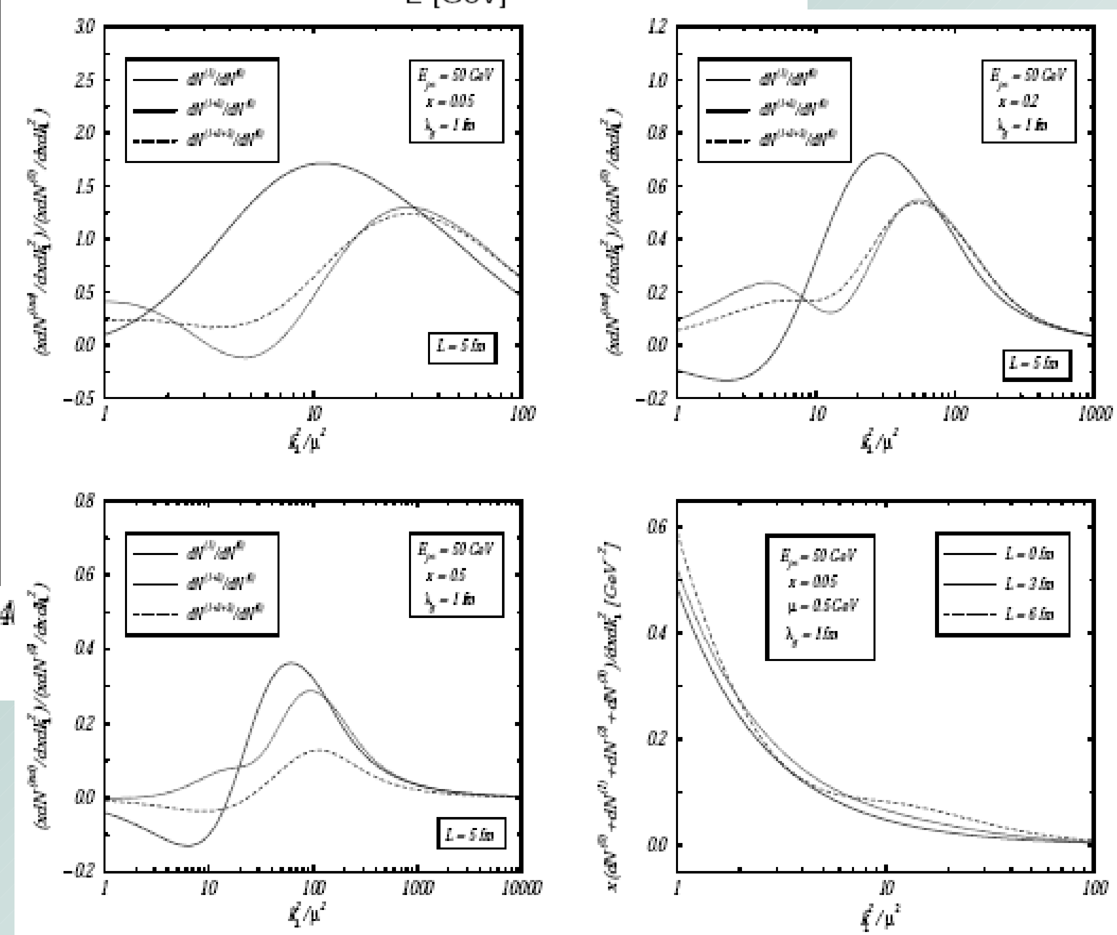
$$x\frac{dN^{(n)}}{dx\,d^2\mathbf{k}} = \frac{C_R\alpha_s}{\pi^2}\frac{1}{n!}\left(\frac{L}{\lambda_g(1)}\right)^n \int \prod_{i=1}^{n}\left(d^2\mathbf{q}_i \left(\frac{\lambda_g(1)}{\lambda_g(i)}\right)\left[\bar{v}_i^2(\mathbf{q}_i) - \delta^2(\mathbf{q}_i)\right]\right)$$
$$\times\left(-2\,\mathbf{C}_{(1,\dots,n)} \cdot \sum_{m=1}^{n} \mathbf{B}_{(m+1,\dots,n)(m,\dots,n)}\right.$$
$$\left.\times\left[\cos\left(\sum_{k=2}^{m}\omega_{(k,\dots,n)}\Delta z_k\right) - \cos\left(\sum_{k=1}^{m}\omega_{(k,\dots,n)}\Delta z_k\right)\right]\right),$$

0^{th} order = 1 diagram
1^{st} order = 13 diagrams
2^{nd} order = 135 diagrams
... 9^{th} order = ???

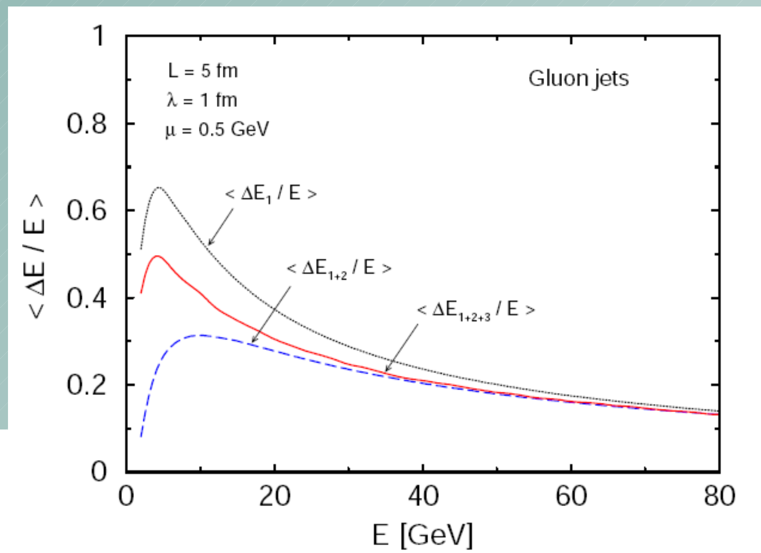
Numerical evaluation 2000/2001

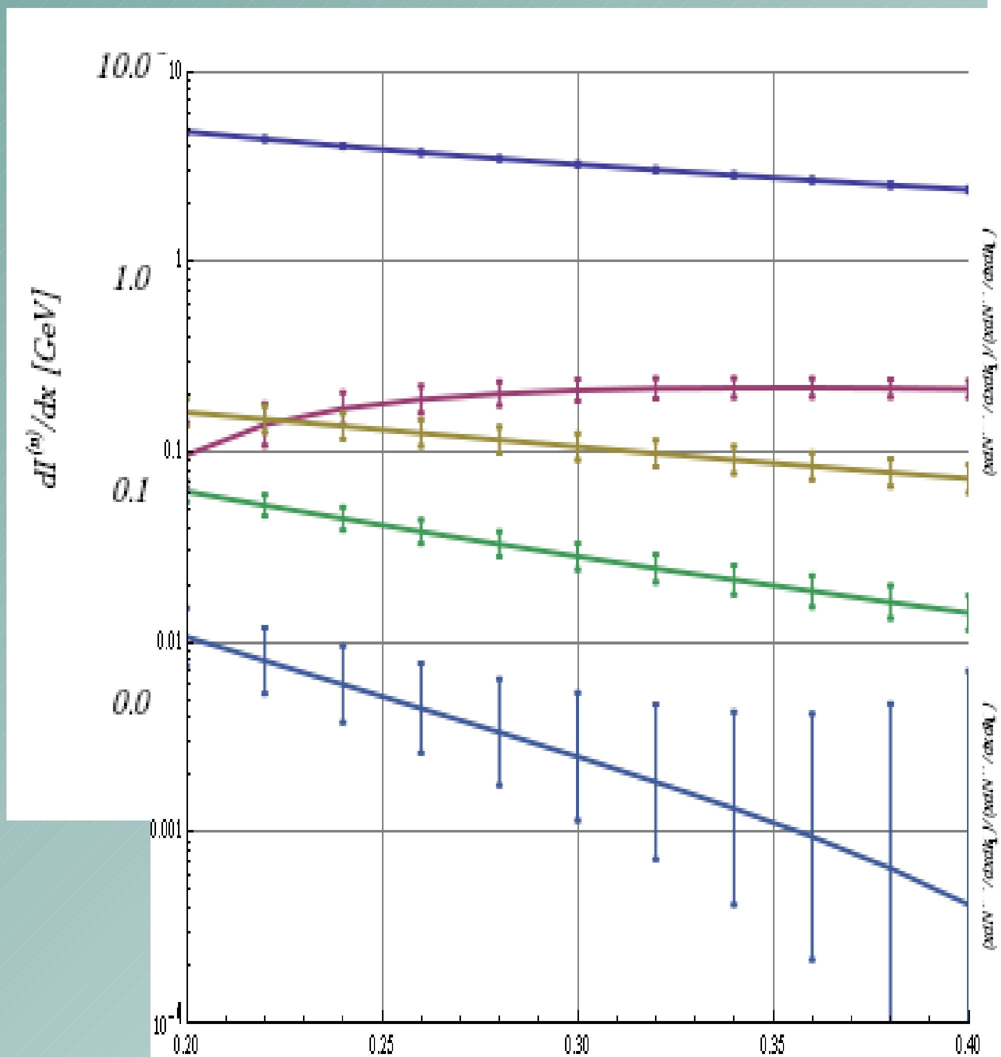


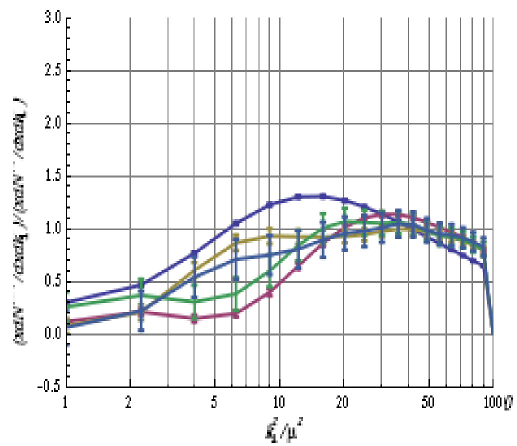


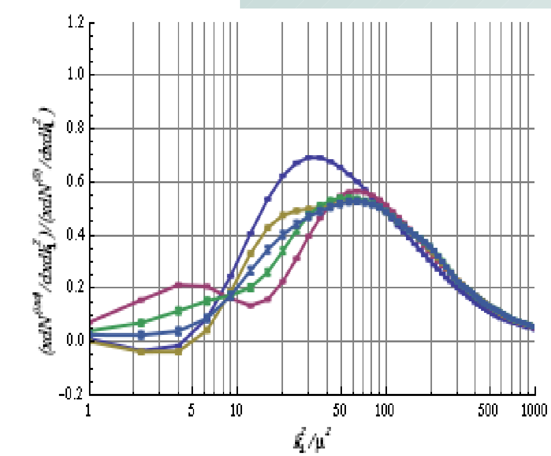


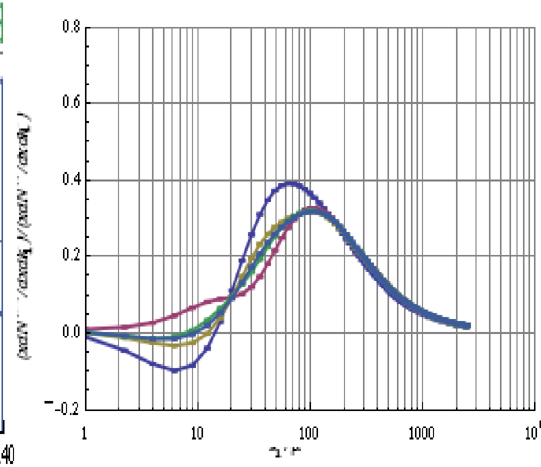
Numerical evaluation 2008

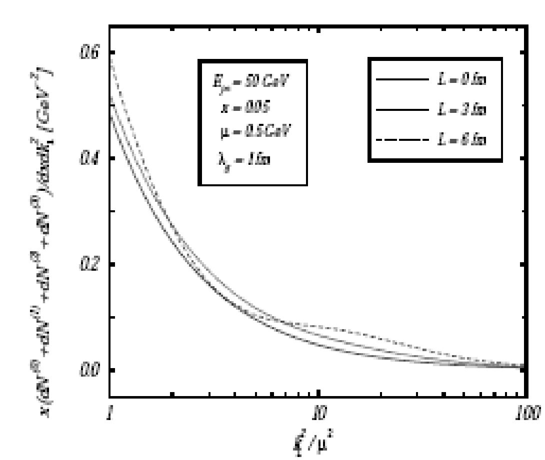












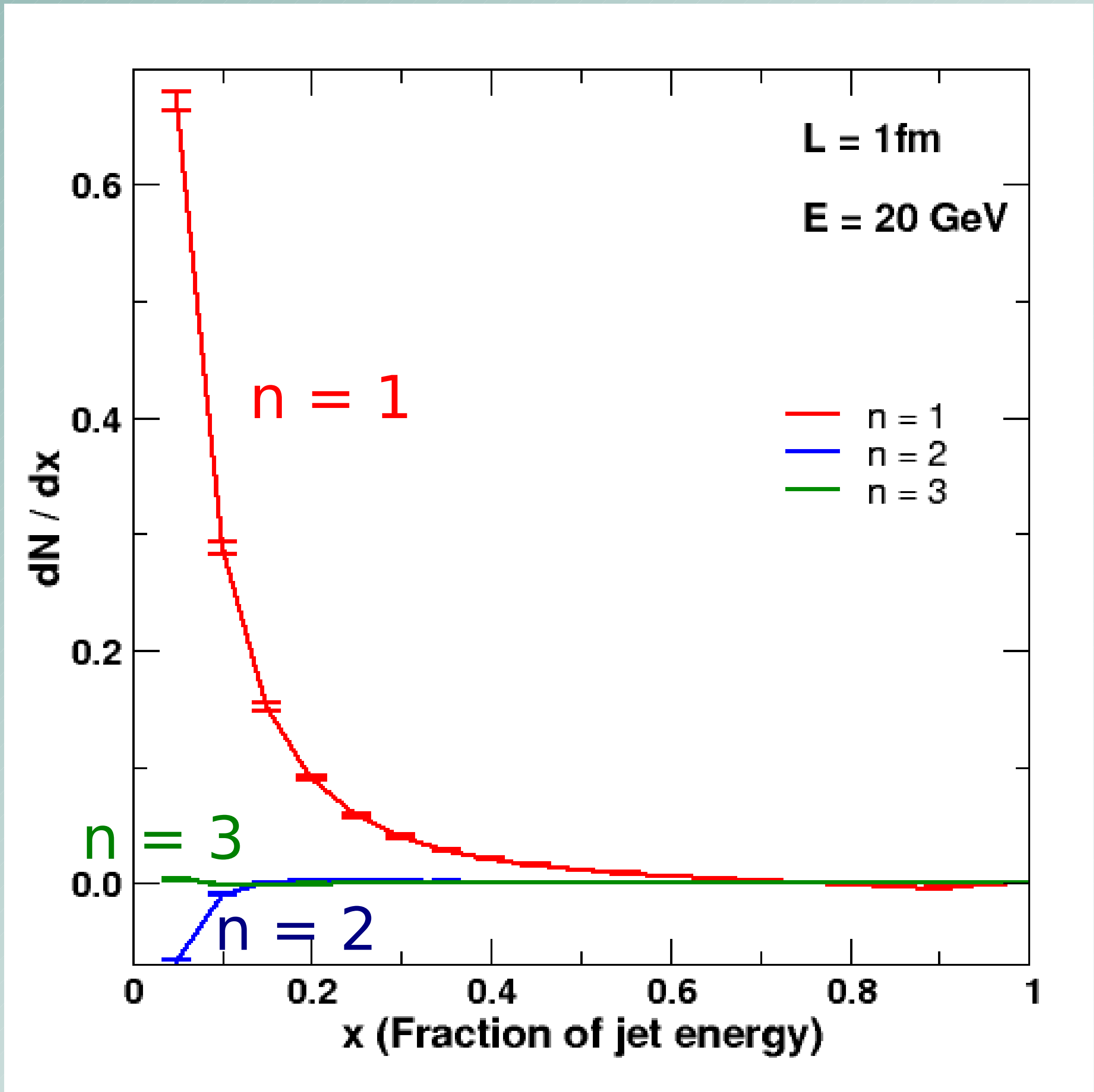
Numerical evaluation – 2008 Monte Carlo

- Evaluate the 2n dimensional integral using Monte Carlo
- The model
  - GLV radiative energy loss only, Gyulassy-Wang model of the medium, soft emission approximation ...
  - Uncorrelated scattering centers
    - It is in fact possible to do correlated scattering centers, arbitrary density profile
  - Static medium, mu=0.5GeV, T=0.25GeV, lambda=1fm, look at sample lengths

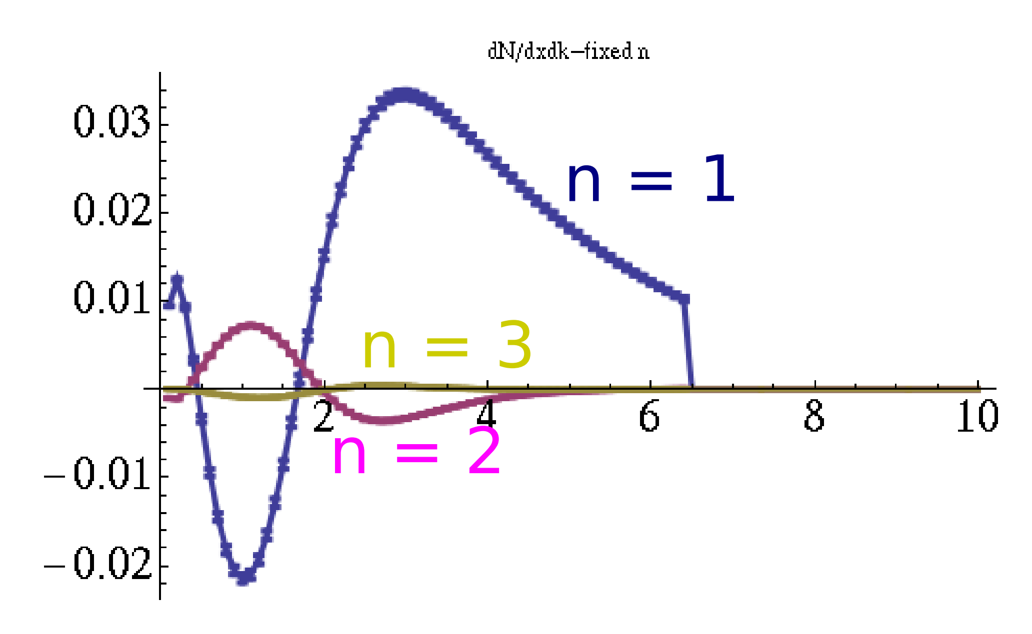
L = 1fm, E = 20GeV - dN/dx

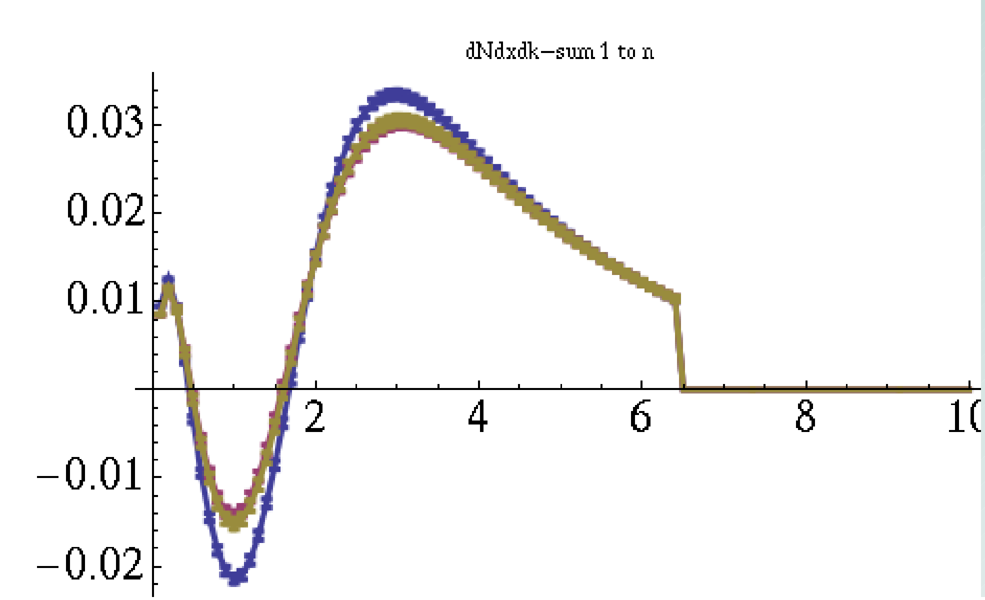
dN/dx
Number of gluons radiated as a function of the radiated gluon energy
(x = gluon / jet energy)

A weighted integral of this function gives R_{AA}



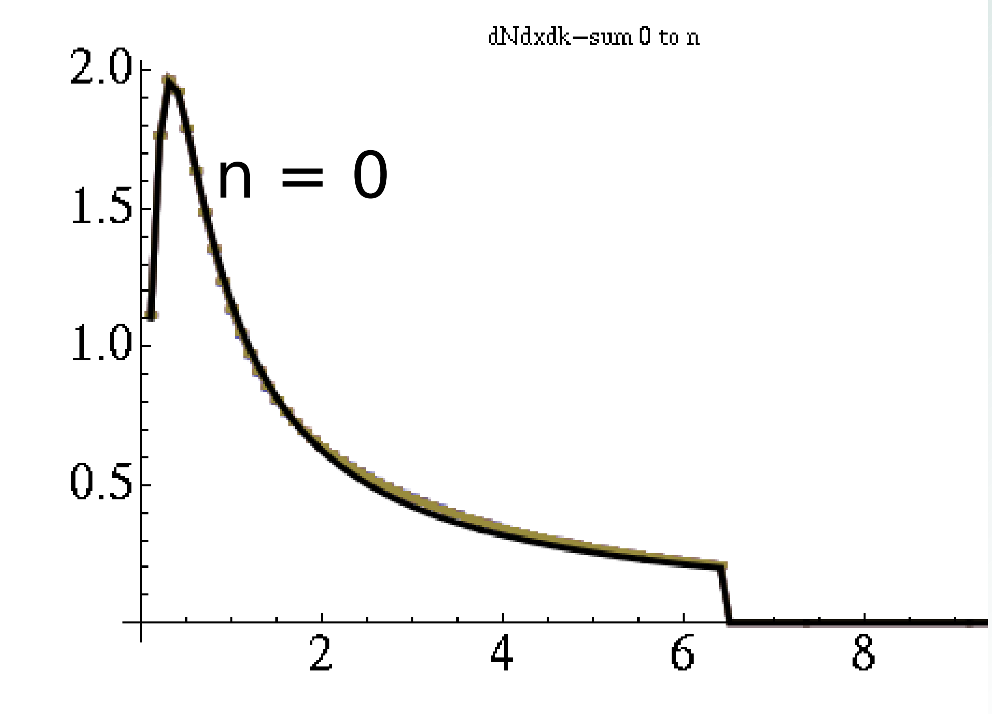
$L = 1fm, E = 20\ GeV - dN/dxdk_T$





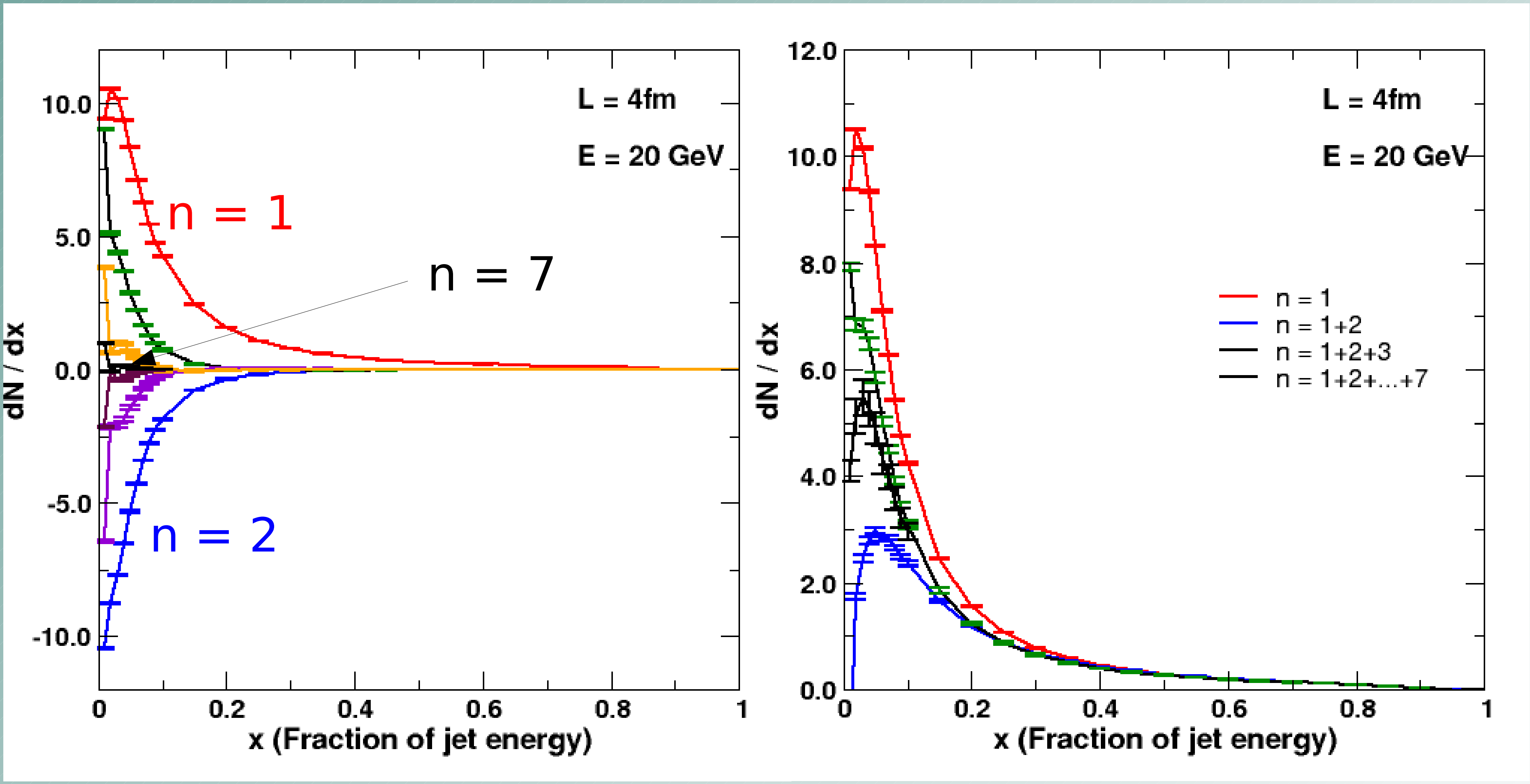
$x = 0.2$

ie emitted gluon=4 GeV



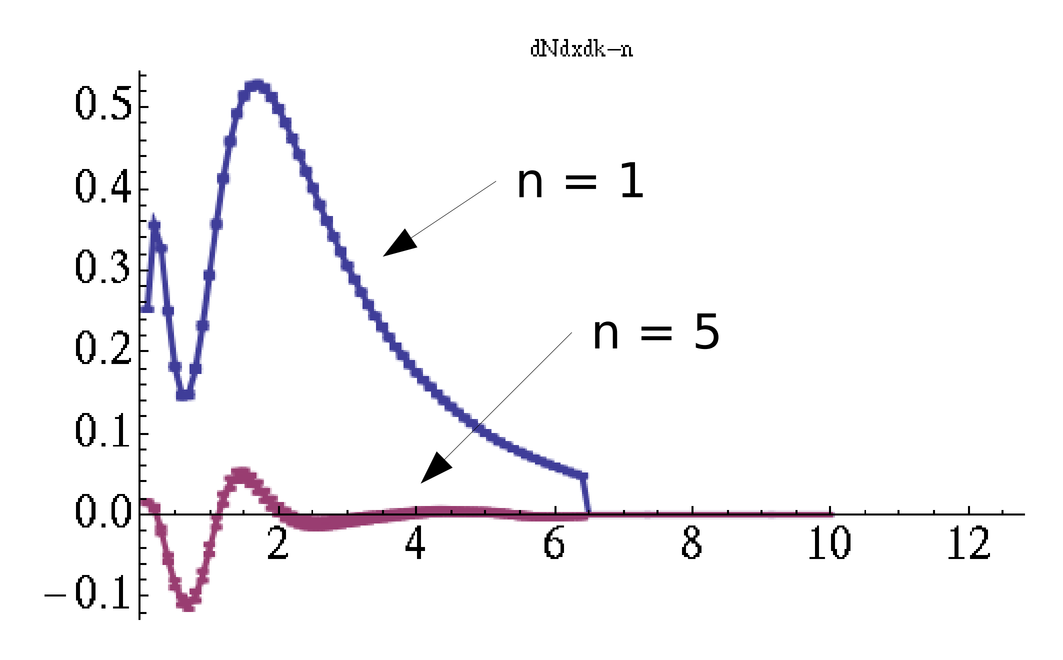
$dN/dxdk_T$ – radiated spectrum with respect to x and k_T, the momentum of the emitted gluon transverse to the direction of the jet

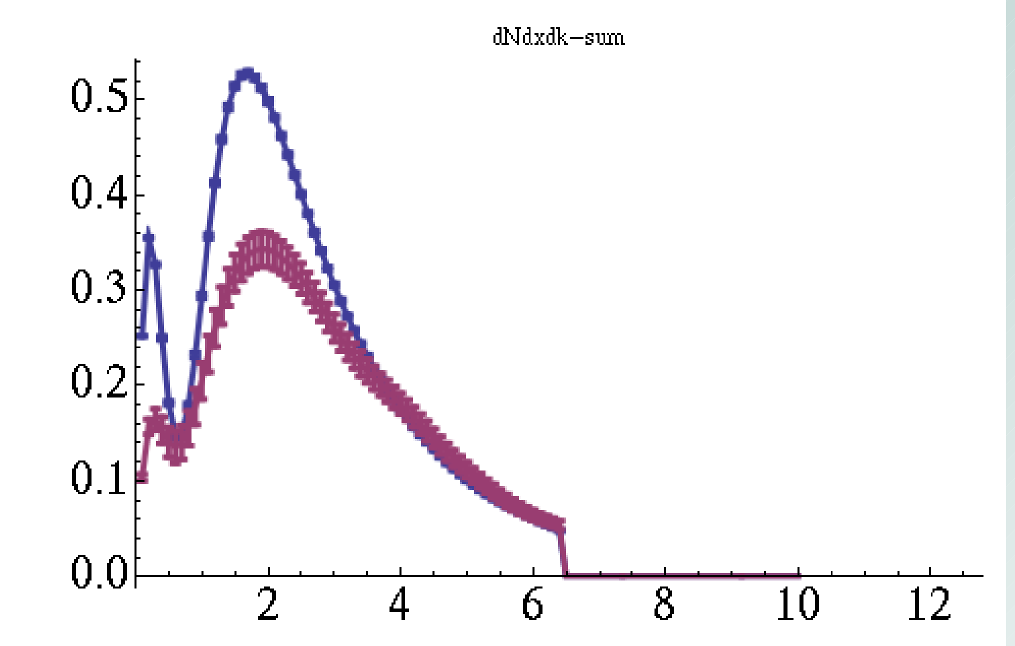
L=4fm, E = 20GeV - dN/dx



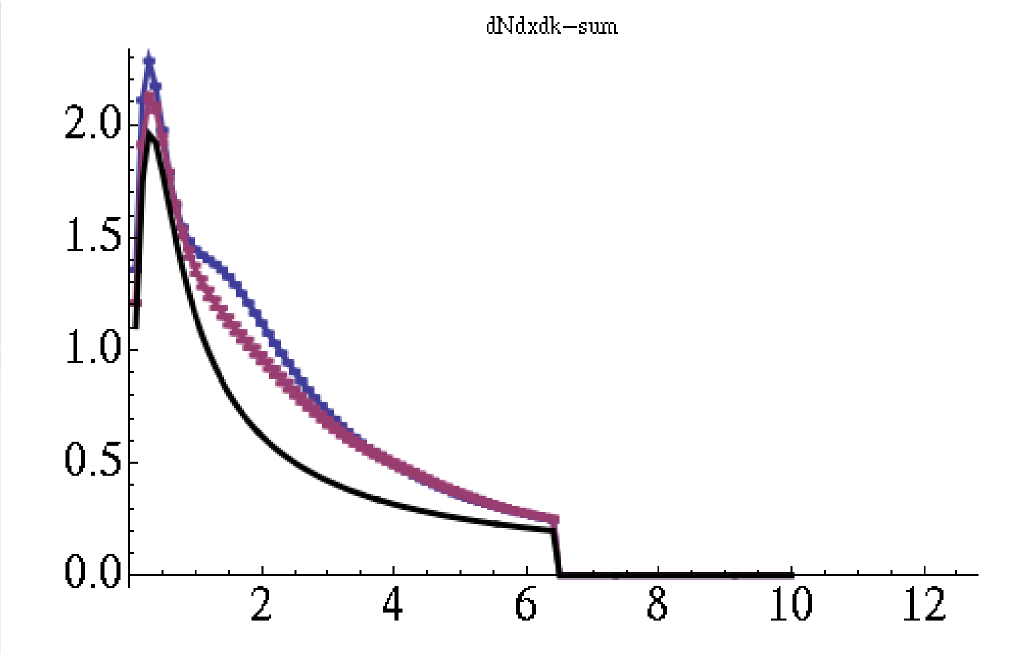
Raa n = 1 = 0.14, n = 2 = 0.27, n = 3 = 0.22

L=4fm, E = 20GeV - dN/dxdk$_T$

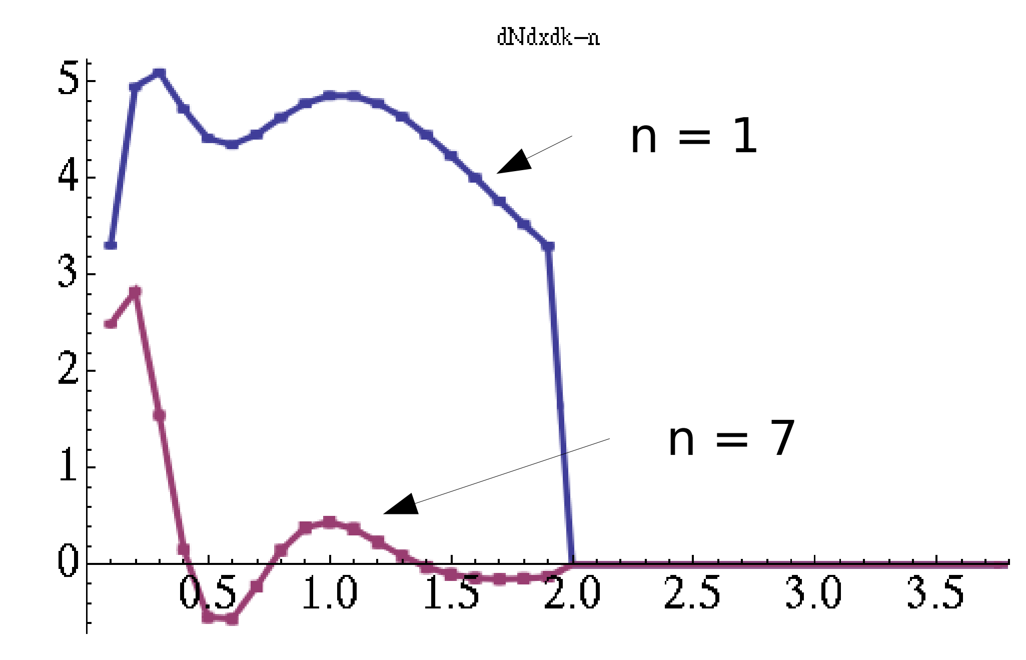


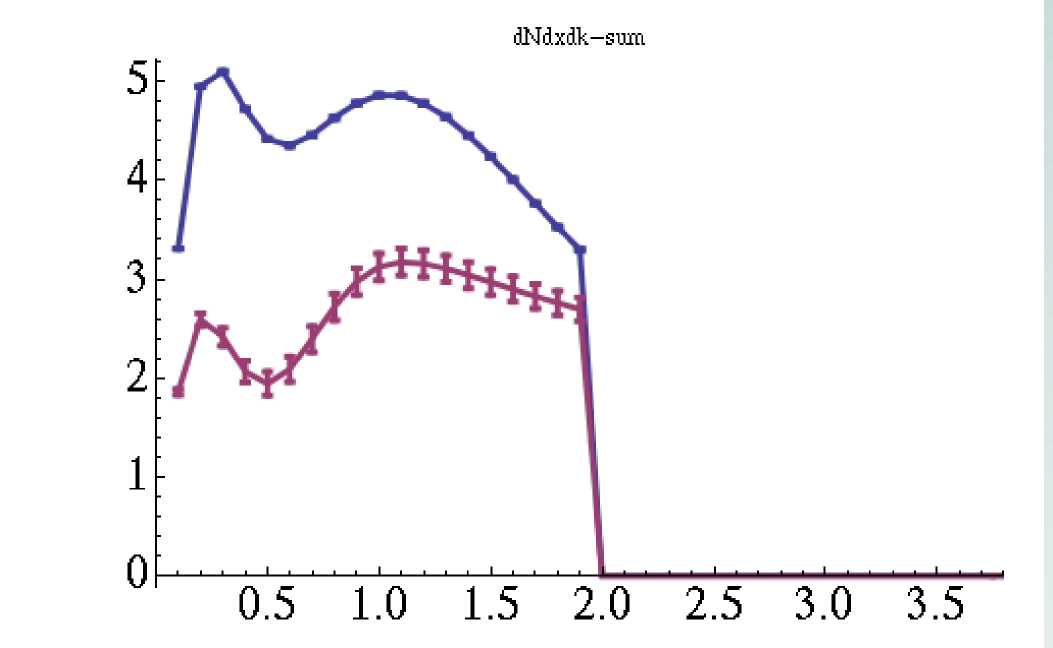


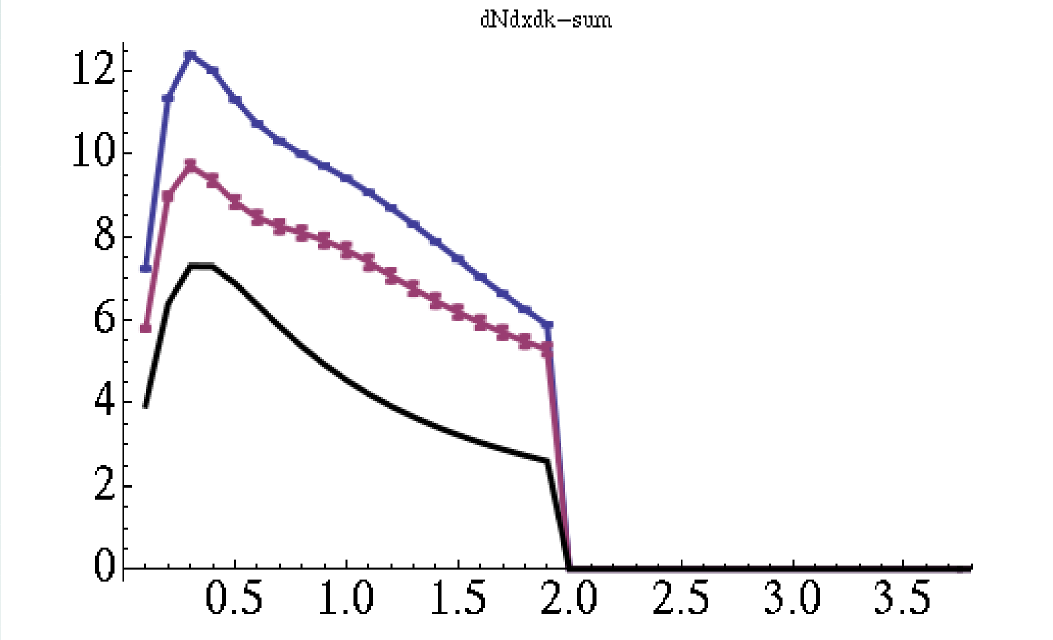
Light quark jet
x = 0.2
ie emitted gluon = 4 GeV



L=4fm, E = 20GeV - dN/dxdk$_T$





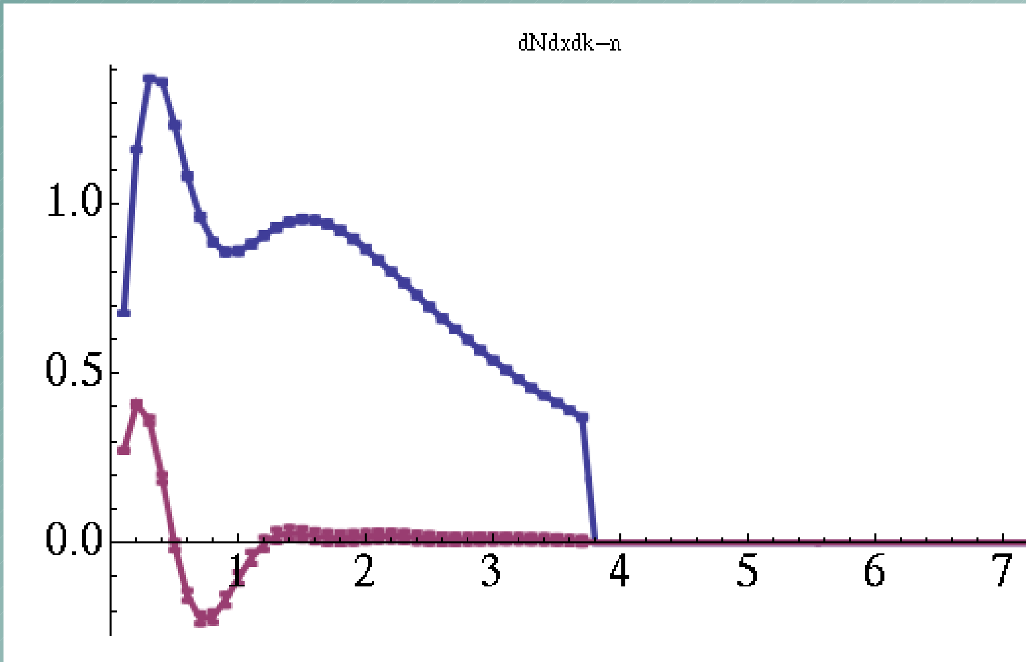


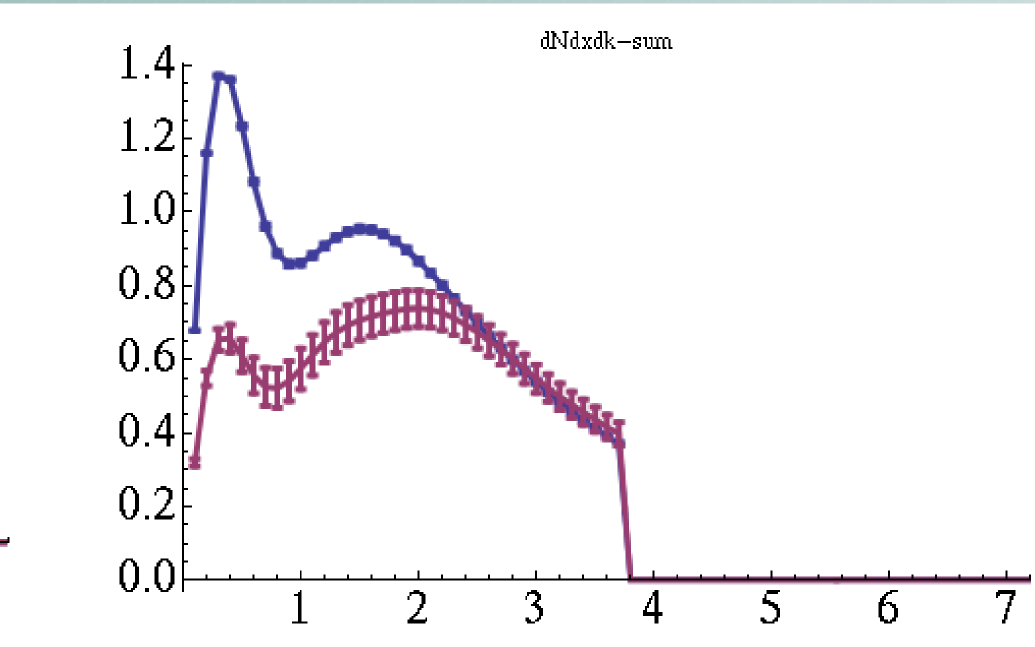
Light quark jet

<u>x = 0.1</u>

ie emitted gluon = 2 GeV

L=4fm, E = 20GeV - dN/dxdk$_T$

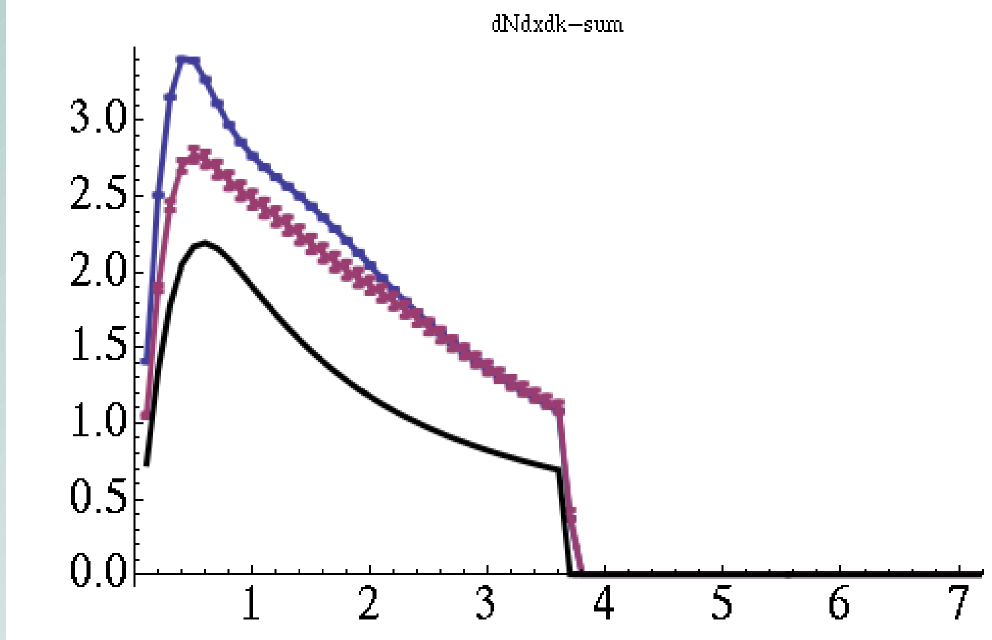




Bottom quark jet

x = 0.2

ie emitted gluon = 4 GeV



L = 4fm

- Qualitatively similar to L = 1fm
  - BUT for same Raa, using n=1 may underestimate necessary density by 40%?
- Higher orders are smaller than lower orders
  - make small alterations to 0^{th}, 1^{st} orders
- No visible approach to random walk in kT

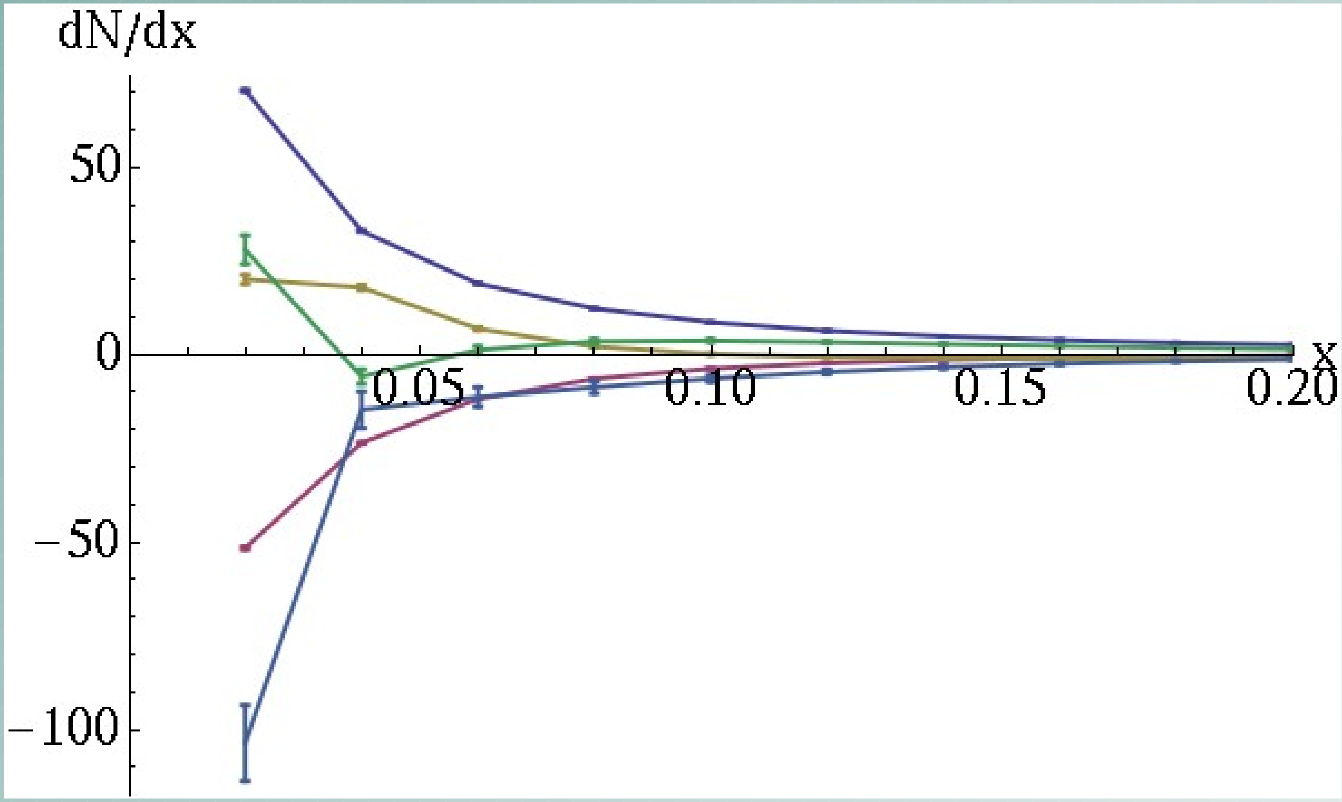
- Is there an effect differential in mass?

$L = 10\text{fm}, E = 100\ \text{GeV}$

The approach to a random walk?

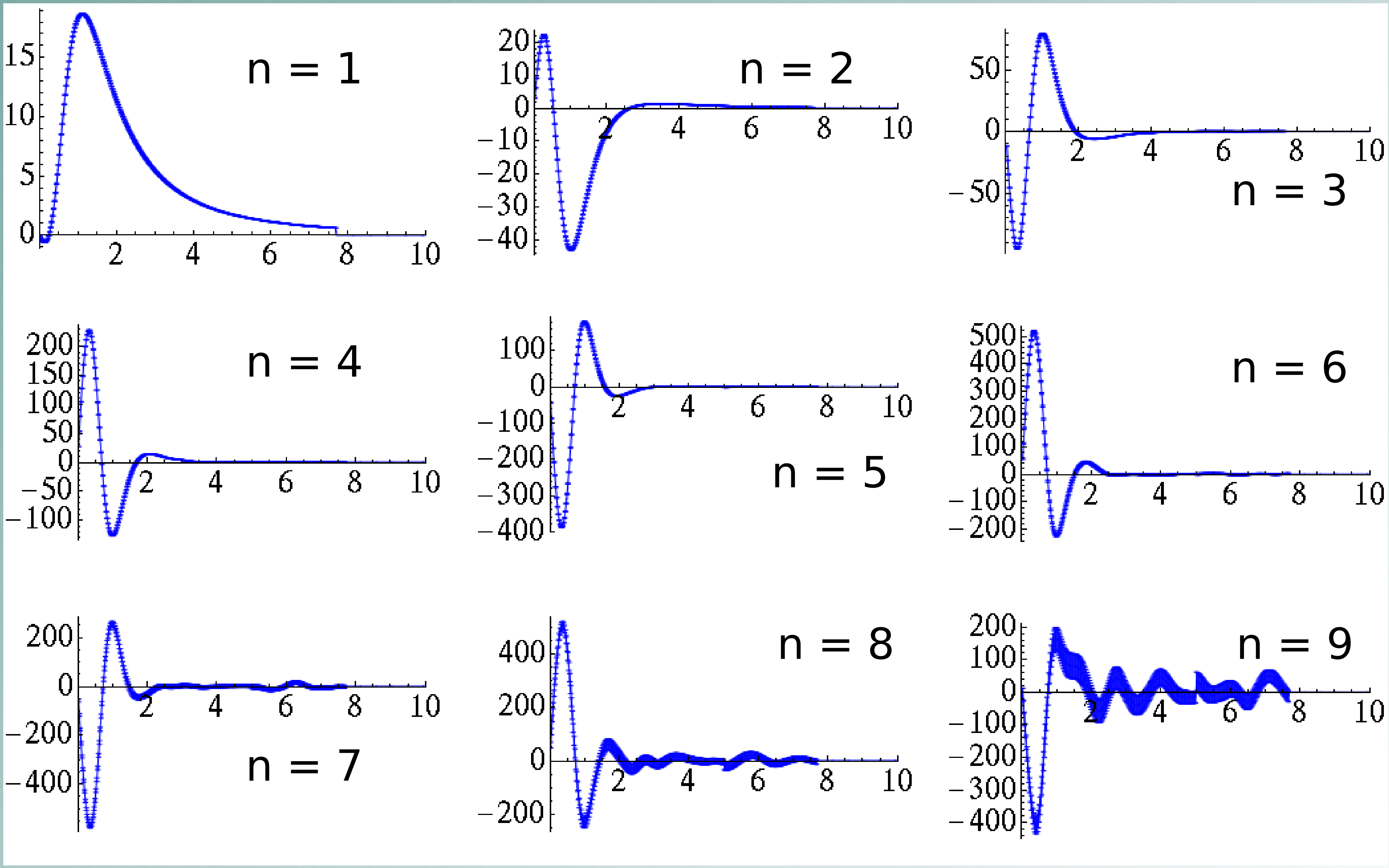
L = 10fm, E = 100 GeV

The approach to random walk?

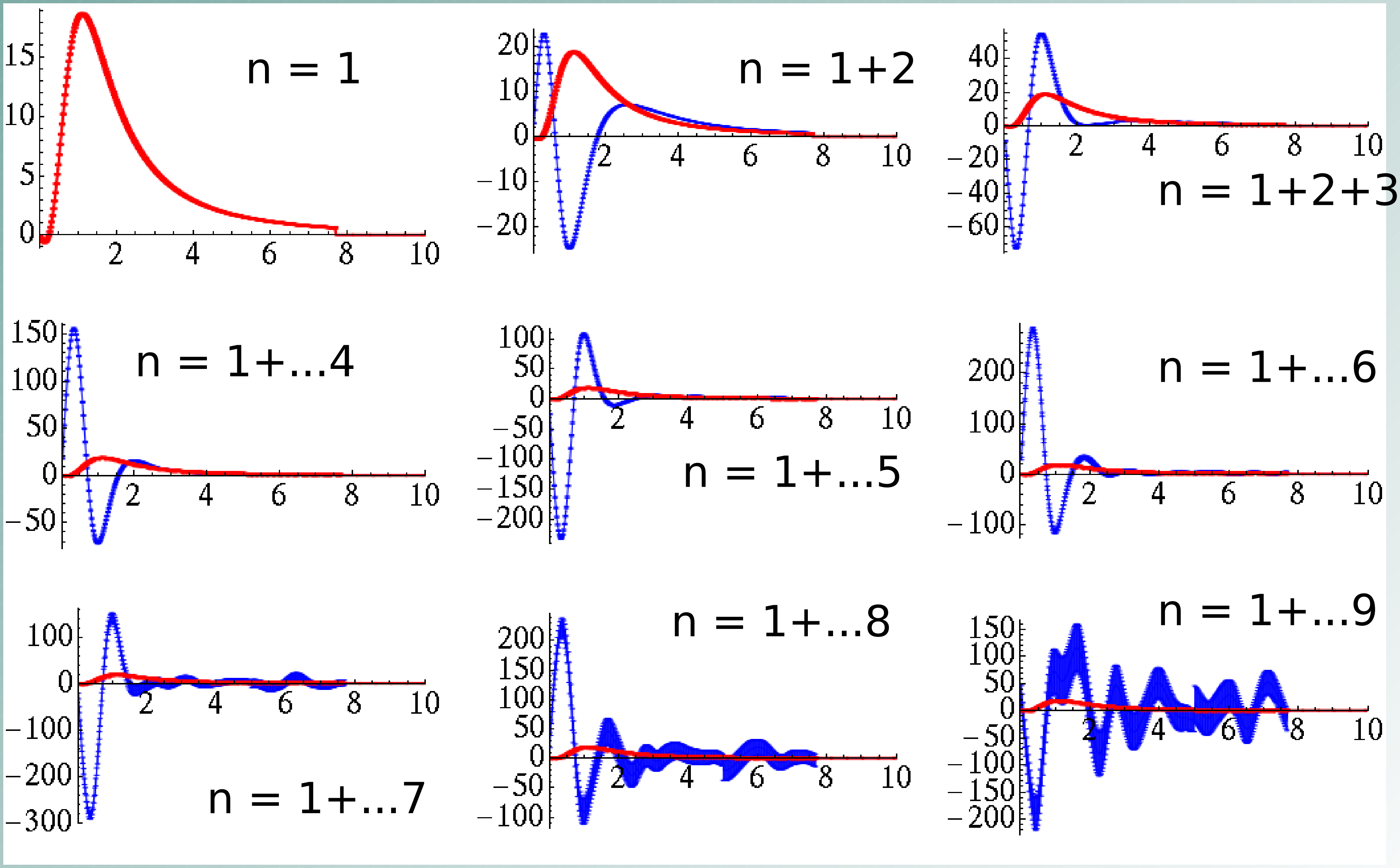


Higher orders are of the same order or greater than 1st order

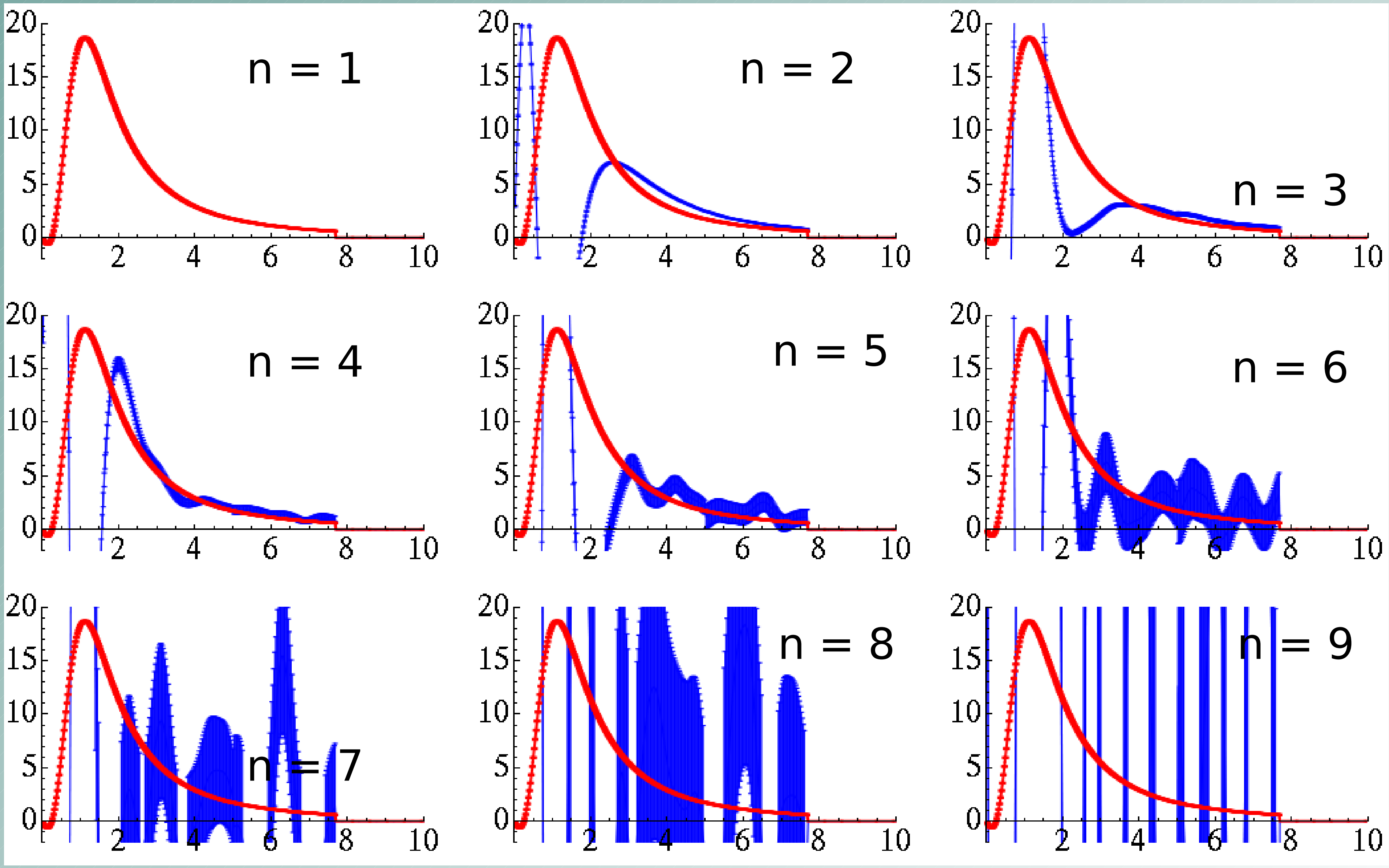
$L = 10\text{fm}, E = 100 \text{ GeV}, x = 0.04$



L = 10fm, E = 100 GeV, x = 0.04



L = 10fm, E = 100 GeV, x = 0.04



L = 10fm

- Complicated cancellations / additions between different orders in opacity
- Largest contribution from orders 5,6,7 ...
  - For the away side and for shapes sensitive to long distances, need to include higher orders
  - Is 10fm close to a n->infinity approximation?

Conclusions

- Here – looked at GLV recursion for GW model
  - Will an improved medium interaction model change the results?
  - How does expansion affect the result?
  - Energy dependence? Mass dependence?

- R_{AA} ok with n=1 approximation (up to 40% uncertainty on extracted parameter)
- Higher correlations may need the explicit summation of the orders in opacity
  - Not in a region of n=1 or n->infinity

- Nice, analytical results that fit on a line (or even a page) are no longer enough on their own – brute force numerical evaluation lets us test our approximations
- **In this way, we can test our energy loss mechanisms on a theoretical level**