Phase Diagram of QCD

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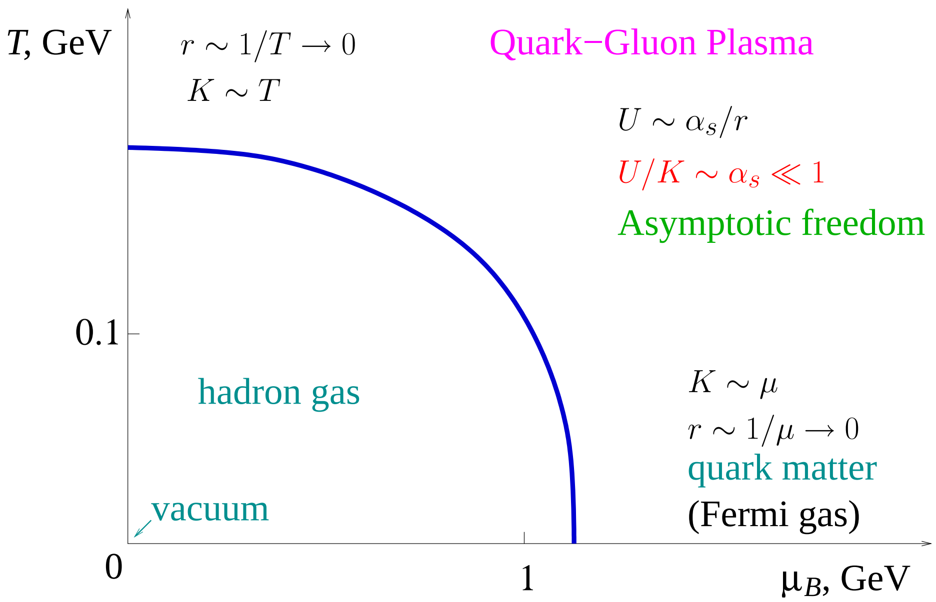
Phase Diagram of QCD

- QCD (asymptotic freedom) predicts a transition at $T \sim \Lambda_{\rm QCD}$, $\mu_B \sim N_c\Lambda_{\rm QCD}$:

Hadron/resonance gas (π,N, resonances) becomes a (color) plasma of quarks and gluons.

Simple arguments lead to the sketch:



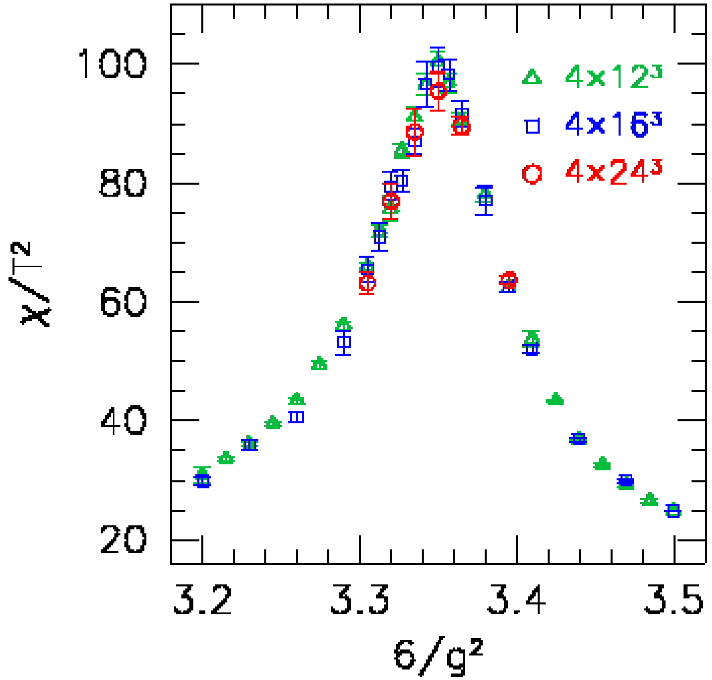
- Order of transition?

Originally, arguments suggested 1st order (discontinuous):
e.g., $S_{\rm QGP} \sim N^2_{\rm color}$, while $S_{\rm HG} \sim N^0_{\rm color}$.

Lattice says: crossover (at $\mu = 0$)

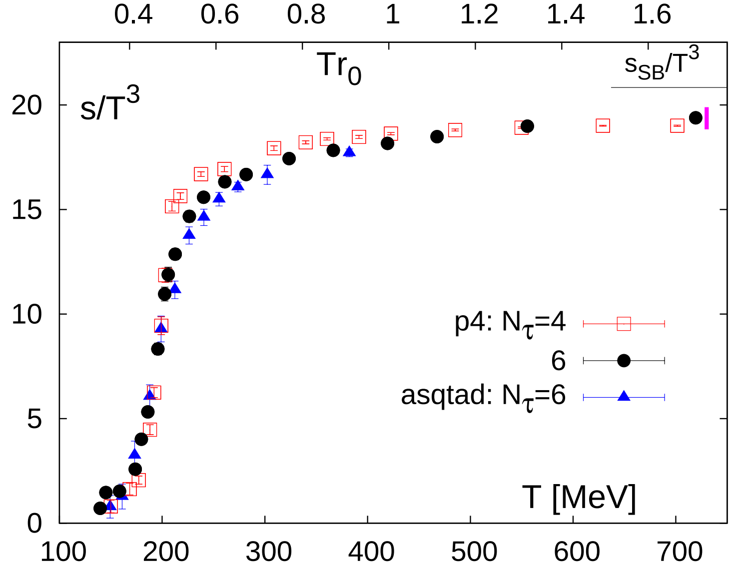
- Earliest: Columbia group, PRL 65(1990)2491
- Recent: Wuppertal-Budapest group, Nature 443(2006)675.

Wuppertal-Budapest:



the peak should have grown $8\times$ for 1st order transition

RBC-BI:

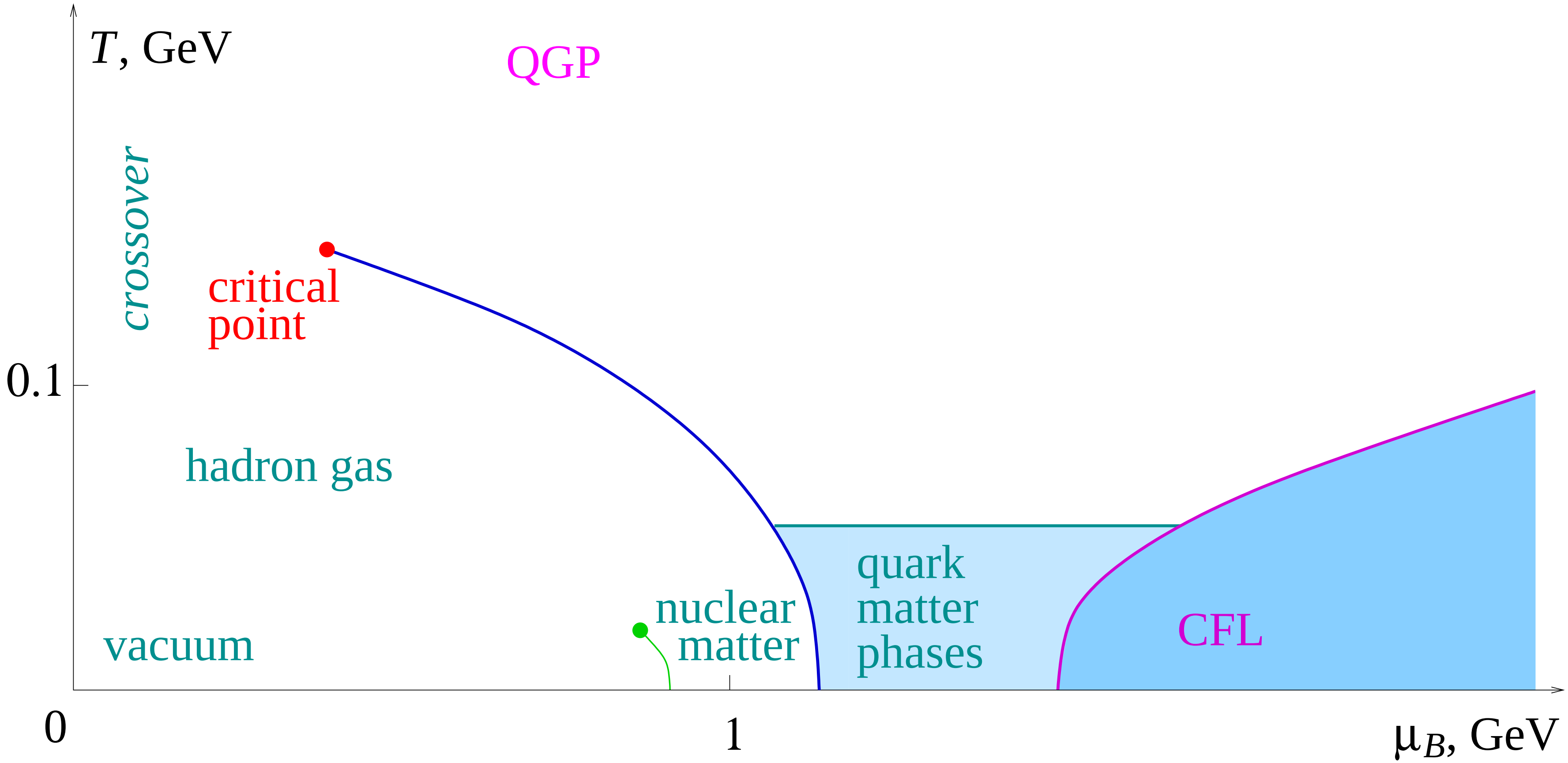


Entropy/$T^3 \sim \#$ of d.o.f. grows (color is liberated) but no discontinuity

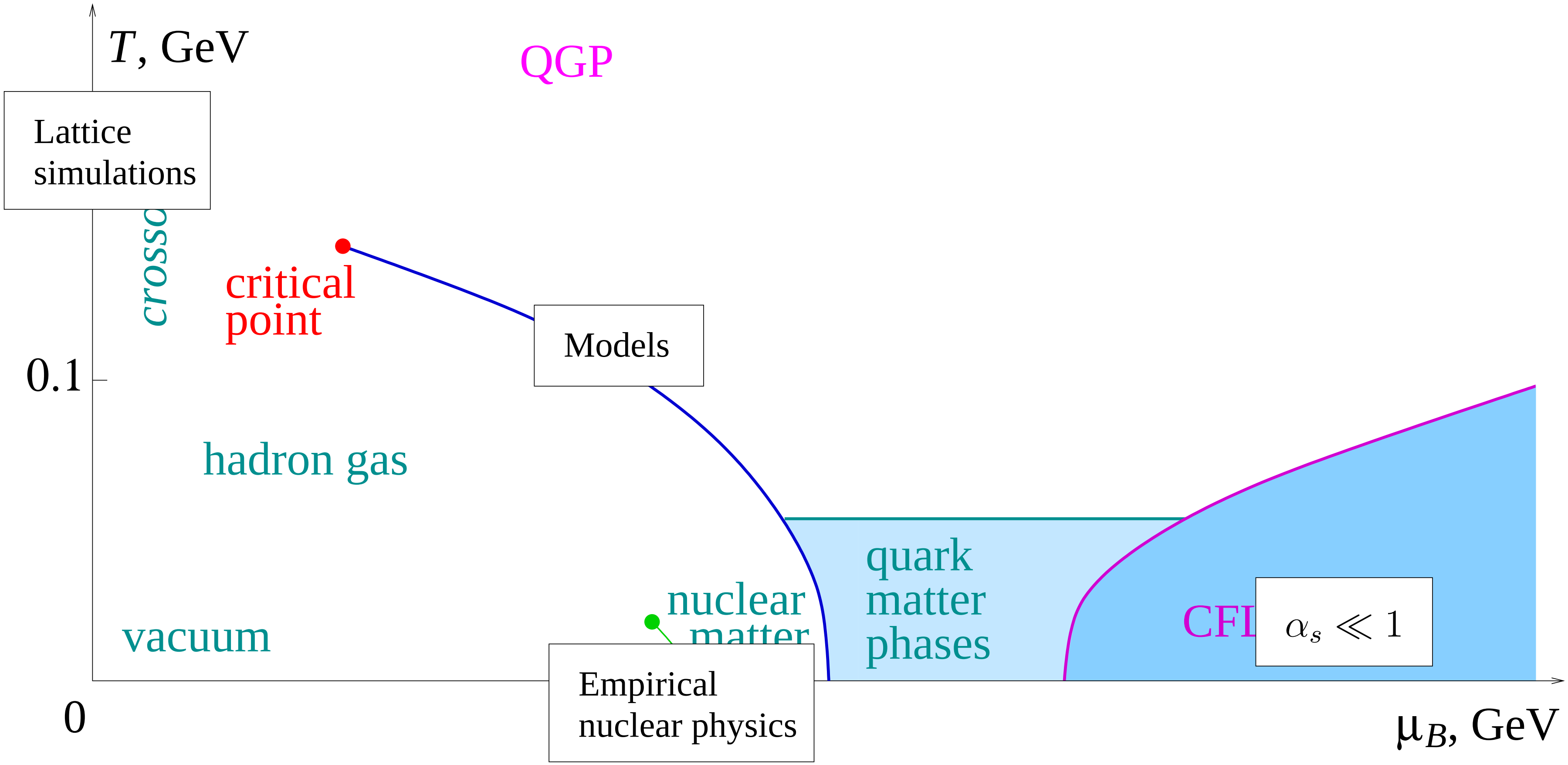
- Quarks are important: w.o. them the transition is 1st order.

Session XI: F. Karsch, Z. Fodor, P. de Forcrand

QCD phase diagram (contemporary view)

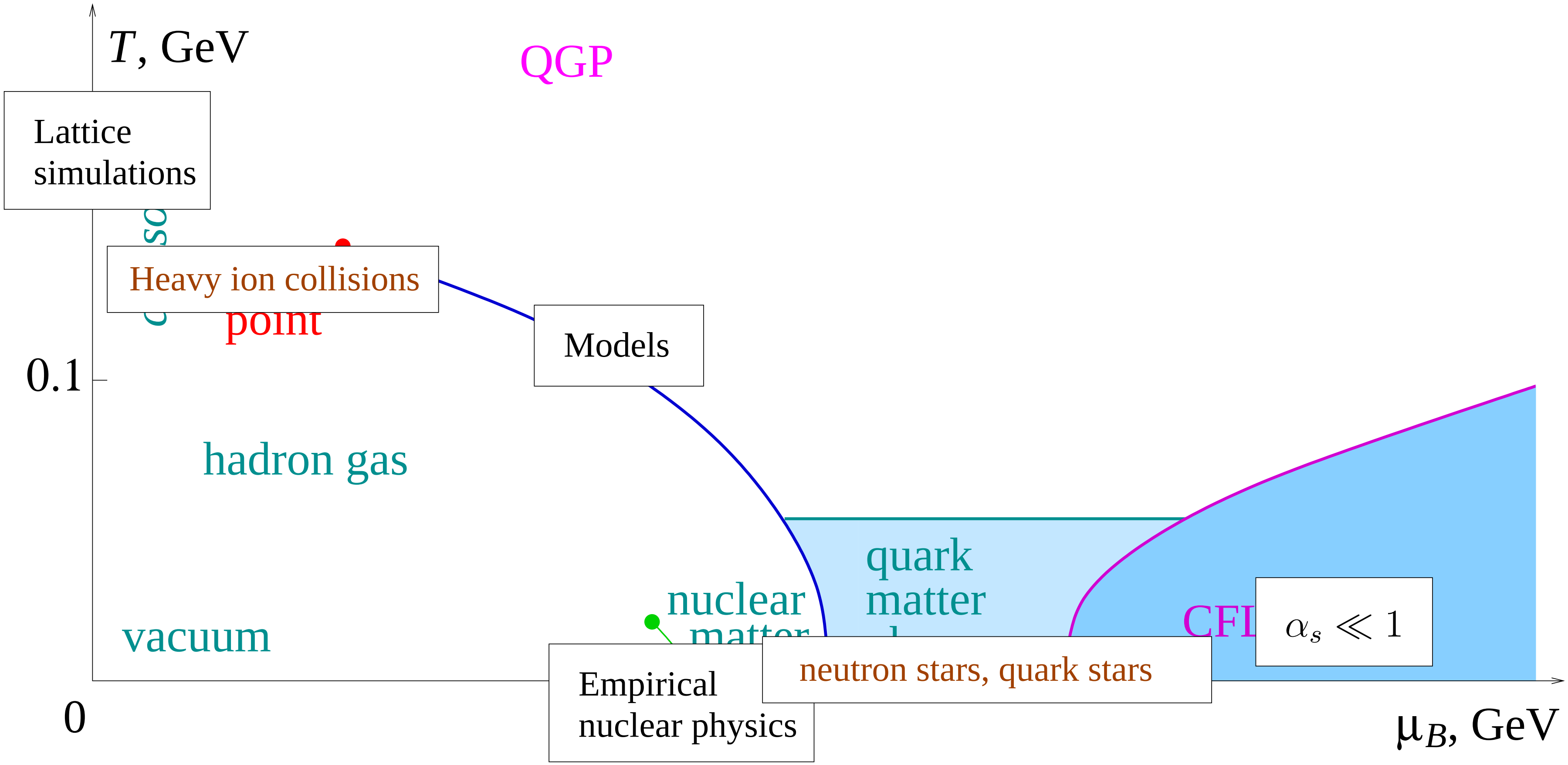


QCD phase diagram (contemporary view)



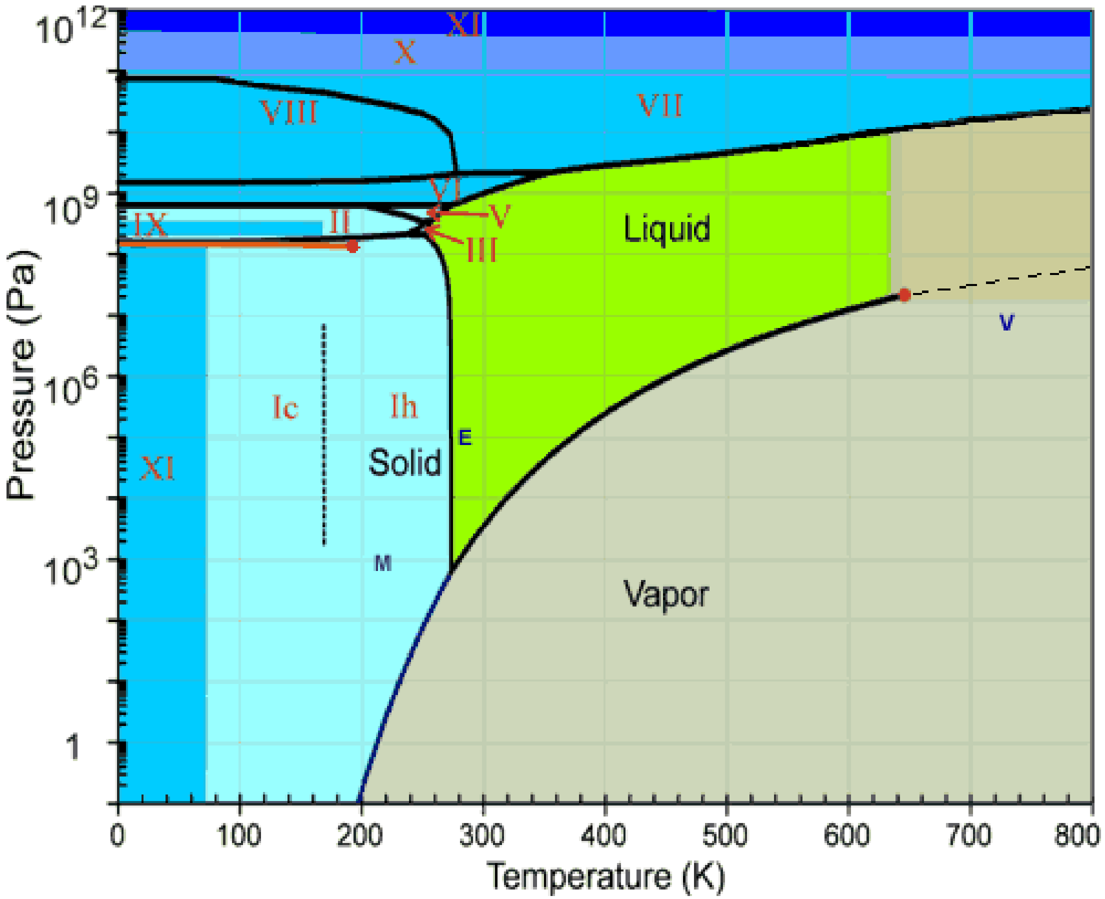
- Models (and lattice) suggest the transition becomes 1st order at some μ_B.
- Large μ_B – CFL ("QCD ice"). Need to translate theor. understanding into predictions (e.g., for neutron stars). Understand mechanical (rigidity), transport (viscosity), magnetic properties.
- More structure possible/expected. More critical points (G. Baym, S. Gupta, VII).

QCD phase diagram (contemporary view)



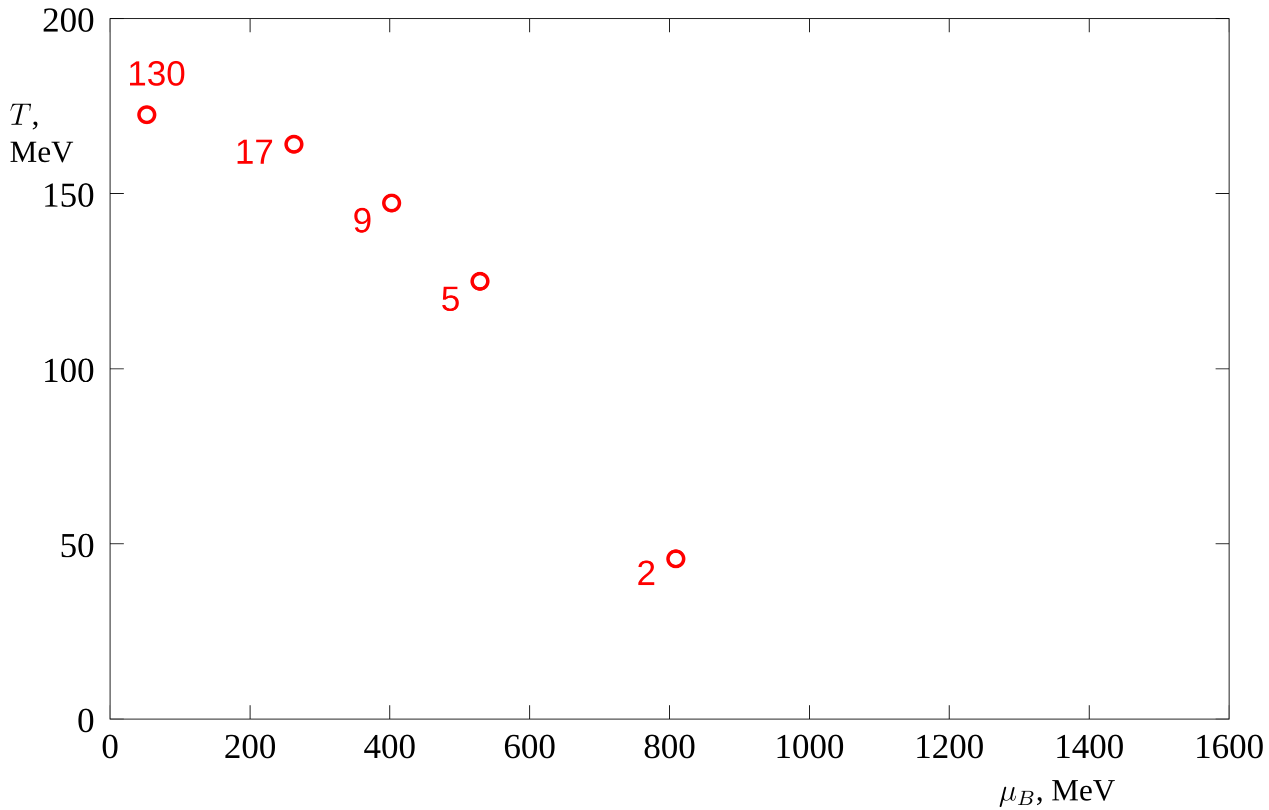
Heavy ion collisions freeze out in the region where the critical point might be found

Water



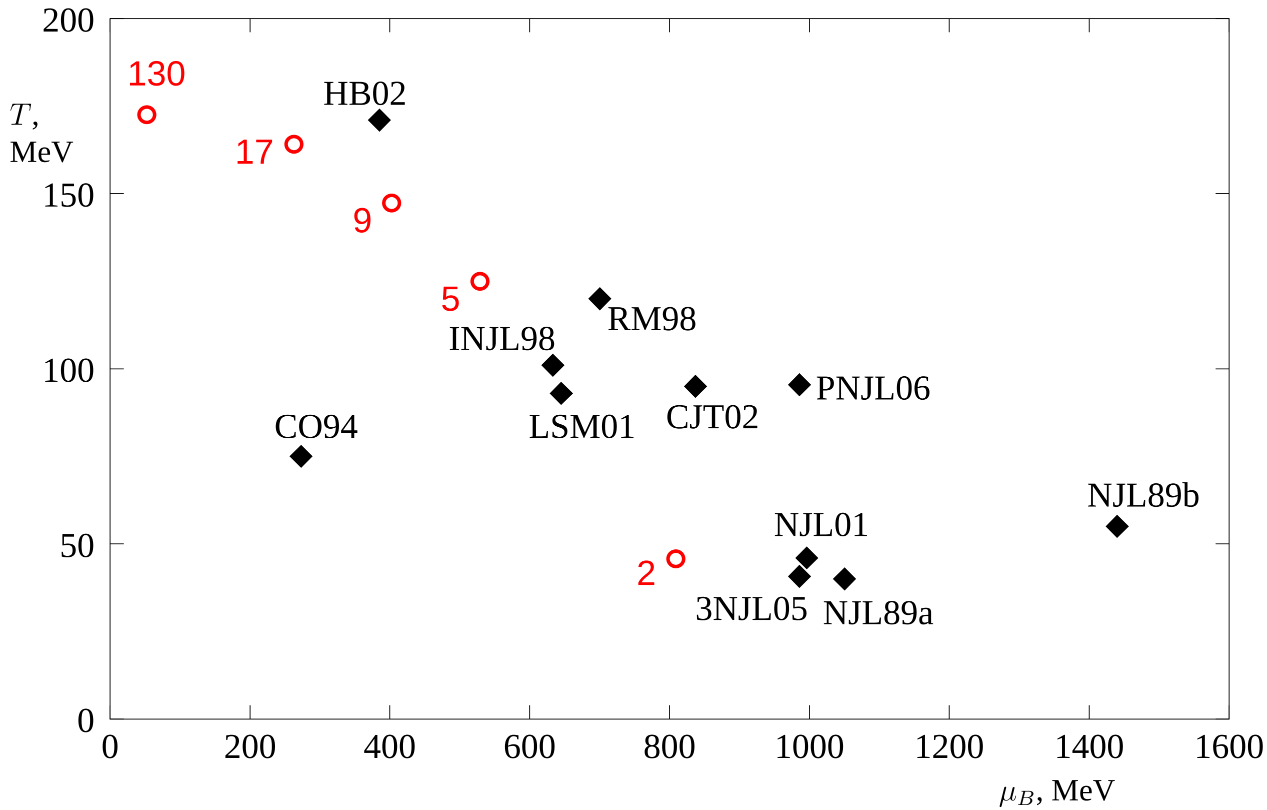
- Critical point is a common feature of liquids

Can we discover the critical point?



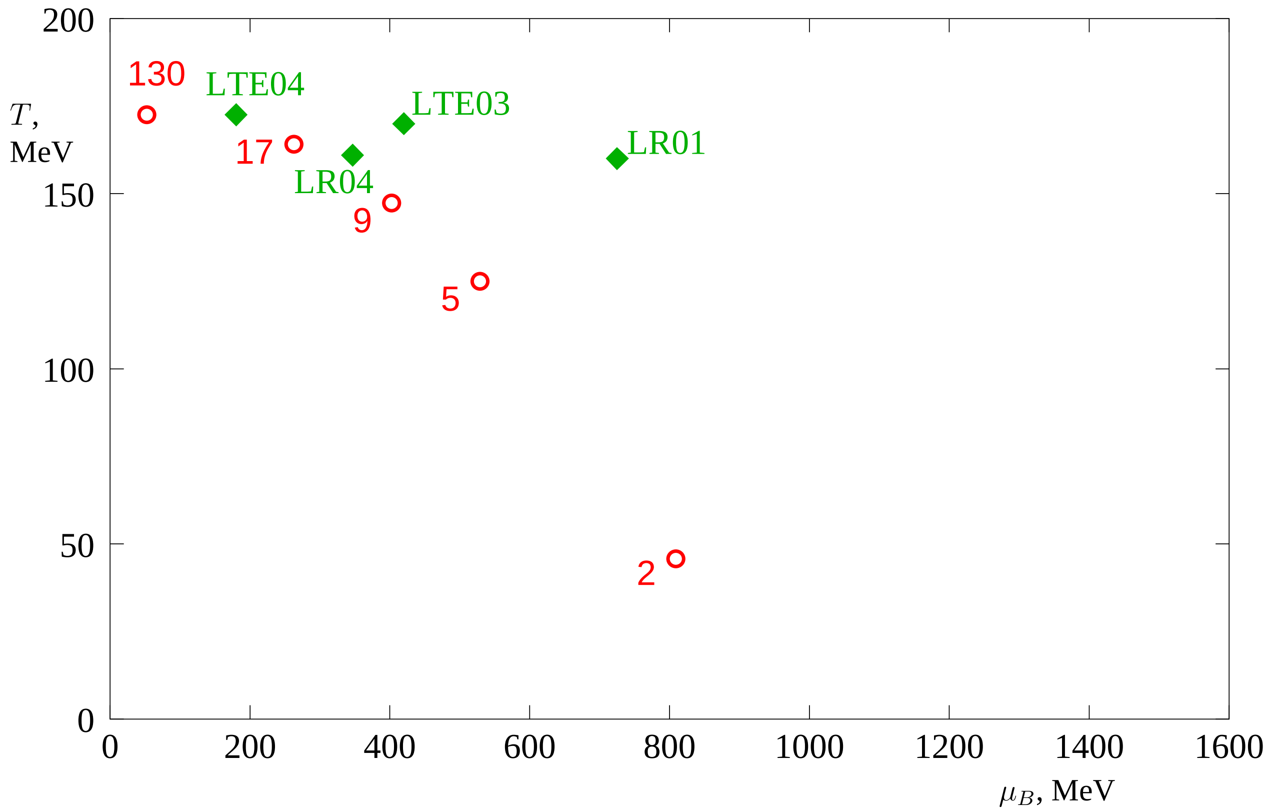
- Freezeout conditions (T,μ_B) depend on $\sqrt{s}$ (GeV)
- What is special about the critical point?
- It is a point where the thermodynamic functions are *singular*
- Signatures: fluctuations $\nearrow\searrow$ near the point *non-monotonically* vs $\sqrt{s}$.

Can we discover the critical point?



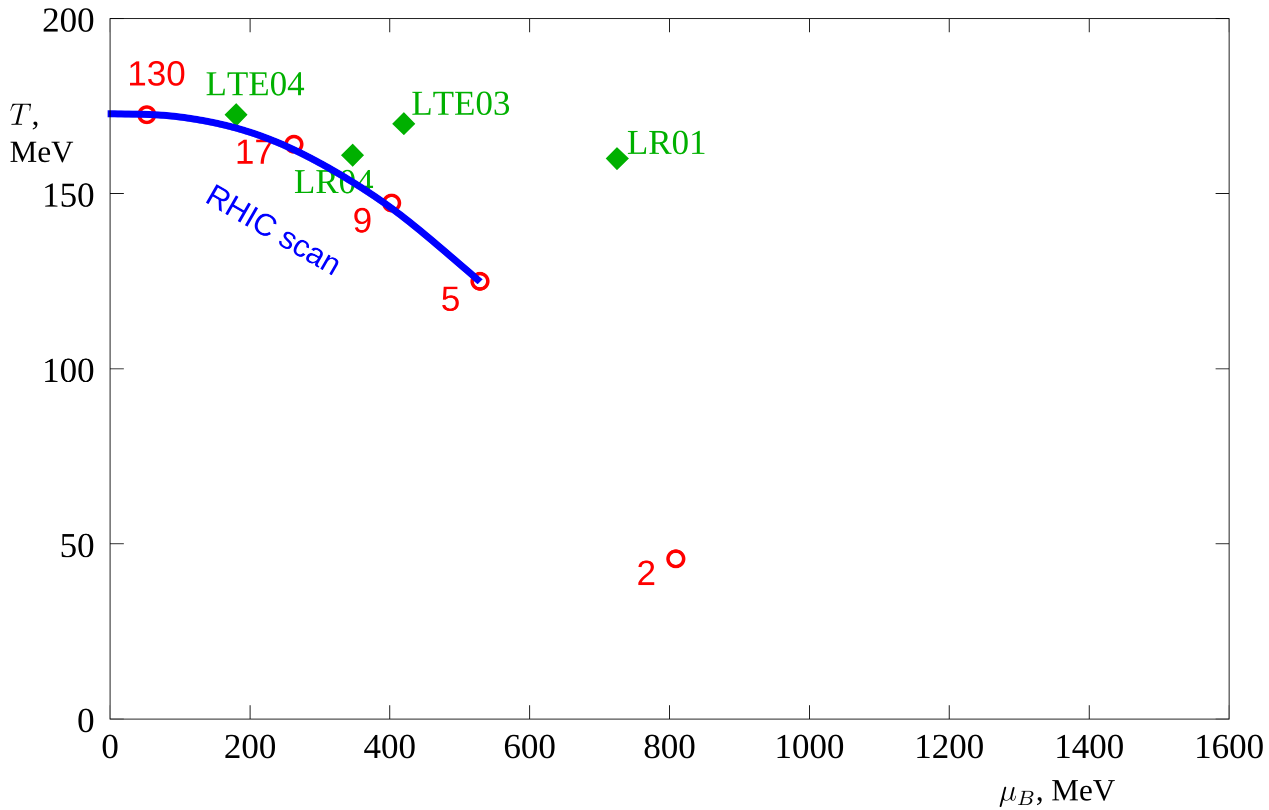
- Although models agree that there is a critical point, they disagree where it is precisely.
- Not surprising: models extrapolate from $T = 0$, $\mu_B = 0$.

Can we discover the critical point?



- The lattice has a sign problem to deal with – traditional Monte Carlo does not work.
- Clever methods to circumvent this problem: e.g., reweighting, Taylor expansion in μ_B, imaginary μ_B, etc.
- In a way, these also extrapolate from $\mu_B = 0$, but there is more information available (e.g., $T \neq 0$) compared to models.

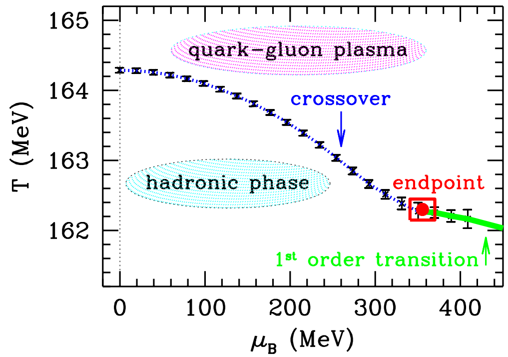
Can we discover the critical point?

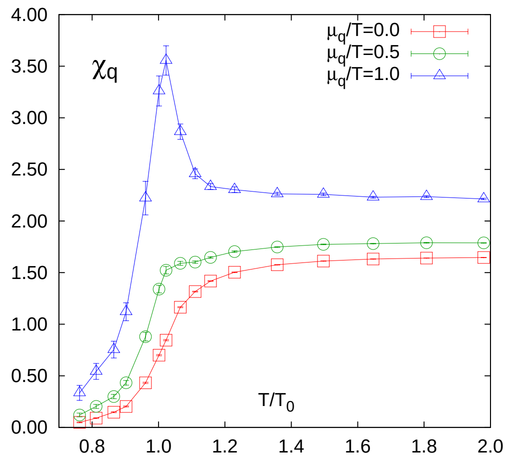


What do we need to discover the critical point:

- Experiment: RHIC, NA61(SHINE), FAIR/GSI
- Improve lattice predictions (both algorithm and CPU), understand systematic errors.
- Understand critical phenomena in the dynamical environment of a h.i.c. (more by V. Koch; C. Nonaka – 3D hydro + c.p.)

Critical point on the lattice





Allton, *et al*: peak in χ_B, but not in χ_I

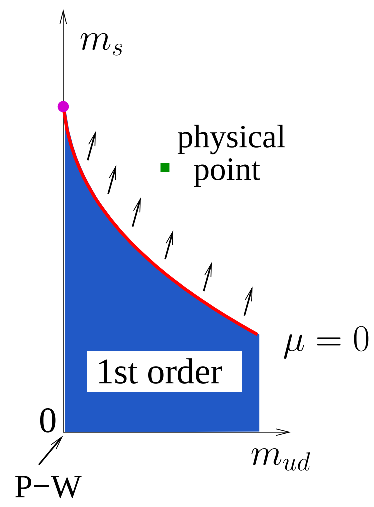
Several approaches:

- Reweighting: Fodor-Katz
  - 2001: $\mu_B \sim 725$ MeV
  - 2004: $\mu_B \sim 360$ MeV (smaller m_q and larger V)
- Taylor expansion: Bielefeld-Swansea (to μ^6)
  - 2003: $\mu_B \sim$ 420 MeV
  - 2005: 300 MeV $\lesssim \mu_B \lesssim$ 500 MeV
- Taylor expansion: Gavai-Gupta (to μ^8)
  - From convergence radius: $\mu_B \sim 180$ MeV (more precisely > 180 MeV)
- Imaginary μ: deForcrand-Philipsen, Lombardo, *et al*
  - Sensitive to m_s, perhaps $\mu_B \gg$ 300 MeV
- Fixed density: deForcrand, Kratochvila; Density of states: Fodor, Katz, Schmidt.
  - ? ($N_f = 4$, small volumes)

de Forcrand-Philipsen scenario at $N_t = 4$

- Logic of the standard scenario:
  - Transition becomes 1st order because $\searrow m_q$ promotes fluctuations of quark fields – approaching Pisarski-Wilczek point.
  - Turning on μ has the same effect. Thus, at fixed m_q, transition turns 1st order for sufficiently large μ.

“Standard”:



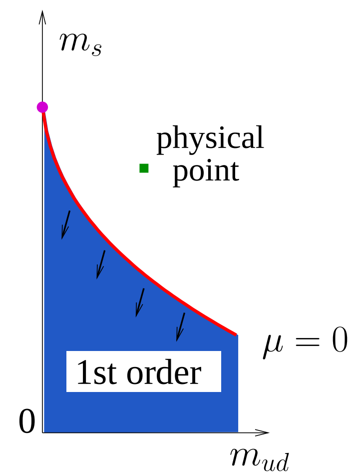
Why is this not seen in de Forcrand-Philipsen data?

- $N_t = 4$ is too small? $a^{-1} = N_t T \sim 800$ MeV – are baryons comfortable? Virtual $N\bar{N}$ pairs?

In fact, the difference in the m_c (red line) is 100% between $N_t = 4$ and 6.

- But if this survives $N_t \to \infty$ – do models supporting standard scenario mislead?

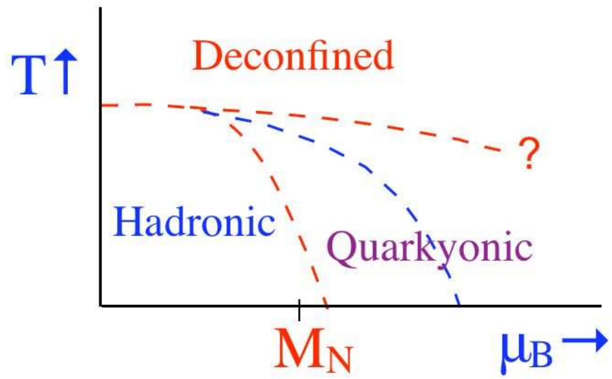
dF-P:



Need to find out!
(more by P. de Forcrand)

Insights from large N_c

McLerran, Pisarski



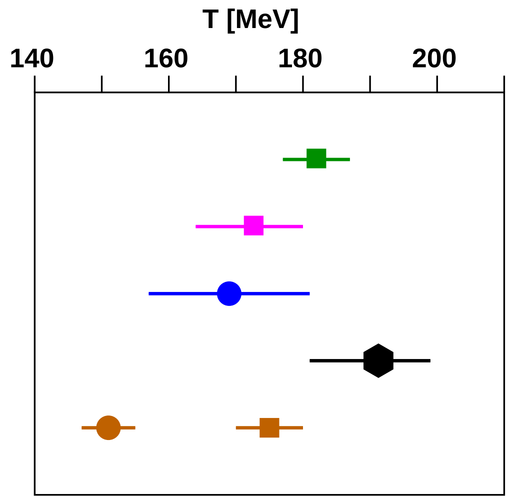
- Large N_c is different from QCD in many aspects:
  - crossover at $\mu = 0$ in QCD, not 1st order;
  - CFL needs $N_c = N_f = 3$, not $N_c \gg N_f$.

- Some features might be present in QCD, however.

Quarkyonic phase (quark matter with baryon-like excitations)?

T_c, chiral restoration vs deconfinement

- The quest to determine the location of the crossover at $\mu_B = 0$ (RHIC, LHC).



(Karsch)

- Different lattice approaches extrapolated to $a \to 0$, $m \to m_{\text{physical}}$ must agree (eventually).

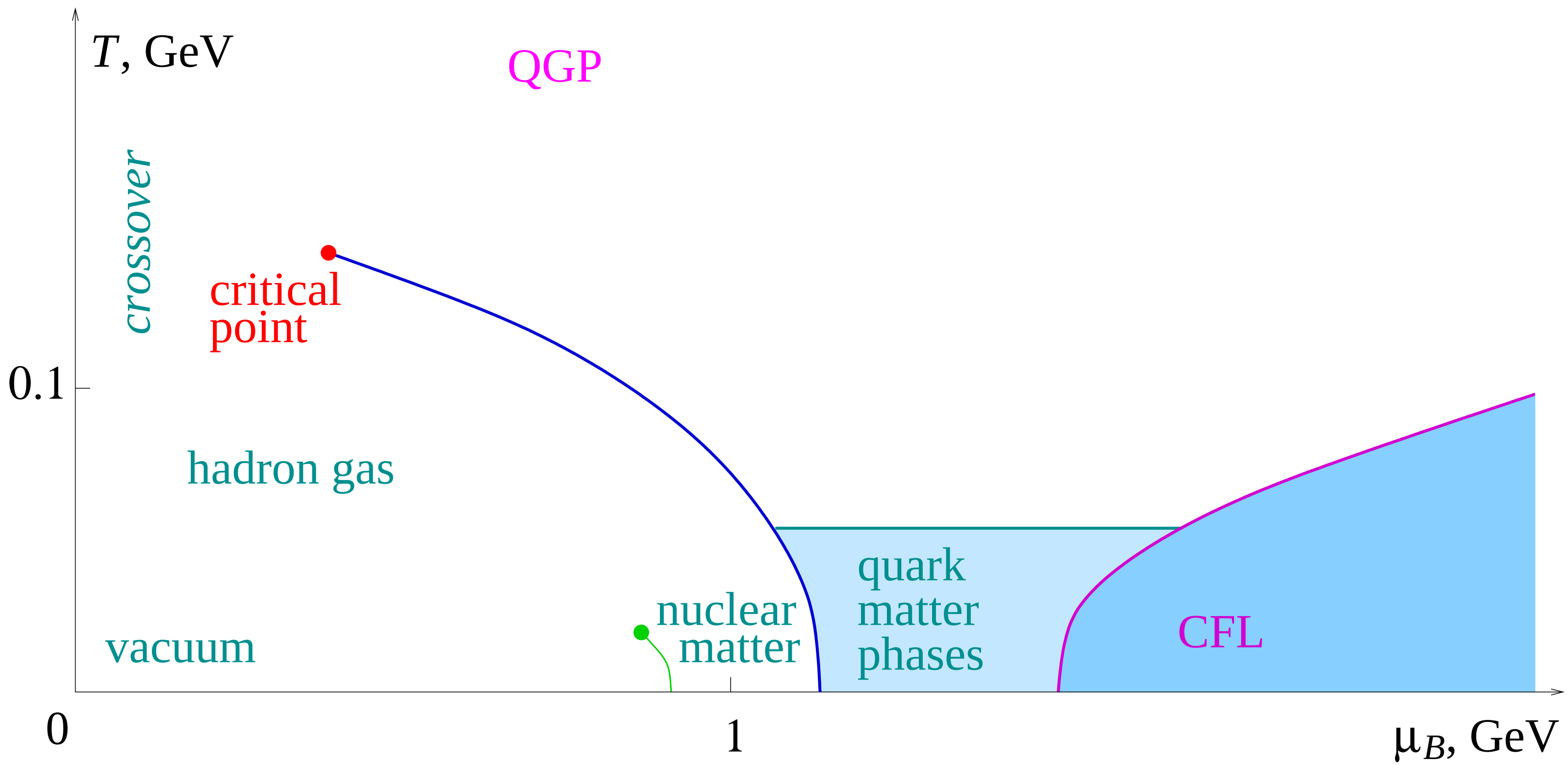
- Wuppertal-Budapest results raise another, physical, issue:
Do chiral (circles) and Polyakov loop (squares) susceptibilities peak at the same T_c?

More by Z. Fodor, F. Karsch.

Crossover

What is the matter in the crossover region made of?

Hadrons? Quarks? Neither?

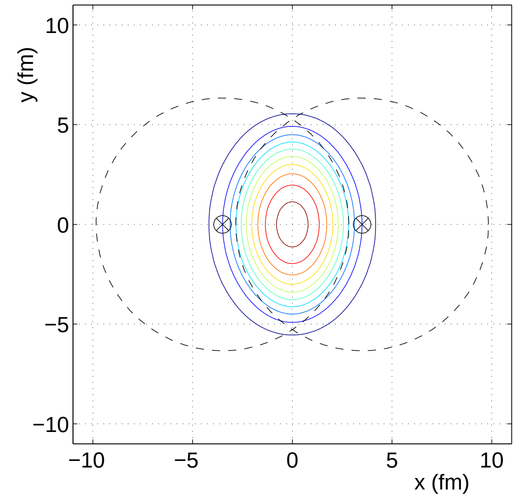


The answer comes, surprisingly, from comparing results of hydrodynamic calculations with RHIC data.

Hydrodynamic modeling and v2

Approach: take an equation of state, initial conditions, and solve hydrodynamic equations to get particle yields, spectra, etc.

from Kolb/Heinz review

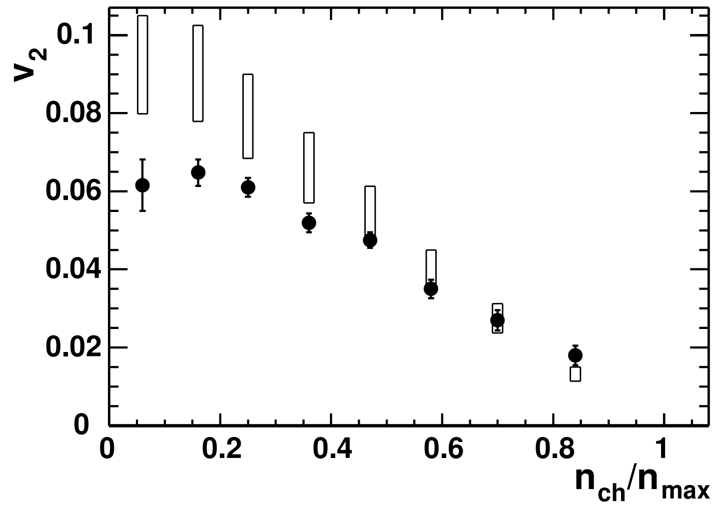


- v2 – a measure of elliptic flow is a key observable.

- Pressure gradient is large in-plane. This translates into momentum anisotropy. To do this the plasma must do work, i.e., pressure$\times \Delta V$

- v2 is large $\rightarrow$ 1st conclusion, there is pressure, and it builds very early.
I.e., plasma thermalizes early ($< 1\mathrm{fm}/c$).

- BIG theory question: HOW does it thermalize? and why so fast/early?



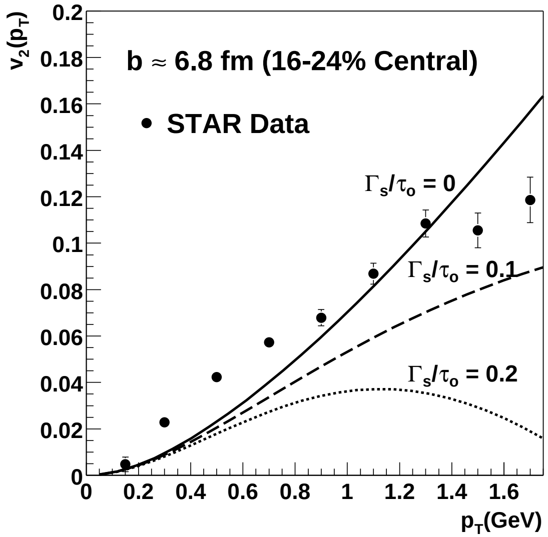
- Need to understand initial conditions
– Color Glass Condensate
(more by R. Venugopalan)

- Mechanism of thermalization? Plasma instabilities?

Small viscosity and sQGP (liquid)

Another surprise: where is the viscosity?

Teaney



- Ideal hydro already agrees with data.
- Adding even a small viscous correction makes the agreement worse $\rightarrow$
- If the plasma was weakly interacting the viscosity $\frac{\eta}{T^3} \sim (\mathrm{coupling})^{-2}$ would be large.
- Conclusion: the plasma must be strongly coupled – it is a liquid.

- Can it be simply an ideal liquid, can $\eta = 0$?
- How small could η be? Policastro, Kovtun, Son, Starinets found that in an $\mathcal{N} = 4$ super-Yang-Mills theory at infinite coupling $\eta = \frac{1}{4\pi} s$. And so is in a class of theories with infinite coupling. Can it be a coincidence, or a universal lower bound?
- If $\frac{\eta}{s} = \frac{1}{4\pi}$ is the lowest bound – data suggests RHIC produced an almost perfect liquid. Need viscous (3D) hydro simulation to confirm.

AdS/CFT approach to sQGP

Viscosity is just an example of problems for which lattice is not an ideal tool (yet).

So how do we approach sQGP theoretically?

- AdS/CFT (gauge/gravity): translate a strong coupling gauge theory problem into a semiclassical gravity problem, solve, and then translate back the results.
- Correspondence is "holographic" – the translation is done on a 4d boundary of the space where the gravity lives.
- Practically proven for pure $\mathcal{N} = 4$ SYM.

Dual to QCD is yet unknown, but we know something about it (E. Shuryak).

Many qualitative and semi-quantitative insights
– more by K. Rajagopal and in Session IV.

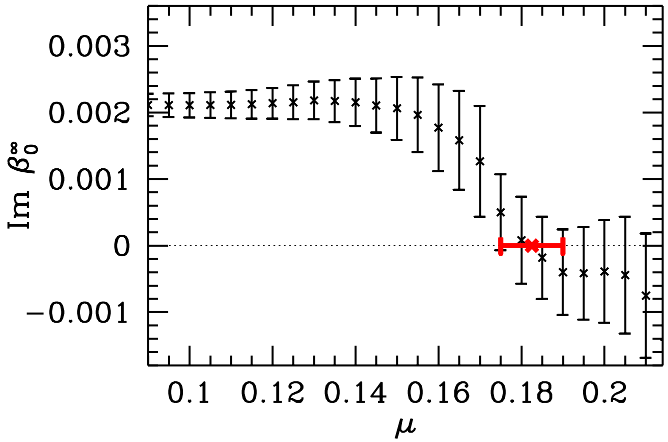
Conclusions

- What have we learned?
  - The transition at zero baryon density ($\mu_B = 0$) is a crossover.
  - We should expect a critical point at some μ_B.
  - The matter just above the crossover is a strongly interacting liquid.
- What do we still want to know?
  - What is the precise (± 5 MeV) value of T_c? Is it above/at/below the chemical freezeout at top RHIC energy (at LHC)?
  - Where on the phase diagram is the critical point where the transition becomes discontinuous (1st order)?

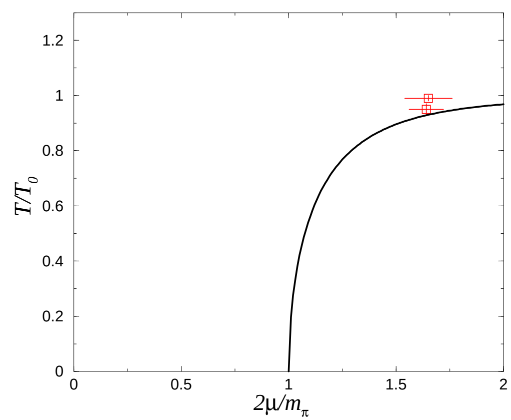
    Experimental discovery of the QCD critical point is crucial to our understanding of the QCD phase diagram!
  - To what extent is the RHIC fluid "perfect"? Is it at LHC?
  - What are the properties of sQGP? Can we study them reliably? E.g., can we take into account non-conformality, confinement, chiral symmetry breaking, finite μ_B?

Appendix

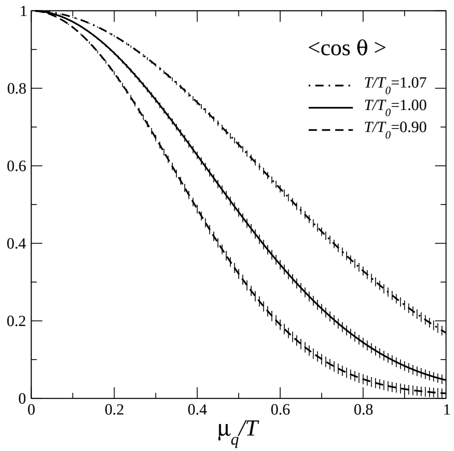
Fodor-Katz critical point



Fodor-Katz: complex singularity reaches real axis – critical point



Splittorff hep-lat/0505001: both F-K points (different m_π) lie on phase quenched transition line. Coincidence?



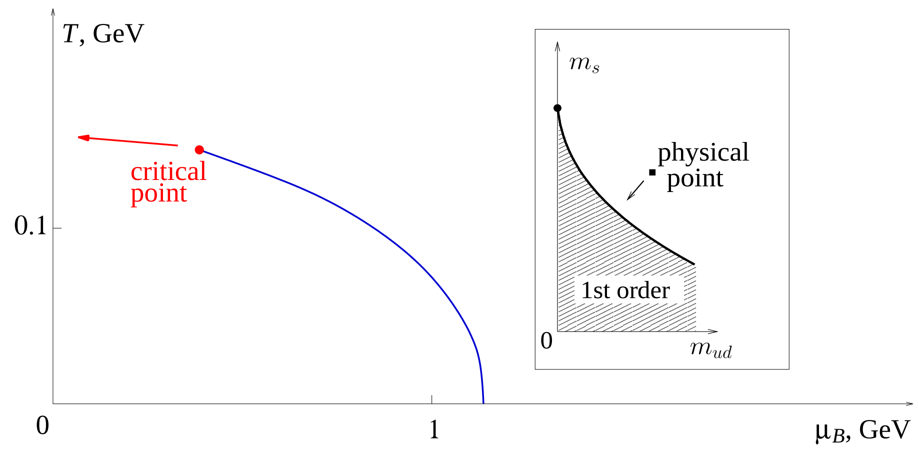
Ejiri hep-lat/0506023: statistical fluctuations of arg(det) cause spurious $\mathrm{Im}\beta = 0$ points. These fluctuations become large at the phase quenched transition line
(see also Splittorff-Verbaarschot).

Golterman *et al* hep-lat/0602026: $\det^{1/4}$ is problematic at $\mu \neq 0$ (Svetitsky, Sharpe)

de Forcrand-Philipsen scenario

For $m_{\rm strange} < m_c(\mu = 0)$ the transition at $\mu = 0$ is a 1st order one
(by universality argument due to Pisarski-Wilczek)

Standard expectation (also in models):



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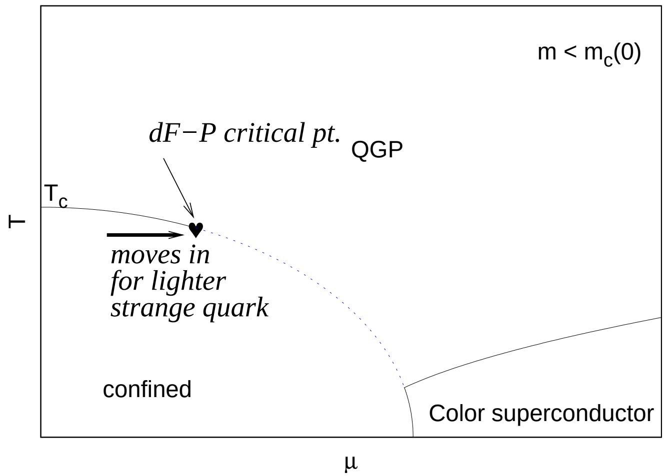

Many caveats (coarse lattices, small volume, etc), but:
If indeed $dm_c(\mu)/d\mu^2 < 0$ then critical point as a function of $m_{\rm strange}$ is not continuously related to the critical point (if it exists) predicted by models.

Unusual feature of the dF-P critical point: the 1st order transition is on the high T side – opposite to normal (e.g. Ising, water).

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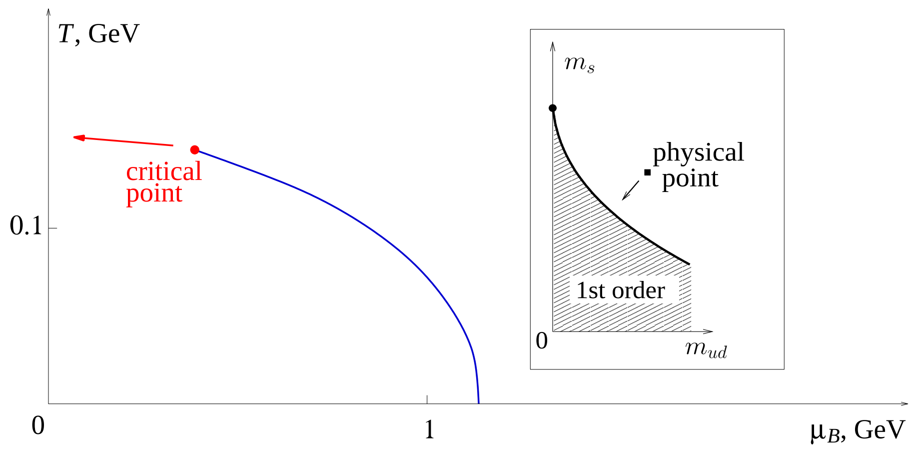

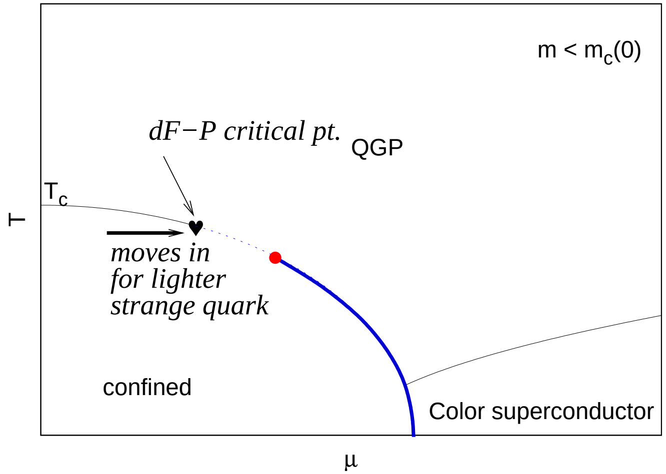

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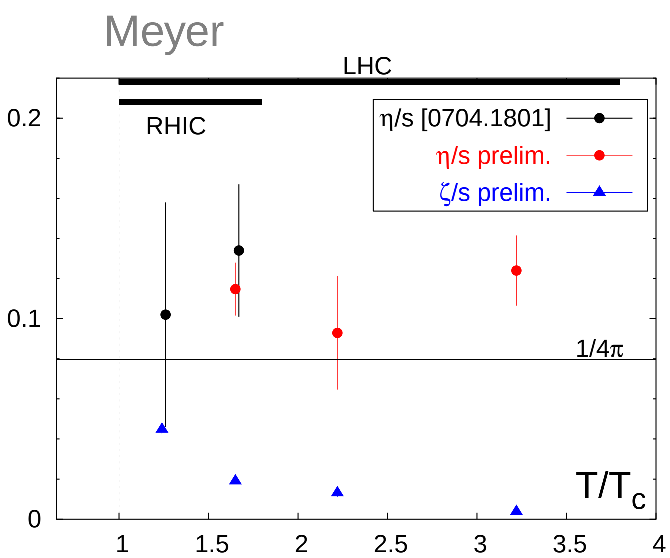
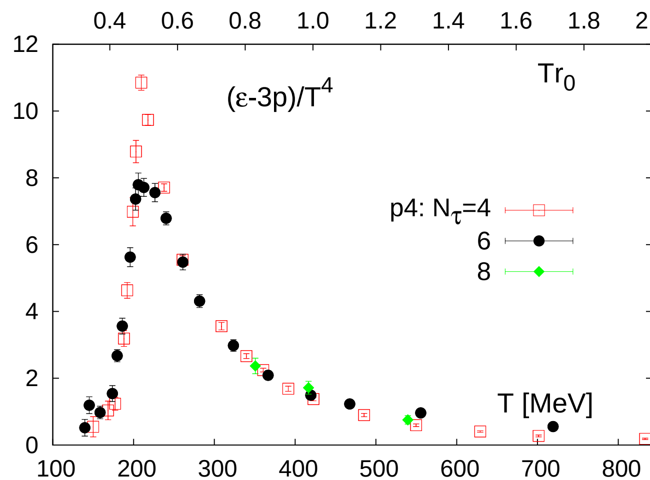
Viscosity on the lattice

- Difficult problem: need to get large *real*-time behavior of a correlation function, from Euclidean (*imaginary*) time measurements.
- Numerical noise must be very low.
- Must assume that extrapolation to large times (low frequencies) is smooth.





- At $T \sim 1 - 3.5\, T_c$ η/s is close to $1/(4\pi)$
- The bulk viscosity vanishes quickly above $T \sim 2T_c$.

The latter is in agreement with trace anomaly calculation by RBC-BI $\rightarrow$

Examples

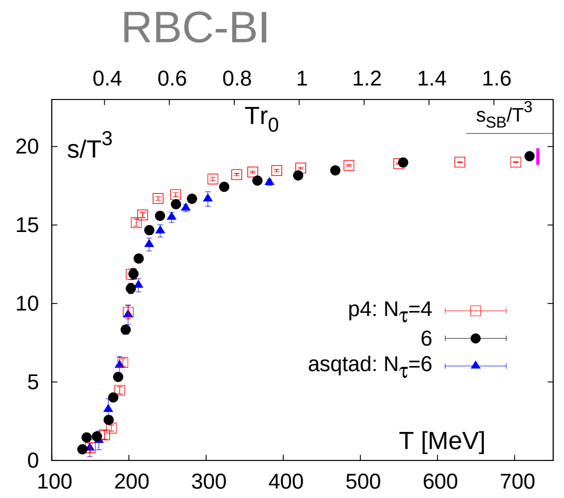
- Example 1: entropy in gauge theory $\leftrightarrow$ entropy of the black hole. Interesting result:

$$S(\lambda = \infty) = 3/4 \cdot S(\lambda = 0).$$

Only small variation with λ.

On the lattice:

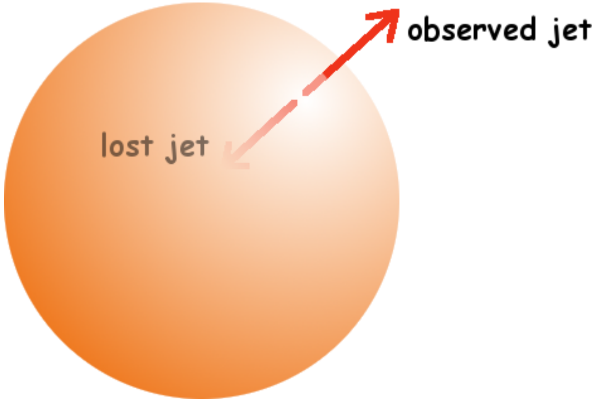
small deviation from S.B. limit $\neq$ weak coupling.



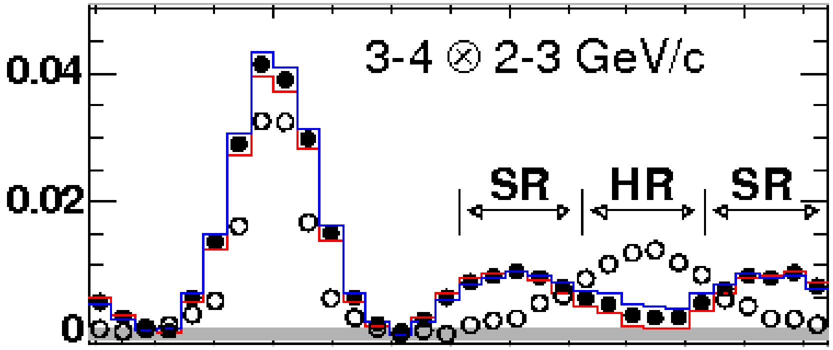
- Example 2: viscosity $\leftrightarrow$ absorption cross section of a graviton on the black hole.

$\sigma_{\rm abs} \sim (\rm b.h.\ area)$, and since $S_{\rm b.h.} \sim (\rm b.h.\ area)$ one gets universal η/s in theories with gravity duals.

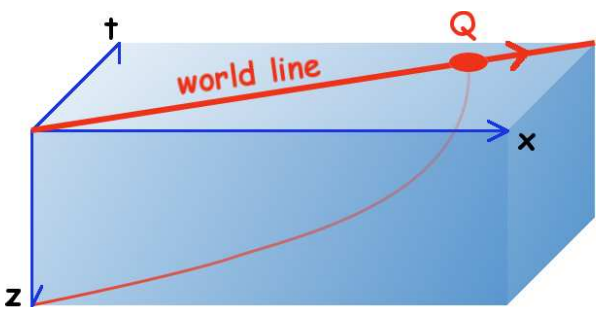
Jet Quenching and AdS/CFT



PHENIX



- Example 3: dragging a quark $\leftrightarrow$ dragging a string



Chesler/Yaffe