UTILIZING THE FLUID NATURE OF QGP

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Quark Matter 2008, Jaipur, Rajastan, India
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High temperature superfluidity at RHIC!

All "realistic" hydrodynamic calculations for RHIC fluids to date have assumed zero viscosity

η = 0 → perfect fluid

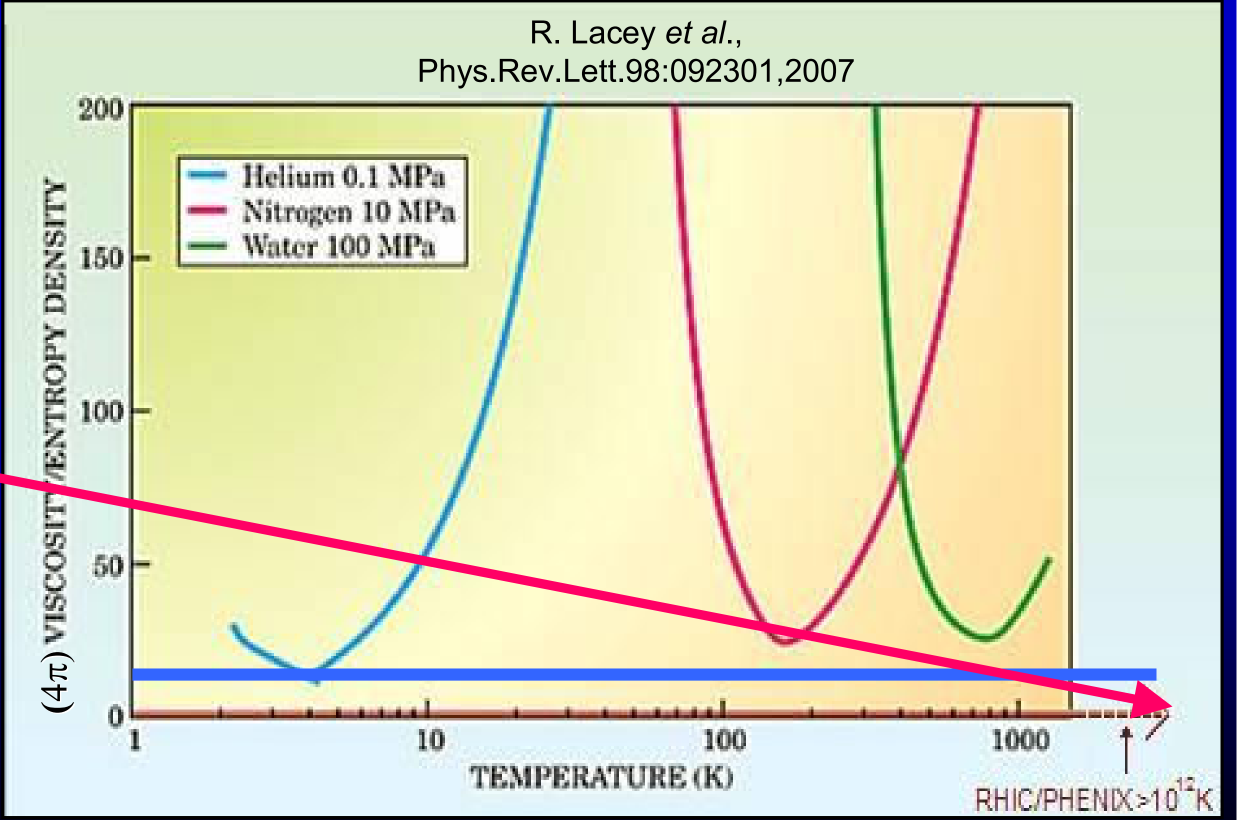
a conjectured quantum limit:

$$\eta \geq \frac{\hbar}{4\pi}(Entropy\ Density) \equiv \frac{\hbar}{4\pi}s$$

P. Kovtun, D.T. Son, A.O. Starinets, hep-th/0405231

How "ordinary" fluids compare to this limit?

$(4\pi)\ \eta/s > 10$



R. Lacey *et al.*, Phys.Rev.Lett.98:092301,2007

RHIC's perfect fluid

$(4\pi)\ \eta/s \sim 1$!

T > 2 Terakelvin

The hottest

& most perfect fluid

ever made…

Relativistic hydrodynamics

Energy-momentum tensor: $T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu}$ $w = \varepsilon + p$

$$\partial_\nu T^{\mu\nu} = 0$$

Relativistic

Euler equation: $w u^\nu \partial_\nu u^\mu = \left(g^{\mu\rho} - u^\mu u^\rho\right) \partial_\rho p$

Energy conservation: $w \partial_\mu u^\mu = -u^\mu \partial_\mu \varepsilon$

Charge conservation: $\sum \mu_i \partial_\mu \left(n_i u^\mu\right) = 0$

Consequence is entropy conservation:

$$\partial_\mu \left(\sigma u^\mu\right) = 0.$$

Context

Renowned exact solutions

- Landau-Khalatnikov solution: dn/dy ~ Gaussian
- Hwa solution (PRD 10, 2260 (1974)) - Bjorken ε_0 estimate (1983)
- Chiu, Sudarshan and Wang: plateaux
- Baym, Friman, Blaizot, Soyeur and Czyz: finite size parameter Δ
- Srivastava, Alam, Chakrabarty, Raha and Sinha: dn/dy ~ Gaussian
  - Revival of interest: Buda-Lund model + exact solutions,
    Biró, Karpenko+Sinyukov, Pratt (2007),
    Bialas+Janik+Peschanski, Borsch+Zhdanov (2007)

New simple solutions

- Evaluation of measurables
  - Rapidity distribution ⟷ Advanced initial energy density
  - HBT radii ⟷ Advanced life-time estimation

Goal

Need for solutions that are:

- explicit
- simple
- accelerating
- relativistic
- realistic / compatible with the data:
  - lattice QCD EoS
  - ellipsoidal symmetry (spectra, v_2, v_4, HBT)
  - finite dn/dy

Report on a new class that satisfies these criteria

but not simultaneously

arXiv:0709.3677v1 [nucl-th] PRC(2008) in press

Self-similar, ellipsoidal solutions

Publication (for example):

T. Csörgő, L.P.Csernai, Y. Hama, T. Kodama, Heavy Ion Phys. A 21 (2004) 73

3D spherically symmetric HUBBLE flow:

$$u^\mu = \frac{x^\mu}{\tau}$$

No acceleration:

$$u^\mu \partial_\mu u_\nu = 0.$$

Define a scaling variable for self-similarly expanding ellipsoids:

$$s = \frac{r_x^2}{\dot{X}_0^2 t^2} + \frac{r_y^2}{\dot{Y}_0^2 t^2} + \frac{r_z^2}{\dot{Z}_0^2 t^2}$$

EoS: (massive) ideal gas

$$\epsilon = mn + \kappa p\,, \quad p = nT.$$

$$\varepsilon_Q = m_Q n_Q + \lambda_\varepsilon n_Q T + B, \quad p_Q = \lambda_p n_Q T - B,$$

$$n(t,\mathbf{r}) = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{V}(s) \qquad T(t,\mathbf{r}) = T_0 \left(\frac{\tau_0}{\tau}\right)^{3/\kappa} \frac{1}{\mathcal{V}(s)} \Rightarrow p(t,\mathbf{r}) = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+3/\kappa}$$

Scaling function $\mathcal{V}$(s) can be chosen freely.

Viscous corrections in NR limit: known analytically.

New, simple, exact solutions

$$\begin{aligned} v &= \tanh \lambda\eta, \\ p &= p_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda d \frac{\kappa+1}{\kappa}} \left(\cosh\frac{\eta}{2}\right)^{-(d-1)\phi_\lambda} \end{aligned}$$

Possible cases (one row of the table is one solution):

Case	λ	d	κ	ϕ_λ
a.)	2	$\in \mathbb{R}$	d	0
b.)	$\frac{1}{2}$	$\in \mathbb{R}$	1	$\frac{\kappa+1}{\kappa}$
c.)	$\frac{3}{2}$	$\in \mathbb{R}$	$\frac{4d-1}{3}$	$\frac{\kappa+1}{\kappa}$
d.)	1	$\in \mathbb{R}$	$\in \mathbb{R}$	0
e.)	$\in \mathbb{R}$	1	1	0

Nagy,CsT, Csanád: arXiv:0709.3677v1

→ New, accelerating, d dimension

↘ d dimensional with $p=p(\tau,\eta)$

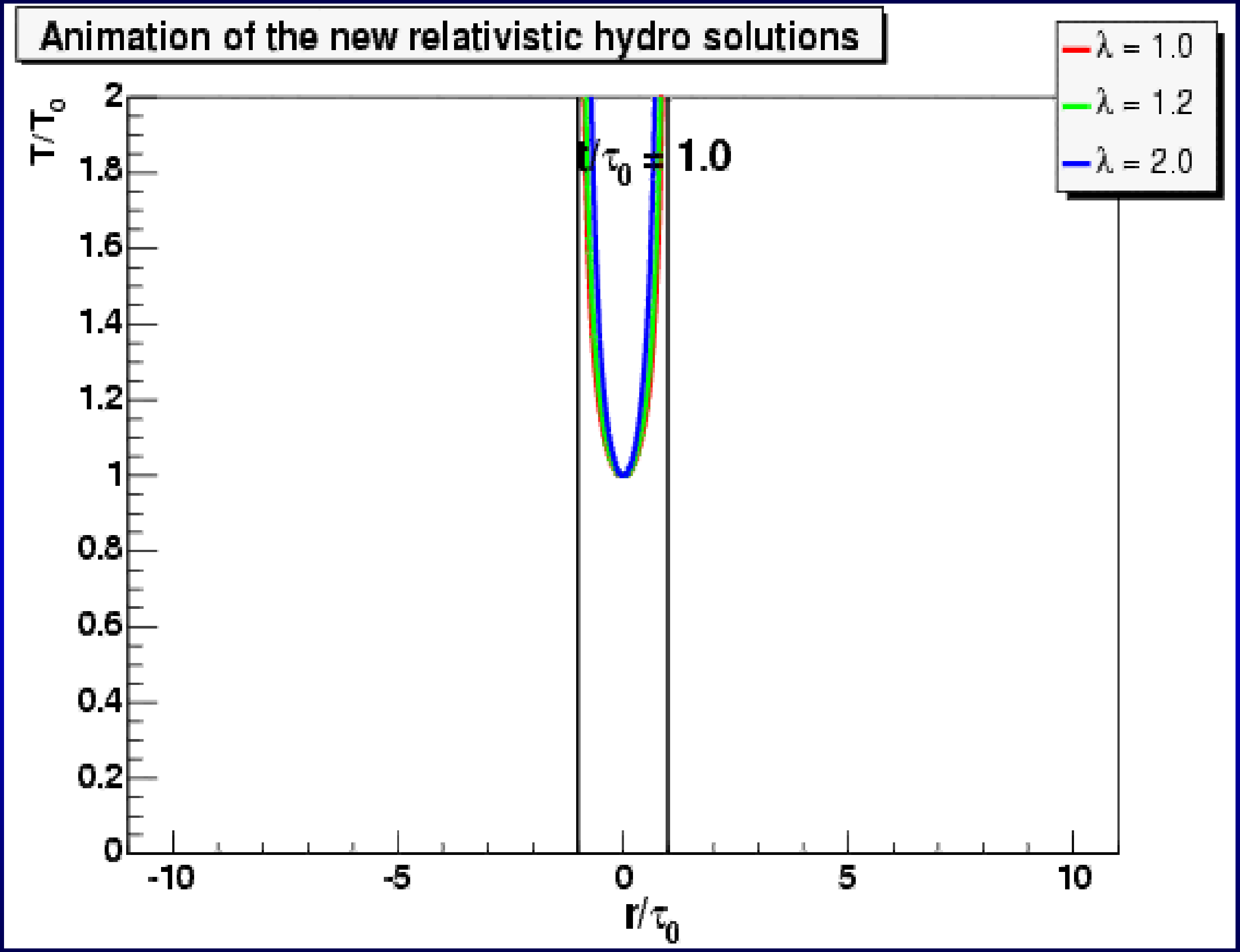
↗ (thanks T. S. Biró)

→ Hwa-Bjorken, Buda-Lund type

→ Special EoS, but general velocity

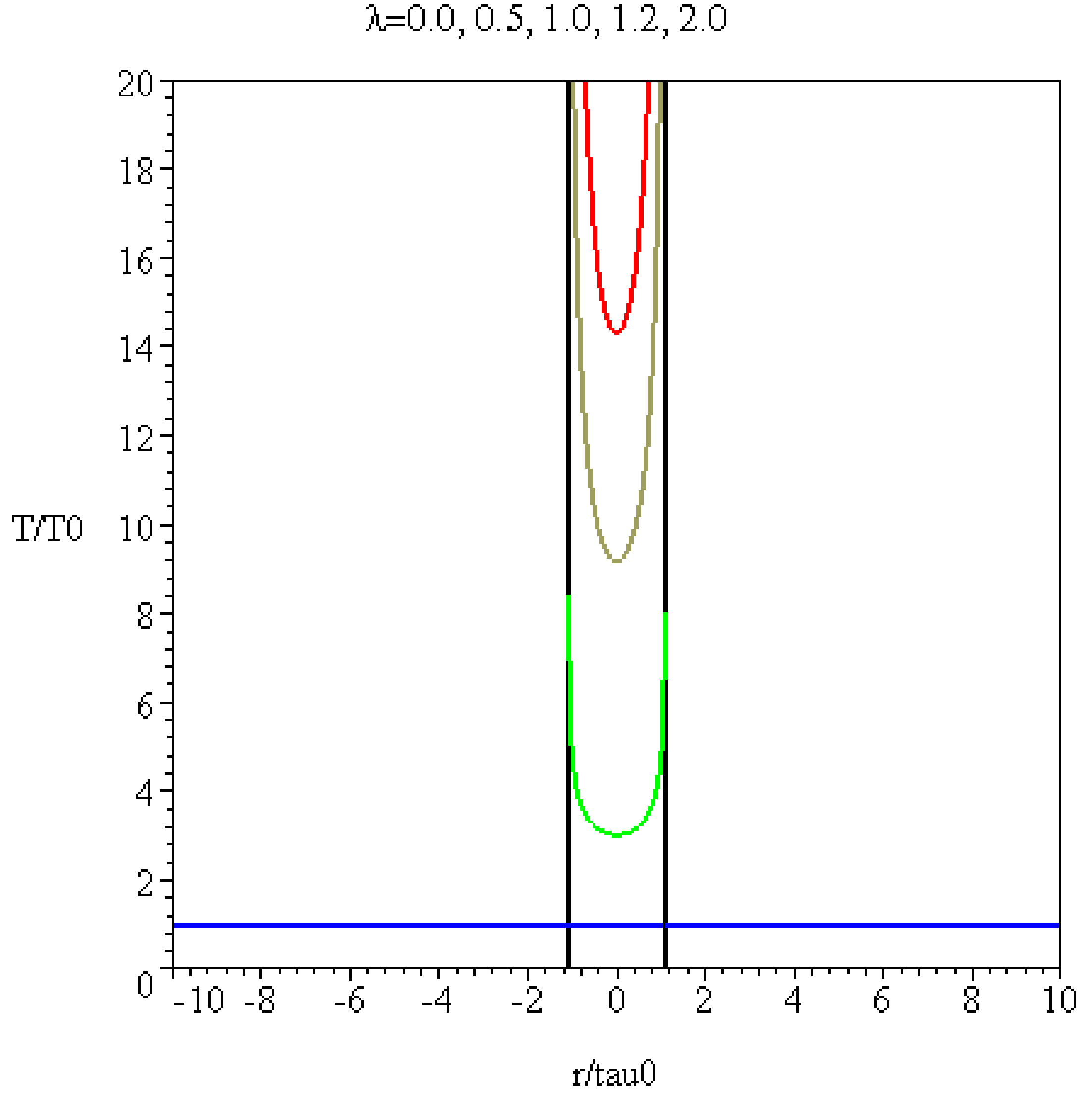
If $\kappa = d = 1$, general solution is obtained, for ARBITRARY initial conditions. It is STABLE !

New simple solutions



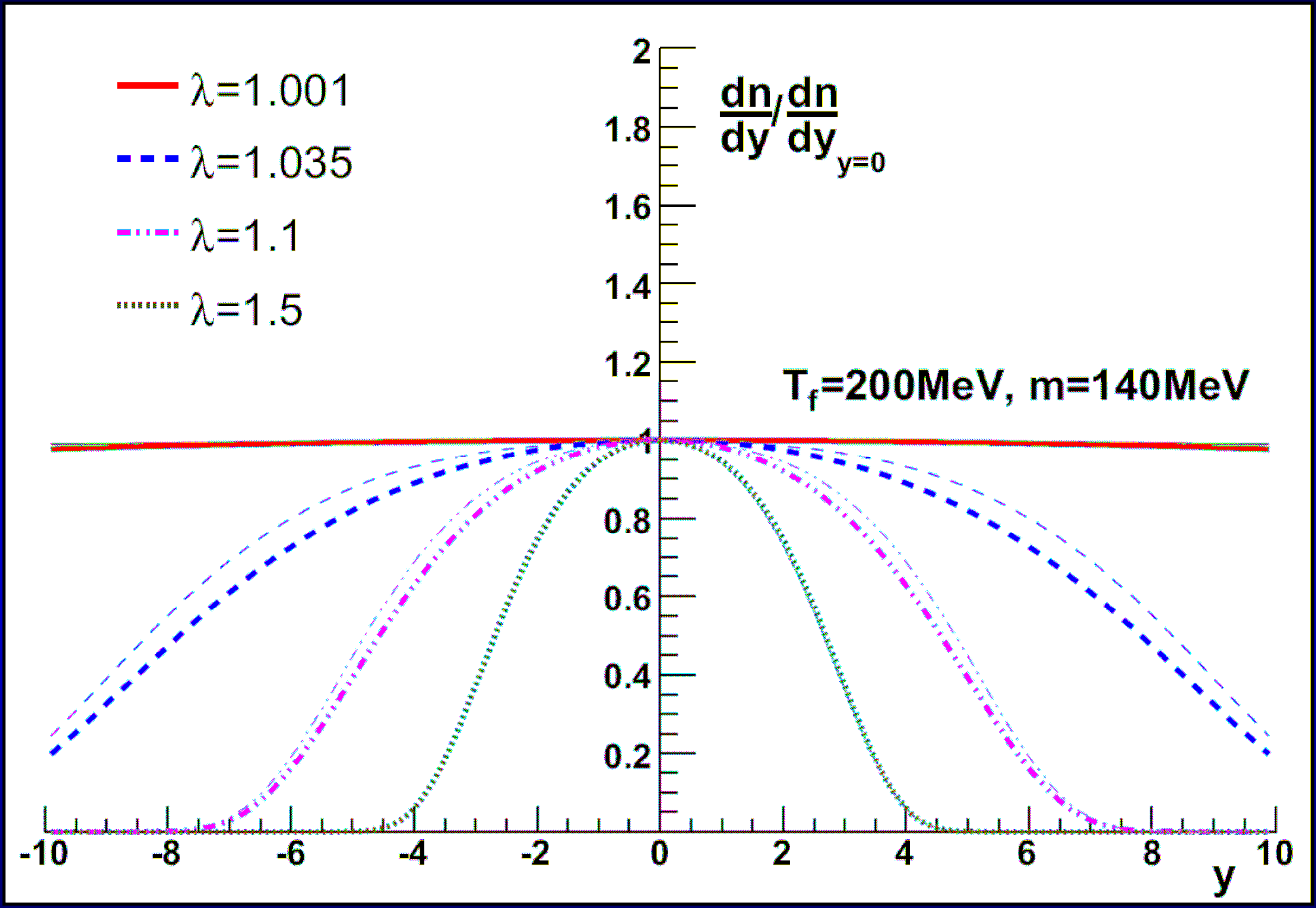
Different final states from similar initial states are reached by varying λ

New simple solutions



Similar final states from different initial states are reached by varying λ

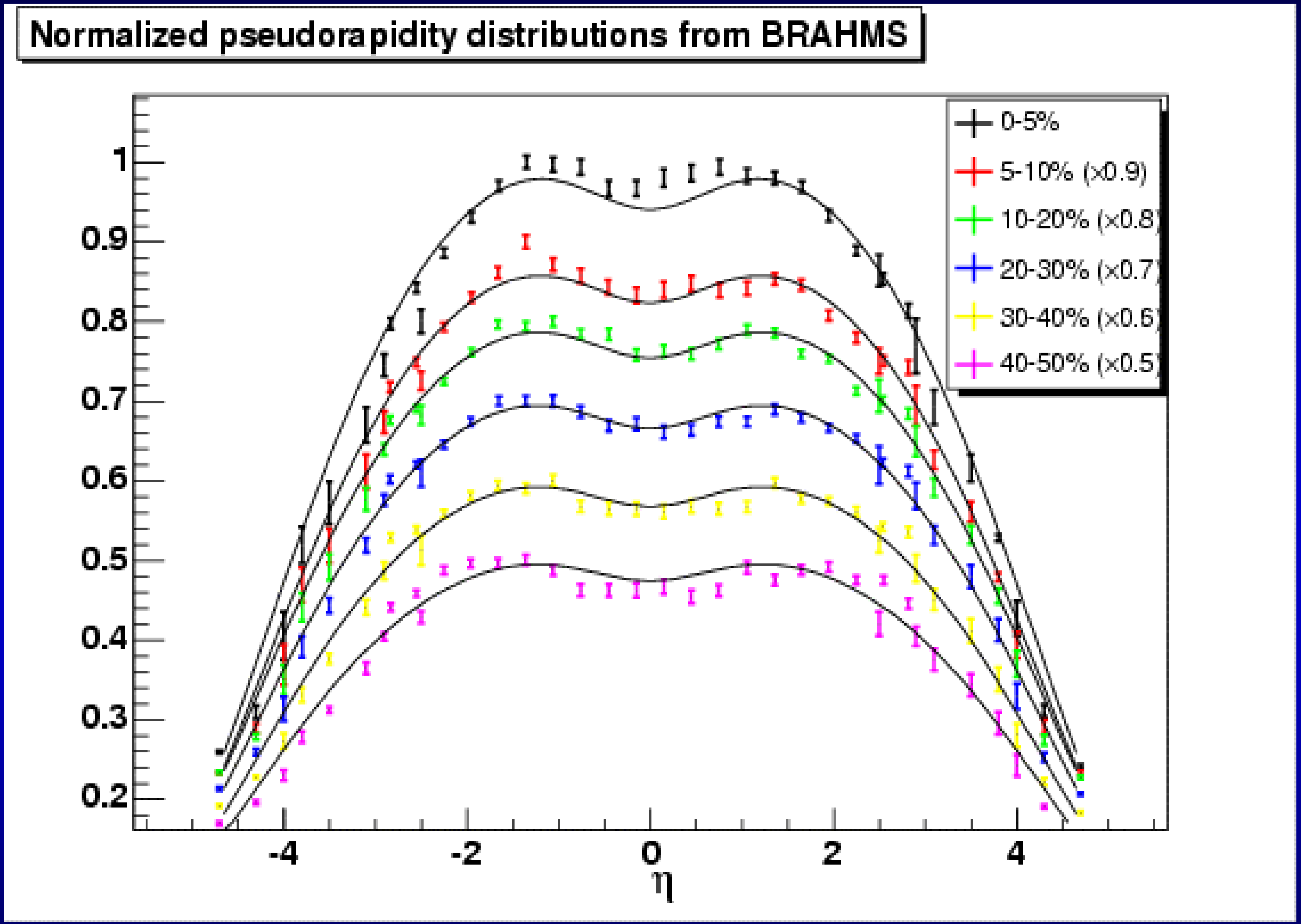
Rapidity distribution



Rapidity distribution from the 1+1 dimensional solution, for $\lambda > 1$.

T_f: slope parameter.

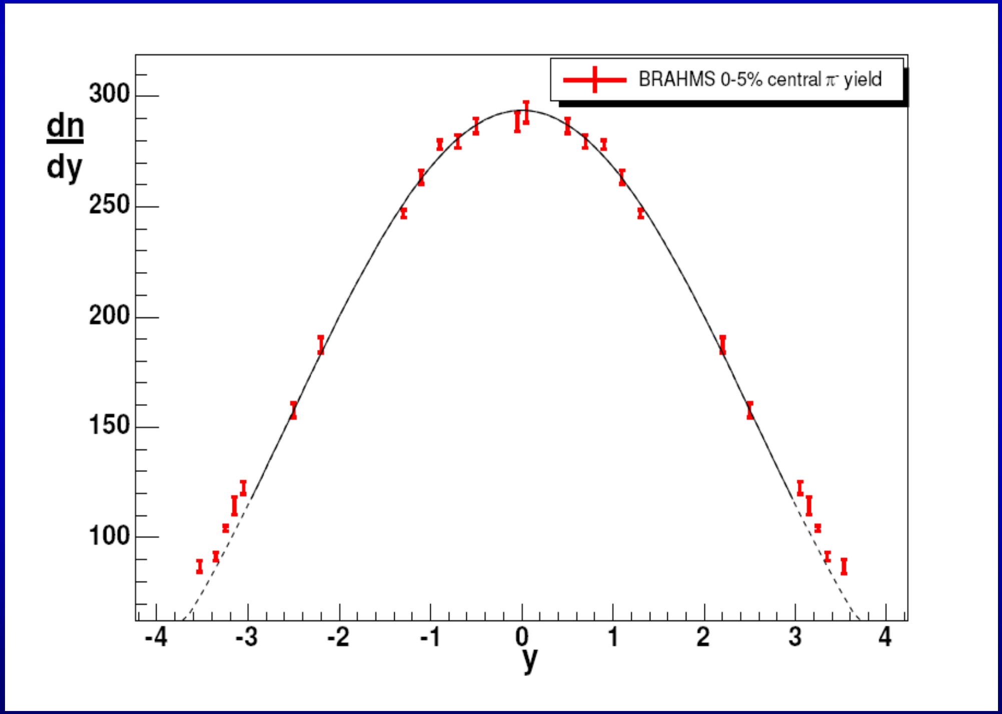
Pseudorapidity distributions



BRAHMS data fitted with the analytic formula of
Additionally: y→η transformation

BRAHMS rapidity distribution

$$\frac{dn}{dy} \approx \left.\frac{dn}{dy}\right|_{y=0} \cosh^{\pm\frac{\alpha}{2}-1}\left(\frac{y}{\alpha}\right) e^{-\frac{m}{T_f}\left[\cosh^{\alpha}\left(\frac{y}{\alpha}\right)-1\right]},$$



$$\lambda = \frac{\alpha-1}{\alpha-2}.$$

α	7.4 ± 0.13
$\left.\frac{dn}{dy}\right\|_{y=0}$	294 ± 1
χ^2/NDF	$30.6/14$
CL	0.6%
T_f (MeV)	200 (fixed)
m (MeV)	140 (fixed)
λ	1.18 ± 0.01 (derived)

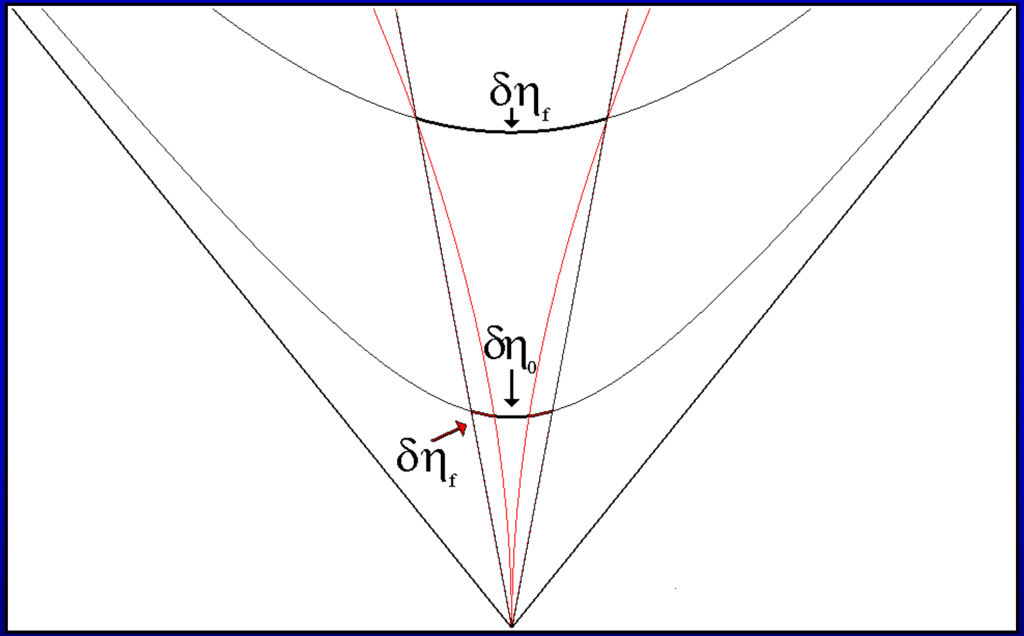
BRAHMS data fitted with the analytic formula of

Advanced energy density estimate

Fit result: $\lambda > 1$

Flows accelerate: ⟺ do work

⟺ initial energy density is higher than Bjorken's



$$\frac{\varepsilon_c}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}$$

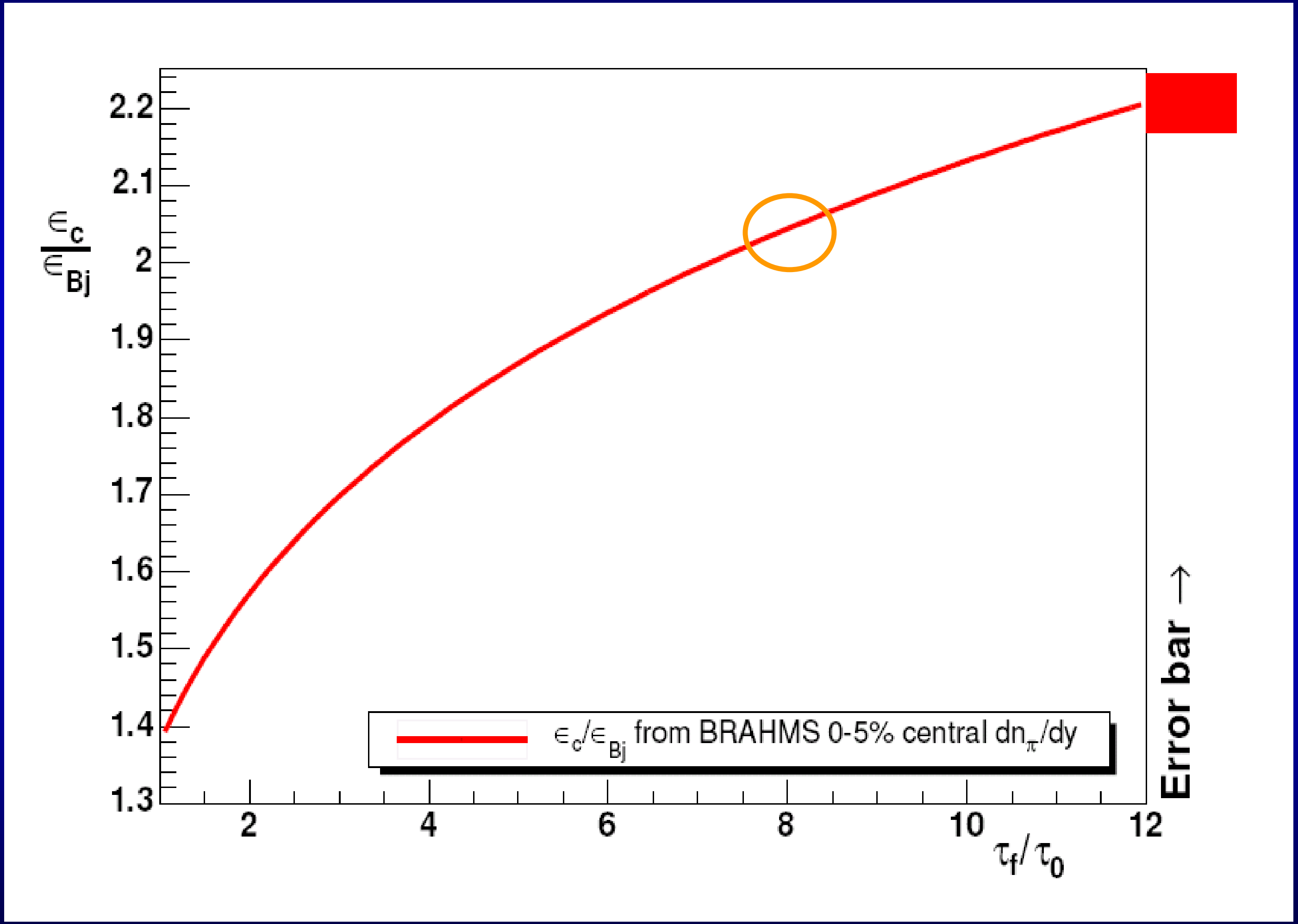
Corrections due to work and acceleration. Ref:

$$\varepsilon_{Bj} = \frac{\langle m_t \rangle}{(R^2\pi)\tau_0}\frac{dn}{dy}$$

For $\lambda > 1$ (accelerating) flows, both factors > 1

Advanced energy density estimate

Correction depends on timescales, dependence is:



With a tipical τ_f/τ_0 of ~8-10, one gets a correction factor of 2!

Conjecture: EoS dependence of ε_0

Four constraints

1) ε_{Bj} is independent of EoS ($\lambda = 1$ case)

2) $c_s^2 = 1$ case is solved for any $\lambda > 0.5$

$$\frac{\varepsilon_c}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}$$

Corrections due to respect these limits.

3) c_s^2 dependence of $\varepsilon(\tau)$ is known

4) Numerical hydro results

Conjectured formula – given by the principle of Occam's razor:

$$\frac{\varepsilon_{c_s^2}}{\varepsilon_{Bj}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}\left(\frac{\tau_f}{\tau_0}\right)^{(\lambda-1)(1-c_s^2)}$$

Using $\lambda = 1.18$, $c_s = 0.35$, $\tau_f/\tau_0 = 10$, we get $e_{cs}/e_{Bj} = 2.9$

$\varepsilon_0 = 14.5$ GeV/fm^3 in 200 GeV, 0-5 % Au+Au at RHIC

Advanced life-time estimate

Life-time estimation: for Hwa-Bjorken type of flows

$$R_{long} = \sqrt{\frac{T_f}{m_t}}\tau_{Bj} \quad \Rightarrow \quad \tau_{Bj} = \sqrt{\frac{m_t}{T_f}}R_{long}.$$

Makhlin & Sinyukov, Z. Phys. C 39, 69 (1988)

Underestimates lifetime (Renk, CsT, Wiedemann, Pratt, …)

Advanced life-time estimate:

width of dn/dy related to acceleration and work

$$R_{\rm long} = \sqrt{\frac{T_f}{m_t}\frac{\tau_{\rm c}}{\lambda}} \quad \Rightarrow \quad \tau_{\rm c} = \lambda\tau_{Bj}.$$

At RHIC energies: correction is about +20%

Conclusions

Explicit simple accelerating relativistic hydrodynamics

- Analytic (approximate) calculation of observables
- Realistic rapidity distributions; BRAHMS data well described

No go theorem: same final states, different initial states

New estimate of initial energy density:

- $\varepsilon_c/\varepsilon_{Bj}$ ~ 2 @ RHIC
- dependence on c_s estimated, $\varepsilon_c/\varepsilon_{Bj}$ ~ 3 for $c_s = 0.35$

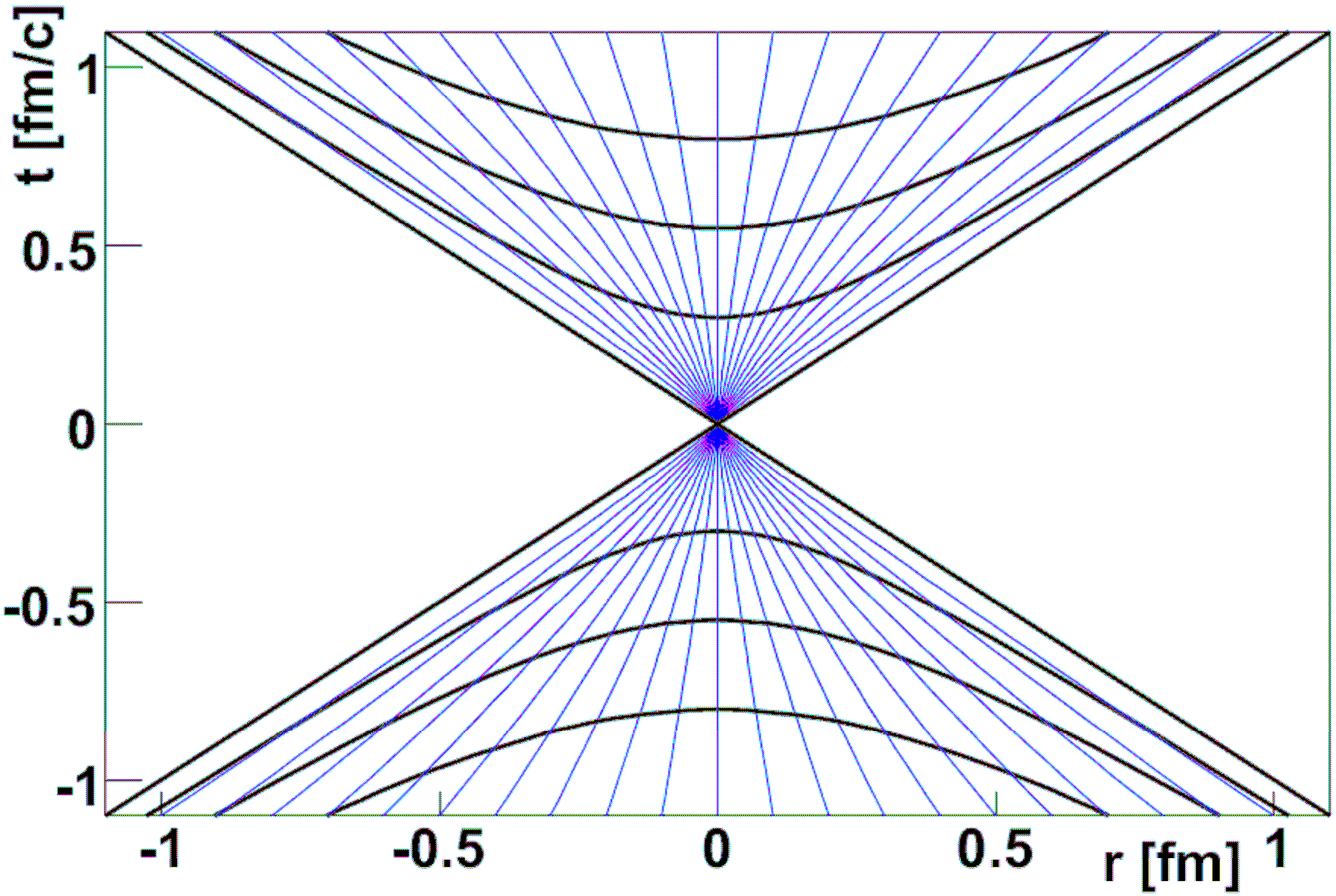
Estimated work effects on lifetime:

- ~ 20% increase @ RHIC

A lot to do …

- more general EoS
- less symmetry, ellipsoidal solutions
- asymptotically Hubble-like flows

New simple solutions in 1+D dim



Back-up Slides

How Perfect is "Perfect" ? Measure η/s !

Damping (flow, fluctuations, heavy quark motion) ~ η/s

FLOW: *Has the QCD Critical Point Been Signaled by Observations at RHIC?*, R. Lacey *et al.*, Phys.Rev.Lett.98:092301,2007 (nucl-ex/0609025)

$$\frac{\eta}{s}=(1.1\pm0.2\pm1.2)\frac{1}{4\pi}$$

The Centrality dependence of Elliptic flow, the Hydrodynamic Limit, and the Viscosity of Hot QCD, H.-J. Drescher *et al.*, (arXiv:0704.3553)

$$\frac{\eta}{s}=(1.9-2.5)\frac{1}{4\pi}$$

FLUCTUATIONS: *Measuring Shear Viscosity Using Transverse Momentum Correlations in Relativistic Nuclear Collisions*, S. Gavin and M. Abdel-Aziz, Phys.Rev.Lett.97:162302,2006 (nucl-th/0606061)

$$\frac{\eta}{s}=(1.0-3.8)\frac{1}{4\pi}$$

DRAG, FLOW: *Energy Loss and Flow of Heavy Quarks in Au+Au Collisions at* $\sqrt{s_{NN}}$ *= 200 GeV* (PHENIX Collaboration), A. Adare *et al.*, Phys.Rev.Lett.98:172301,2007 (nucl-ex/0611018)

$$\frac{\eta}{s}=(1.3-2.0)\frac{1}{4\pi}$$

C
H
A
R
M
!

Landau-Khalatnikov solution

Publications:

- L.D. Landau, Izv. Acad. Nauk SSSR 81 (1953) 51
- I.M. Khalatnikov, Zhur. Eksp.Teor.Fiz. 27 (1954) 529
- L.D.Landau and S.Z.Belenkij, Usp. Fiz. Nauk 56 (1955) 309

Implicit 1D solution with approx. Gaussian rapidity distribution

Basic relations: $u^0 = \cosh\Omega(t,x), u^1 = \sinh\Omega(t,x)$

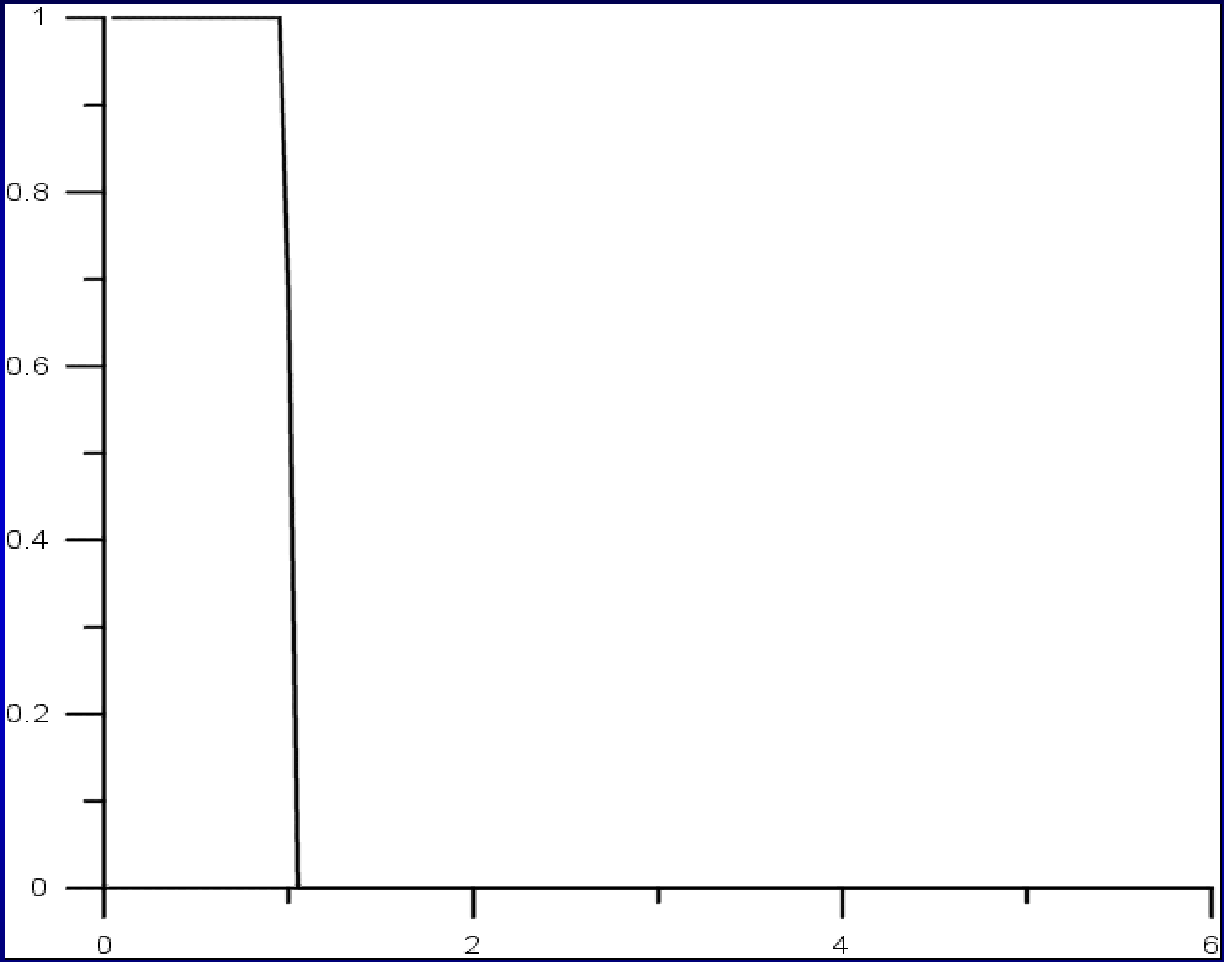
Unknown variables: $T(t,x),\ \Omega(t,x)$

Auxiliary function: $\chi(\Omega,T)$

$$t = \frac{\partial\chi}{\partial T}\cosh\Omega - \frac{1}{T}\frac{\partial\chi}{\partial\Omega}\sinh\Omega, \quad x = \frac{\partial\chi}{\partial T}\sinh\Omega - \frac{1}{T}\frac{\partial\chi}{\partial\Omega}\cosh\Omega$$

Expression of $\chi(\Omega,T)$ is a true „tour de force”

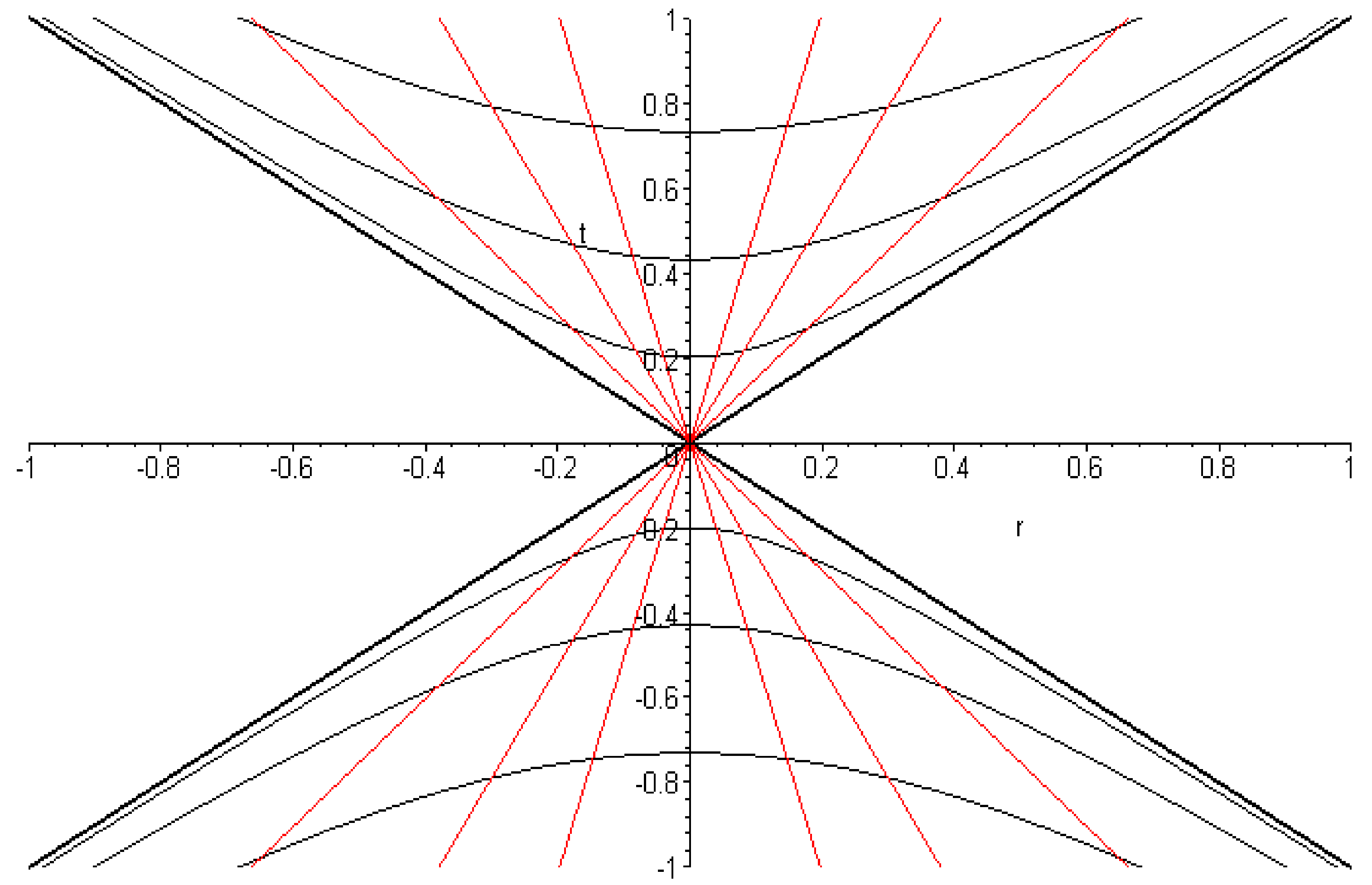
Landau-Khalatnikov solution



Temperature distribution (animation courtesy of T. Kodama)

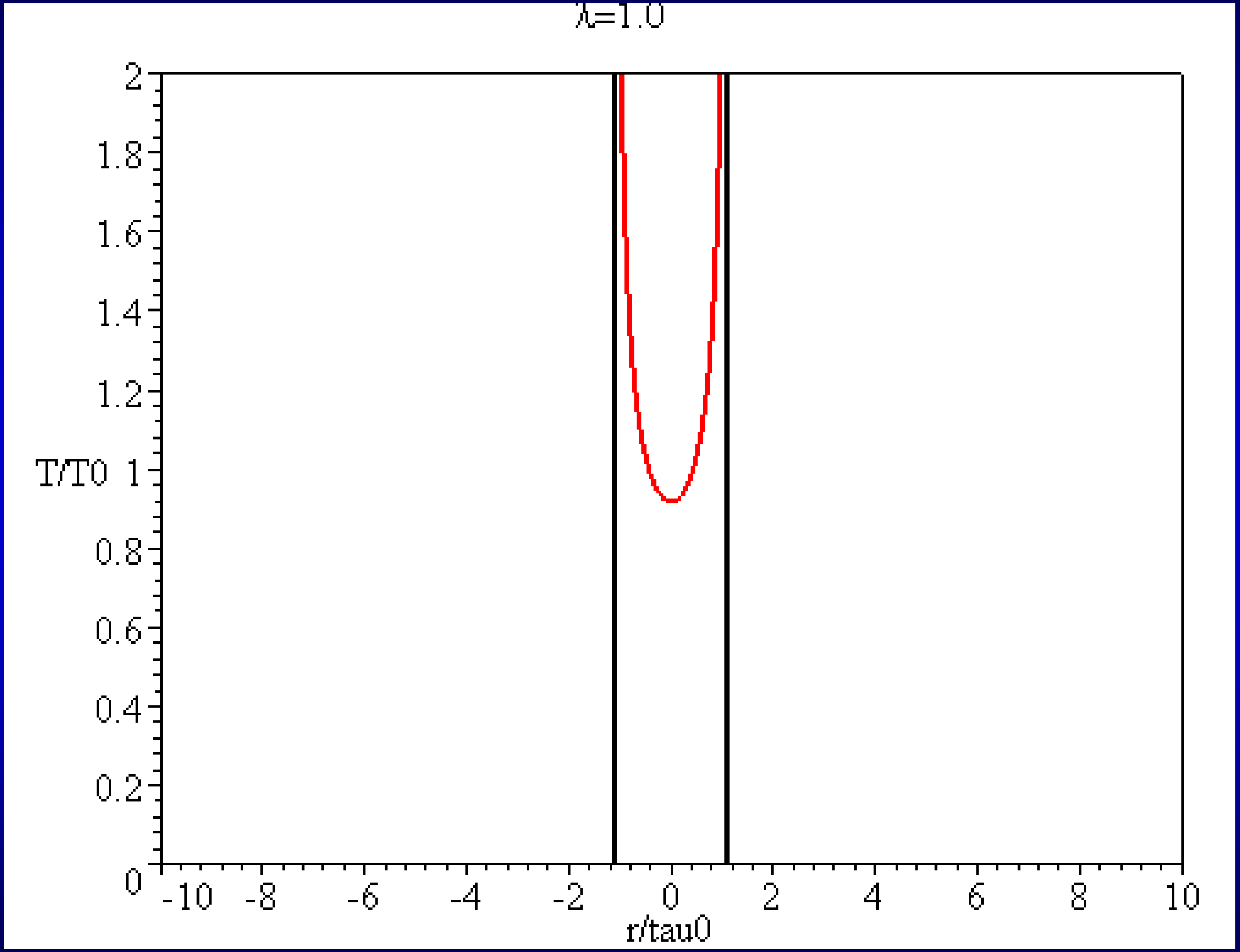
„Tour de force” implicit solution: t=t(T,v), r=r(T,v)

Hwa-Bjorken solution



The Hwa-Bjorken solution / Rindler coordinates

Hwa-Bjorken solution



The Hwa-Bjorken solution / Temperature evolution

Bialas-Janik-Peschanski solution

Publications:

A. Bialas, R. Ja

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i

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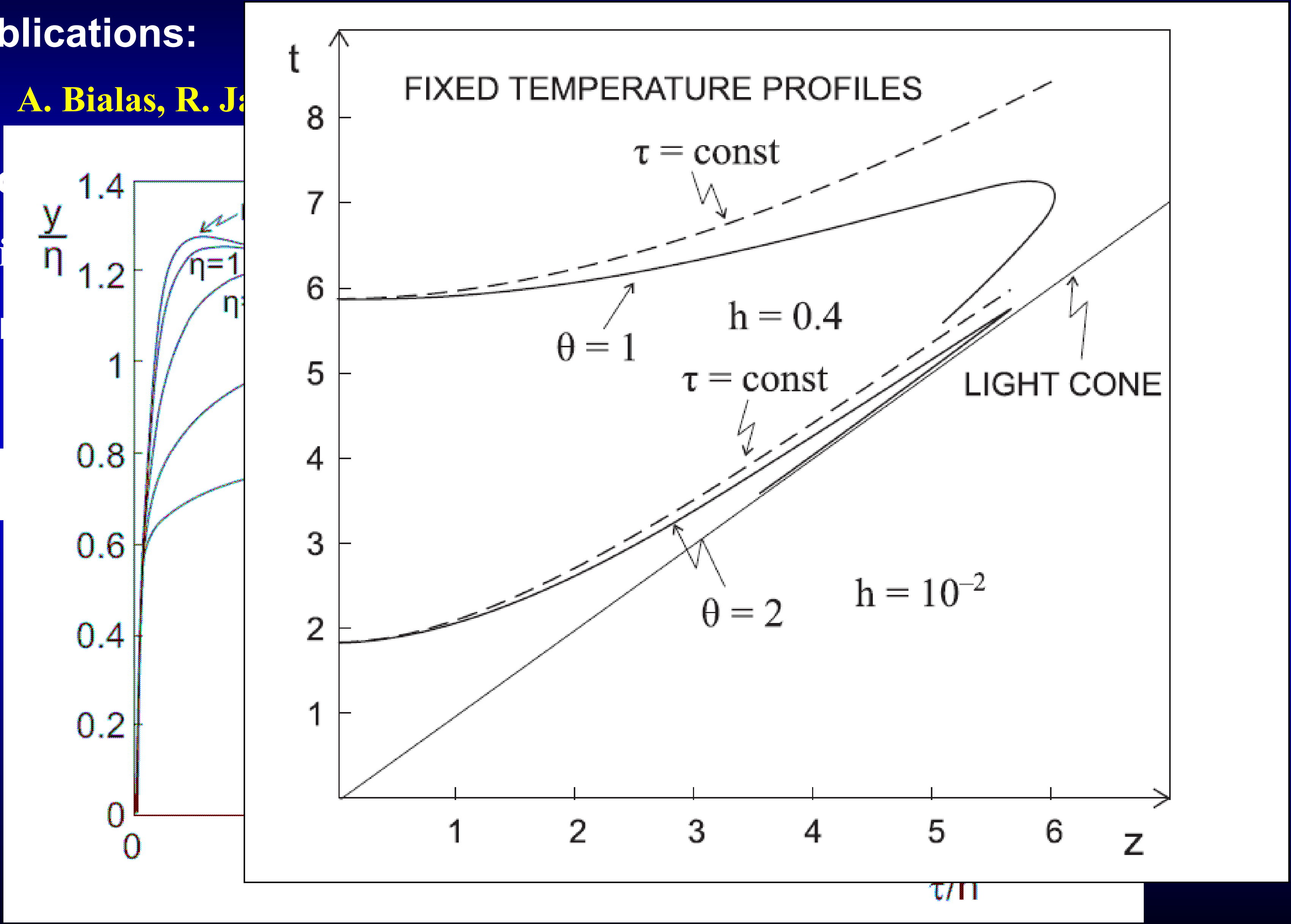
$2y =$



Hwa-Bjorken solution

Publications:

R.C. Hwa, Phys. Rev. D10, 2260 (1974)

J.D. Bjorken, Phys. Rev. D27, 40(1983)

Accelerationless, expanding 1D simple boost-invariant solution

Rindler coordinates: $t = \tau \cosh \eta, \quad r = \tau \sinh \eta$

$$\eta = \operatorname{arctanh} \frac{r}{t}, \tau = \sqrt{t^2 - r^2} = \sqrt{x_\mu x^\mu}$$

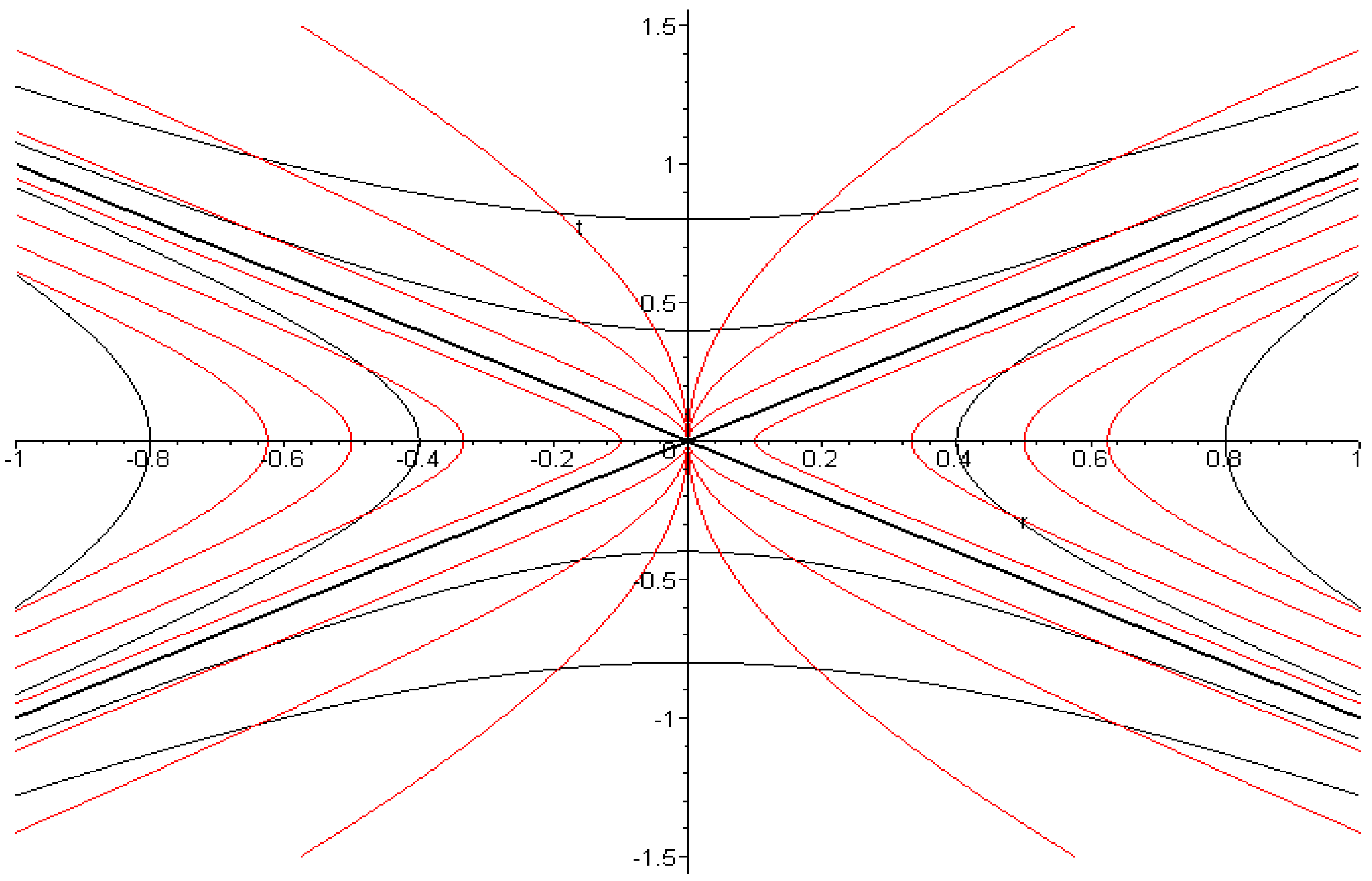
Boost-invariance (valid for asymptotically high energies):

$$\varepsilon = \varepsilon(\tau),\ p = p(\tau),\ s = s(\tau),\ u^\mu = \frac{x^\mu}{\tau},\ v = \frac{z}{t} \Rightarrow \frac{d\mathbf{v}}{dt} = 0$$

$$\partial_\mu (s u^\mu) = 0 \qquad \frac{s}{s_0} = \frac{\tau_0}{\tau}$$

$T = T(\tau)$ depends on EoS, e.g. $\varepsilon = \kappa p,\ \kappa = D,\ D = 1 \Rightarrow T(\tau) = T_0 \frac{\tau_0}{\tau}$

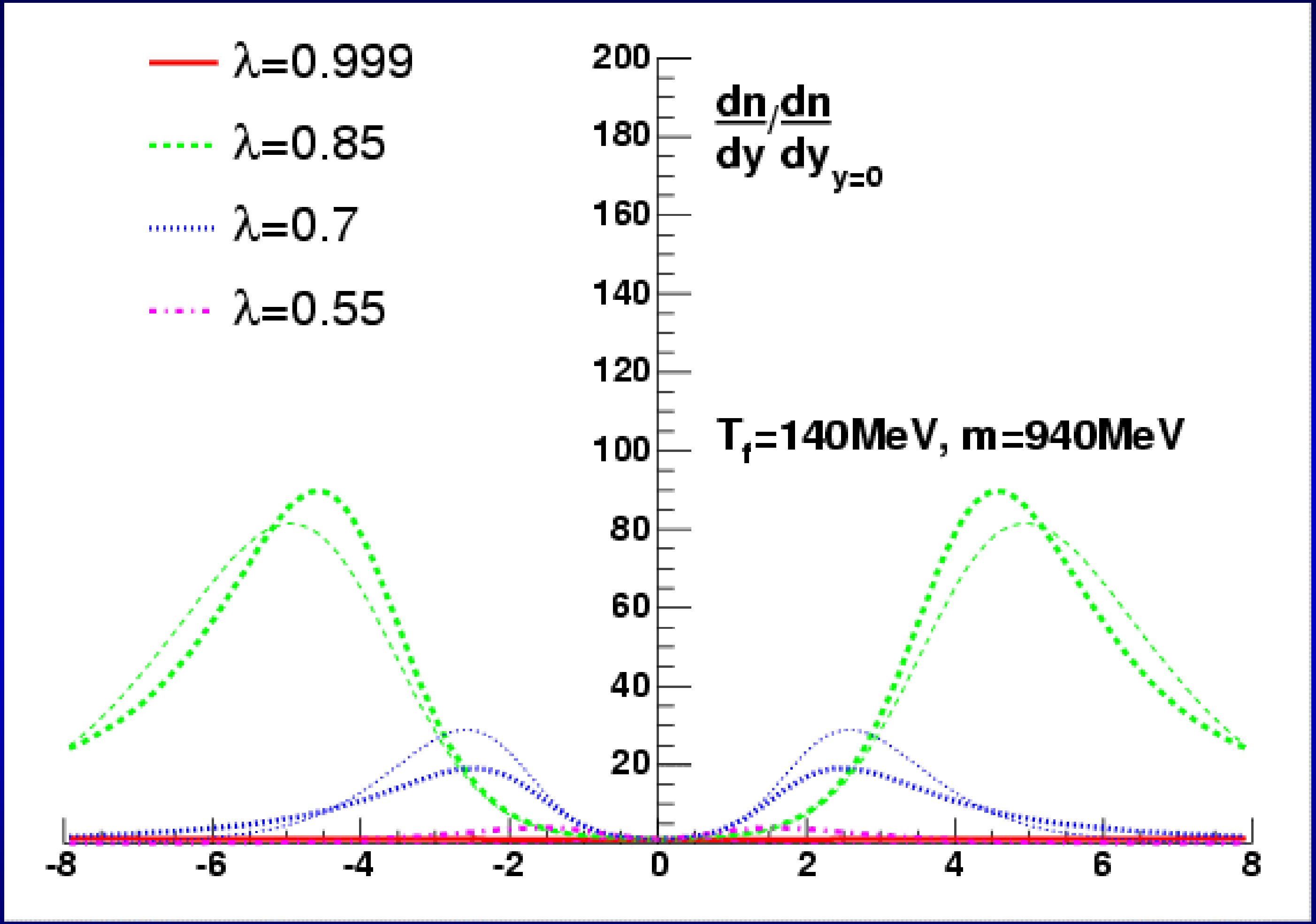
New simple solutions in 1+d dim



The fluid lines (red) and the pseudo-orthogonal freeze-out surface (black)

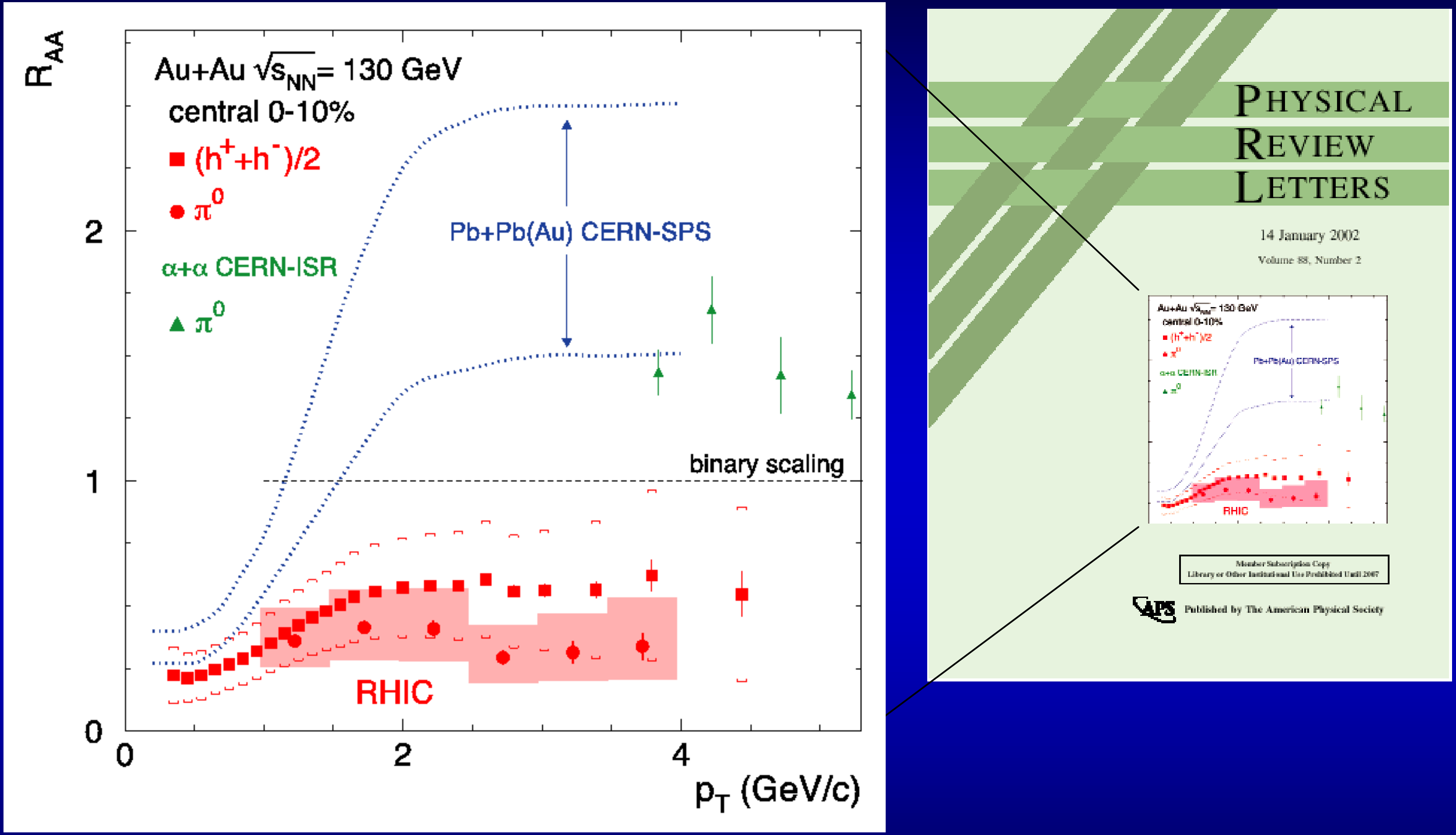
$A = const.$

Rapidity distribution



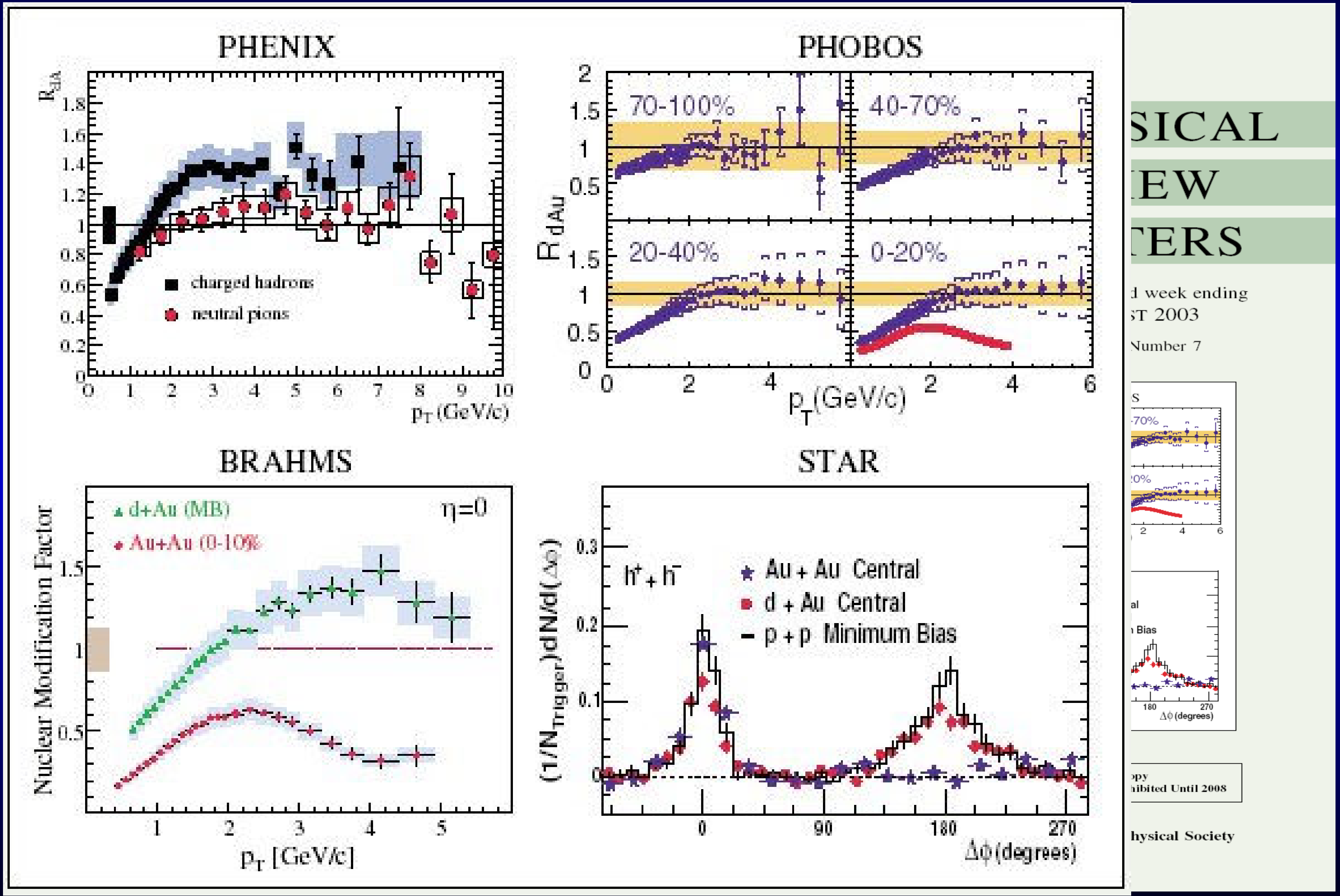
Rapidity distribution from the 1+1 dimensional solution, for $\lambda < 1$

1^{st} milestone: new phenomena

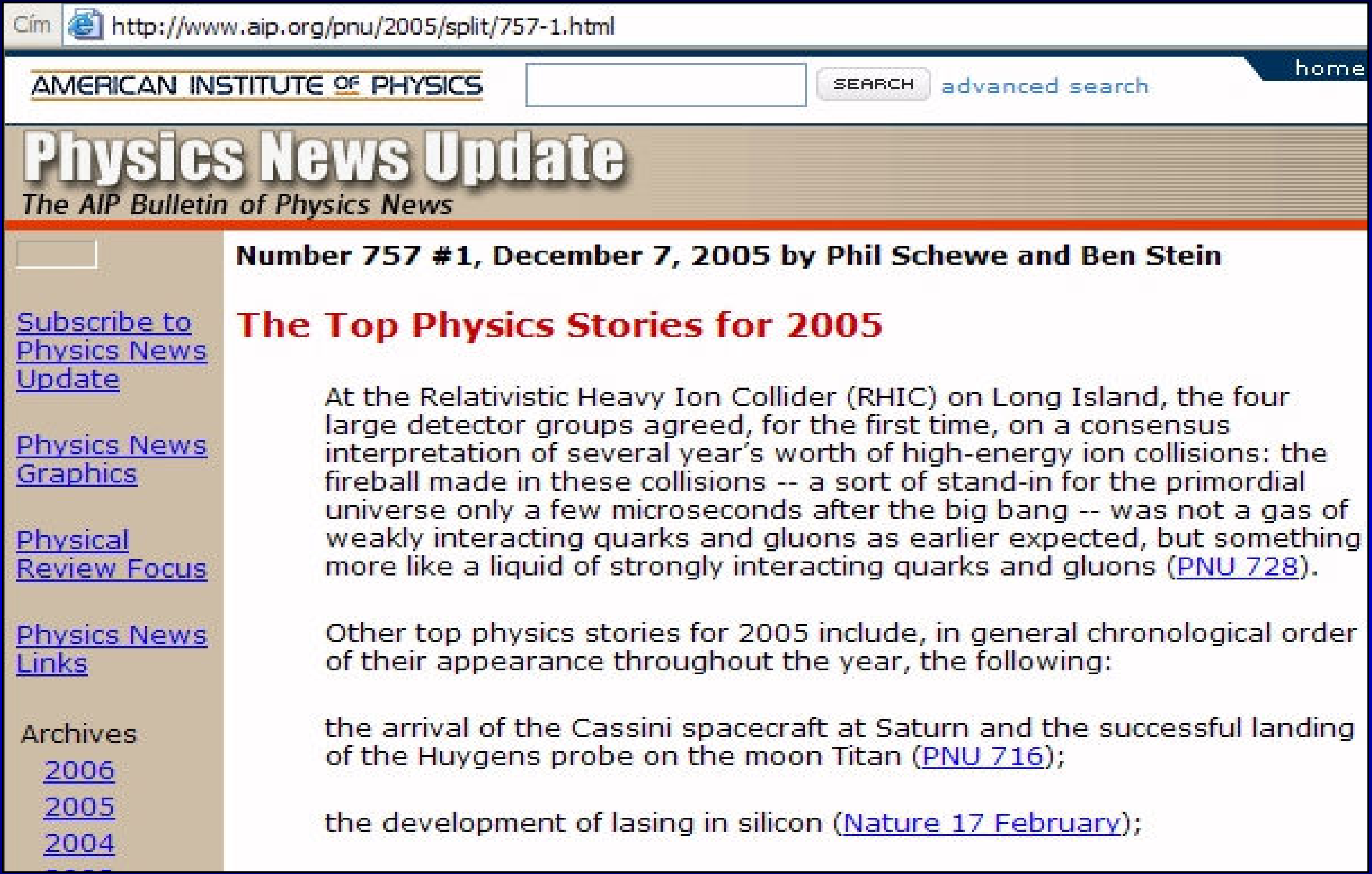


Suppression of high p_t particle production in Au+Au collisions at RHIC

2nd milestone: new form of matter



3rd milestone: Top Physics Story 2005



Physics News Update

The AIP Bulletin of Physics News

Number 757 #1, December 7, 2005 by Phil Schewe and Ben Stein

The Top Physics Stories for 2005

At the Relativistic Heavy Ion Collider (RHIC) on Long Island, the four large detector groups agreed, for the first time, on a consensus interpretation of several year's worth of high-energy ion collisions: the fireball made in these collisions -- a sort of stand-in for the primordial universe only a few microseconds after the big bang -- was not a gas of weakly interacting quarks and gluons as earlier expected, but something more like a liquid of strongly interacting quarks and gluons (PNU 728).

Other top physics stories for 2005 include, in general chronological order of their appearance throughout the year, the following:

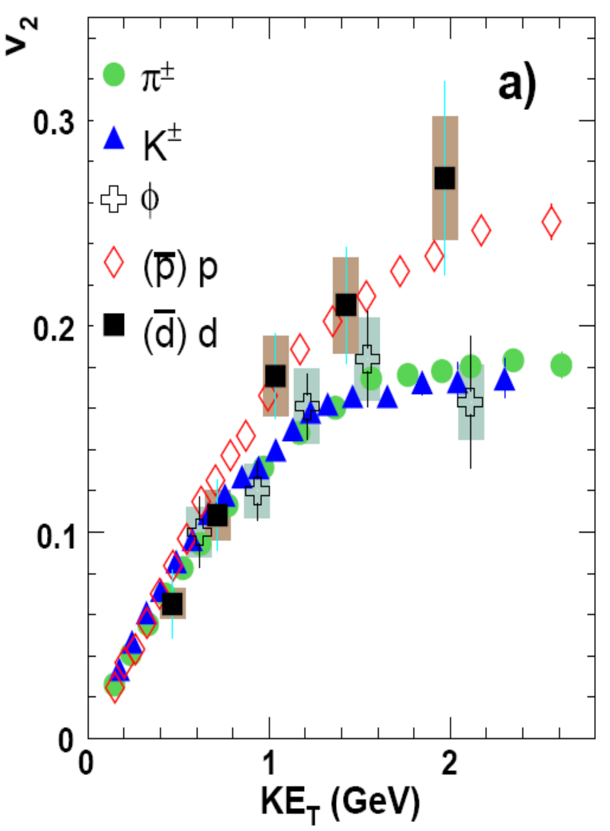
the arrival of the Cassini spacecraft at Saturn and the successful landing of the Huygens probe on the moon Titan (PNU 716);

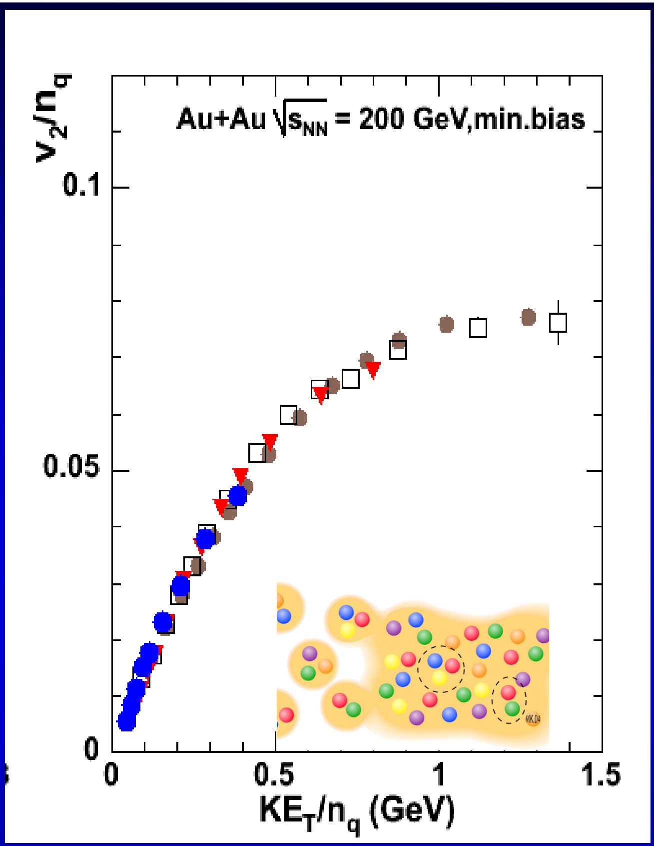
the development of lasing in silicon (Nature 17 February);

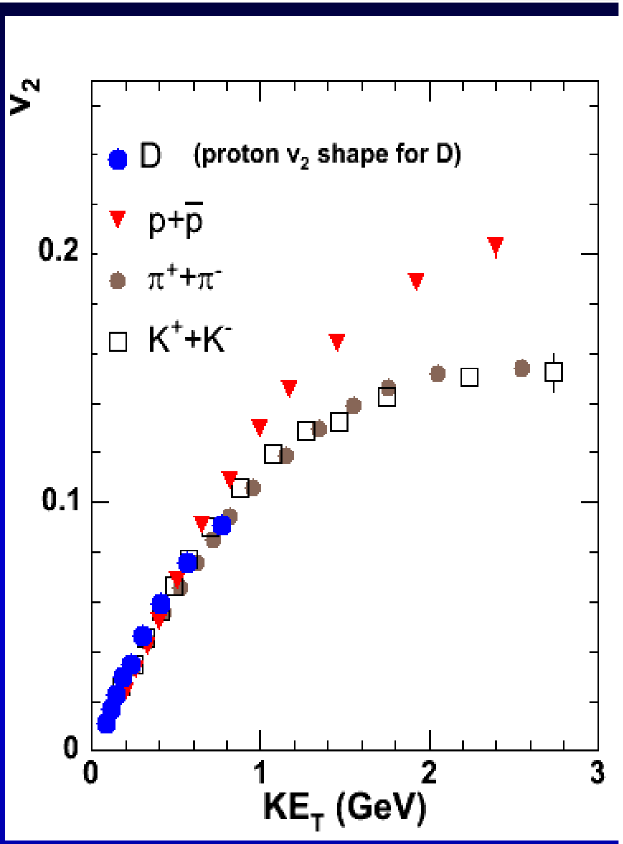
http://arxiv.org/abs/nucl-ex/0410003

PHENIX White Paper: second most cited in nucl-ex during 2006

4th Milestone: A fluid of quarks







v_2 for the φ follows that of other mesons

$$v_2^{hadron}\left(KE_T^{hadron}\right) \approx n v_2^{quark}\left(KE_T^{quark}\right)$$
$$KE_T^{hadron} \approx n KE_T^{quark}$$

v_2 for the D follows that of other mesons

Strange and even charm quarks participate in the flow