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Baryon number Strangeness and electric Charge fluctuations at zero and non zero Density

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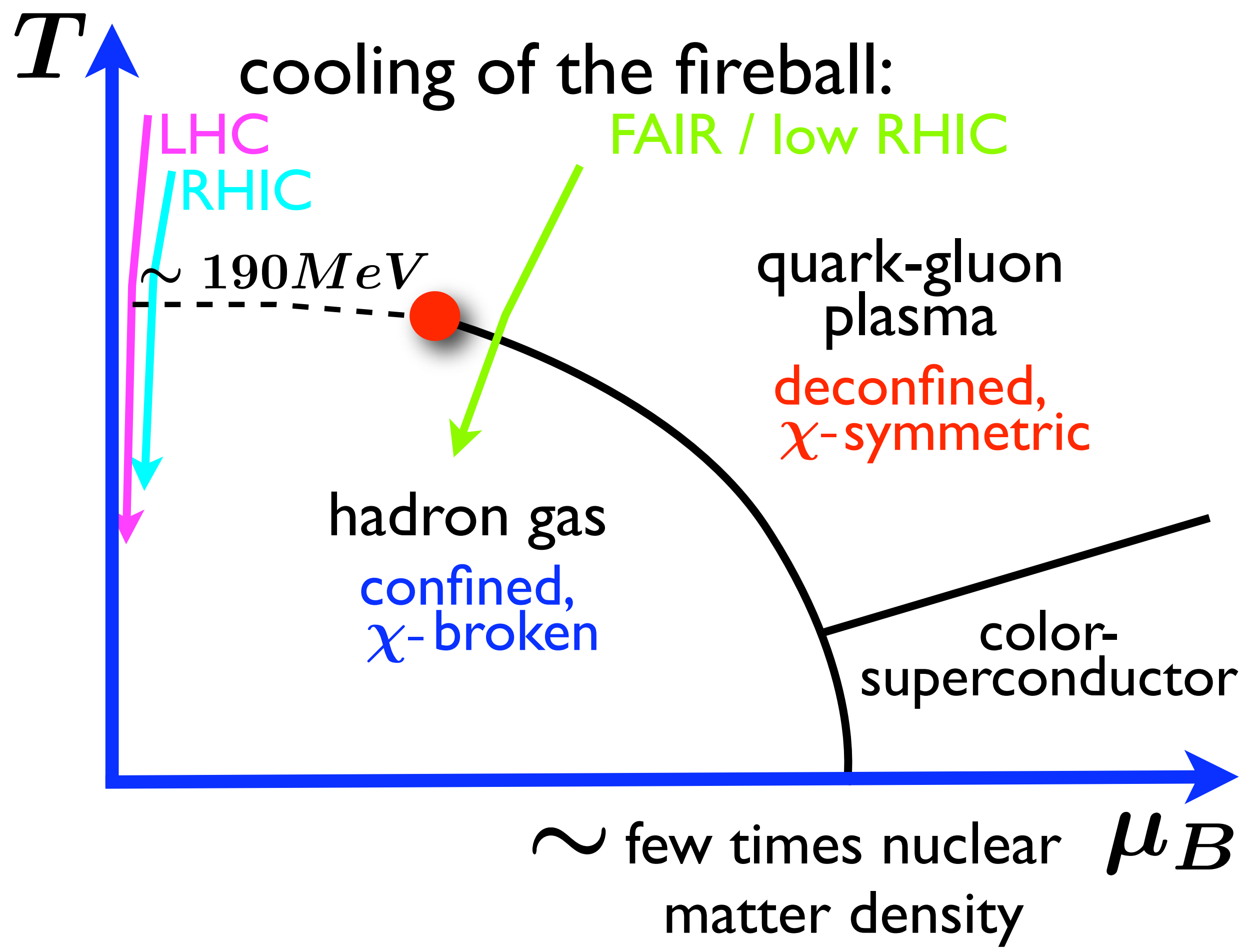
Outline

- Introduction
- Taylor expansion in B, S, Q chemical potentials
  - $\longrightarrow$ ***definition:*** cumulants at $\mu_{B,S,Q} = 0$
  - $\longrightarrow$ ***expectations:*** the resonance gas at low T, universal scaling at T_c
  - $\longrightarrow$ ***good observables:*** ratios of cumulants
- Baryon, strangeness and charge fluctuations at $\mu_B > 0 \quad (\mu_S = \mu_Q = 0)$
- Correlations between charges
- Conclusions

Introduction:
The phase diagram of QCD

- Fluctuations of B, S, Q can be measured experimentally and **indicate criticality**
- LGT at $\mu = 0$
  $\longrightarrow$ RHIC, LHC
- LGT at $\mu > 0$
  $\longrightarrow$ RHIC at low energies, FAIR@GSI



$$\mu_S = \mu_Q = 0$$

Taylor expansion in: $\mu_{B,S,Q}$

QCD is naturally formulated with quark chemical potentials $\mu_{u,d,s}$

- we start from Taylor expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_u,\mu_d,\mu_s) = \sum_{i,j,k} c_{i,j,k}^{u,d,s} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

- use unbiased, noisy estimators to calculate $c_{i,j,k}^{u,d,s}$
  $\longrightarrow$ see C. Miao, CS, PoS (Lattice 2007) 175.

- Line of constant physics: $m_q = m_s/10$
  (physical strange quark mass)

- measure currently up to $\mathcal{O}(\mu^8) \quad \longleftrightarrow \quad (N_t = 4)$
  $\mathcal{O}(\mu^4) \quad \longleftrightarrow \quad (N_t = 6)$

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- expansion coefficients $c_{i,j,k}^{u,d,s}$ are related to B,S,Q-fluctuations

$$\begin{aligned} n_B &= \frac{\partial(p/T^4)}{\partial(\mu_B/T)} = \frac{1}{3}(n_u + n_d + n_s) \\ n_S &= \frac{\partial(p/T^4)}{\partial(\mu_S/T)} = -n_s \\ n_Q &= \frac{\partial(p/T^4)}{\partial(\mu_Q/T)} = \frac{1}{3}(2n_u - n_d - n_s) \end{aligned}$$

$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S \end{aligned}$$

- choice of $\mu_u \equiv \mu_d$ is equivalent to $\mu_Q \equiv 0$

Hadronic fluctuations $(\mu_B = 0)$

In general we have:

- $\dfrac{\partial^p (p/T^4)}{\partial(\mu_X/T)^p}$ related to → $< n_X^p >_{\mu=0}$
- $\dfrac{\partial^{p+q} (p/T^4)}{\partial(\mu_X/T)\partial(\mu_Y/T)^q}$ related to → $< n_X^p n_Y^q >_{\mu=0}$

to be more precise:

$$
\begin{aligned}
2c_2^X = \frac{\partial^2 (p/T^4)}{\partial(\mu_X/T)^2} &= \frac{1}{VT^3}\left\langle (\delta N_X)^2 \right\rangle = \frac{1}{VT^3}\left\langle N_X^2 \right\rangle \\
24c_4^X = \frac{\partial^4 (p/T^4)}{\partial(\mu_X/T)^4} &= \frac{1}{VT^3}\left(\left\langle (\delta N_X)^4 \right\rangle - 3\left\langle (\delta N_X^2) \right\rangle^2 \right)_{\mu=0} \\
&= \frac{1}{VT^3}\left(\left\langle N_X^4 \right\rangle - 3\left\langle N_X^2 \right\rangle^2 \right)_{\mu=0} \\
c_{11}^{XY} = \frac{\partial^2 (p/T^4)}{\partial(\mu_X/T)\partial(\mu_Y/T)} &= \frac{1}{VT^3}\left(\left\langle N_X N_Y \right\rangle - \left\langle N_X \right\rangle \left\langle N_Y \right\rangle \right)_{\mu=0}
\end{aligned}
$$

with: $\delta N_X = N_X - \langle N_X \rangle$

The Resonance gas $(T < T_c)$

- the pressure is given by the free quantum gas pressure, summed over all particles

$$\ln Z(T,V,\mu_B,\mu_S,\mu_Q) = \sum_{i\in hadrons} \ln Z_{m_i}(T,V,\mu_B,\mu_S,\mu_Q)$$
$$\sum_{i\in mesons} \ln Z^B_{m_i}(T,V,\mu_S,\mu_Q) + \sum_{i\in baryons} \ln Z^F_{m_i}(T,V,\mu_B,\mu_S,\mu_Q)$$

- the contribution of particle i with mass m_i and quantum numbers B_i, S_i, Q_i

mesons:

$$\frac{p_i}{T^4} = \frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 \sum_{l=1}^{\infty}(+1)^{l+1} l^{-2} K_2(lm_i/T) \cosh(lS_i\mu_S/T + lQ_i\mu_Q/T)$$

$$K_2(x) \approx \sqrt{(\pi/2x)}\exp(-x), \quad x >> 1$$

baryons:

$$\frac{p_i}{T^4} = \frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 \sum_{l=1}^{\infty}(-1)^{l+1} l^{-2} K_2(lm_i/T) \cosh(lB_i\mu_B/T + lS_i\mu_S/T + lQ_i\mu_Q/T)$$

$\longrightarrow$ for a dilute baryonic gas the Boltzmann approximation is valid:
(only $l = 1$ contributes for $(m_N - \mu_B) >> T$)

$$\frac{p_i}{T^4} = \frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \cosh(B\mu_B/T + S\mu_S/T + Q\mu_Q/T)$$

The Resonance gas $(T < T_c)$

- Baryon number fluctuations in Boltzmann approximation:

$$2c_2^B = \frac{\partial^2 (p/T^4)}{\partial(\mu_X/T)^2} = F(m,T)\, B^2 \cosh(B\mu_B/T)|_{\mu_B=0}$$
$$24c_4^B = \frac{\partial^4 (p/T^4)}{\partial(\mu_X/T)^4} = F(m,T)\, B^4 \cosh(B\mu_B/T)|_{\mu_B=0}$$

with $F(m,T) = \sum_{i \in baryons} \frac{d_i}{\pi^2} \left(\frac{m_i}{T}\right)^2 K_2(m_i/T)$

$\longrightarrow$ μ_B-dependence factorize (all baryons have $B_i = 1$)

$\longrightarrow$ ratio of fourth and second order cumulant gives „unit charge“

$$12\frac{c_4^B}{c_2^B} = B^2$$

mass and temperature independent ratio

- Strangeness and electric charge fluctuations are more difficult: multiple charged particles, light mesons (Boltzmann approximation not valid)

Critical scaling $(T = T_c)$

At $\mu = 0$ and $m_{u,d}$ small, we expect O(4)-critical behavior

- scaling field:

$$t = \left|\frac{T - T_c}{T_c}\right| + A\left(\left(\frac{\mu_u}{T_c}\right)^2 + \left(\frac{\mu_d}{T_c}\right)^2\right) + B\frac{\mu_u}{T_c}\frac{\mu_d}{T_c}$$

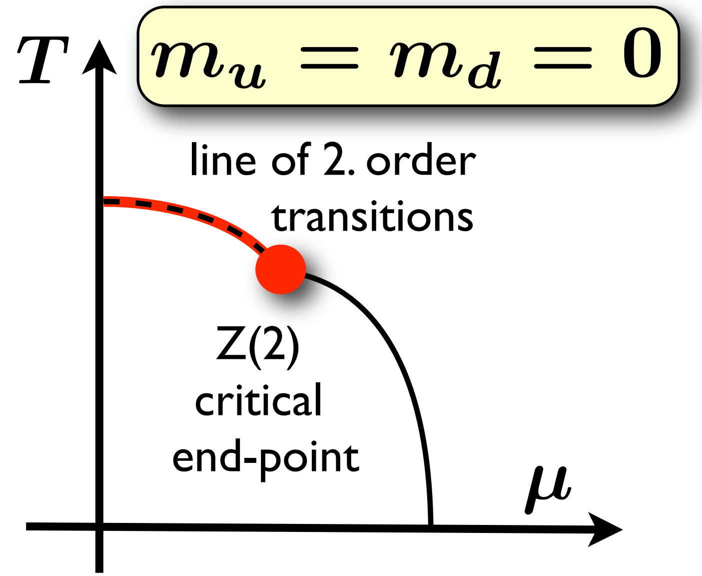
- singular part:

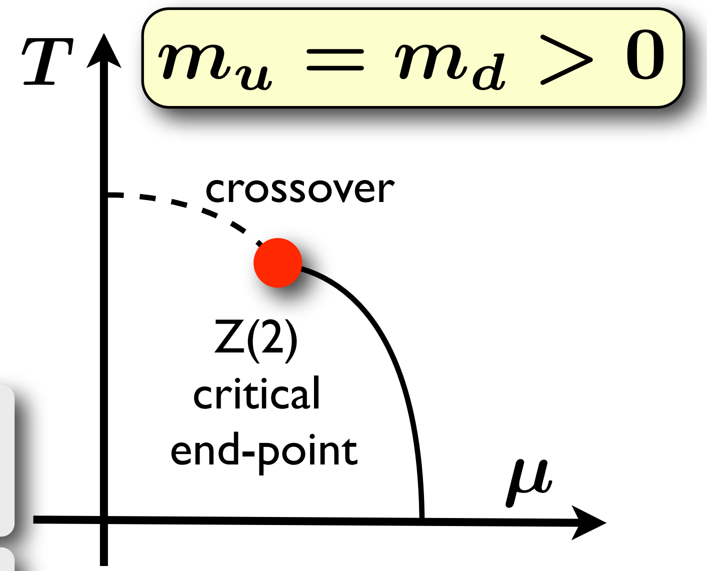
$$f_s(T, \mu_u, \mu_d) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{(2-\alpha)}$$

- O(4)/O(2): $\alpha < 0$, small

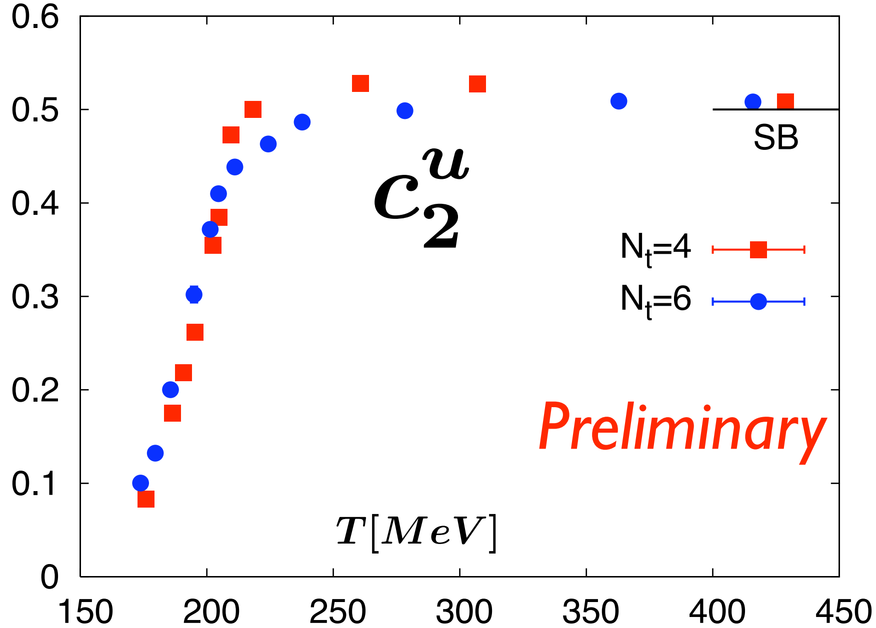
$$\left\langle (\delta n_{u,d})^2 \right\rangle \longrightarrow$$ dominated by T dependence of regular part

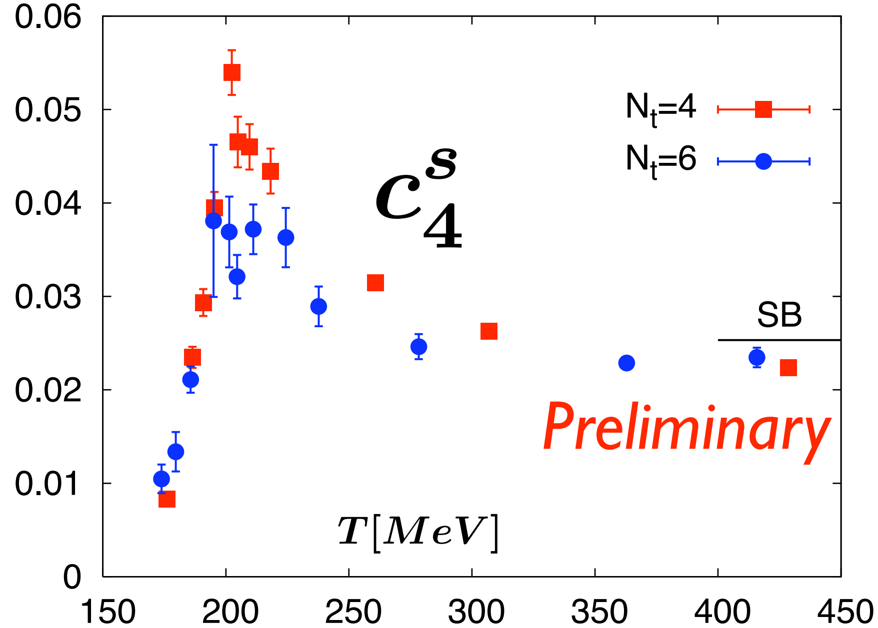
$$\left\langle (\delta n_{u,d})^4 \right\rangle \longrightarrow$$ develops a cusp!

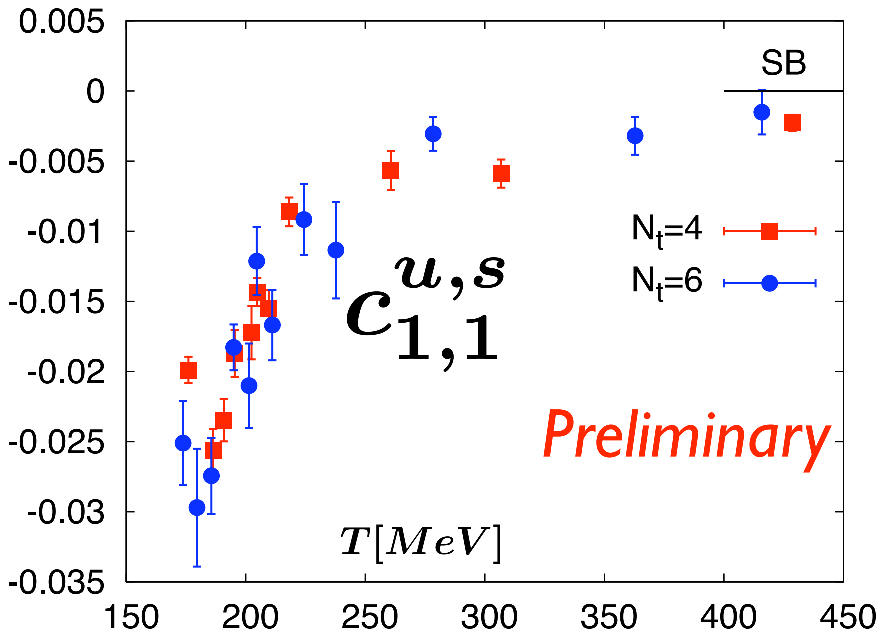




Results for expansion coefficients $c^{u,d,s}_{i,j,k}$







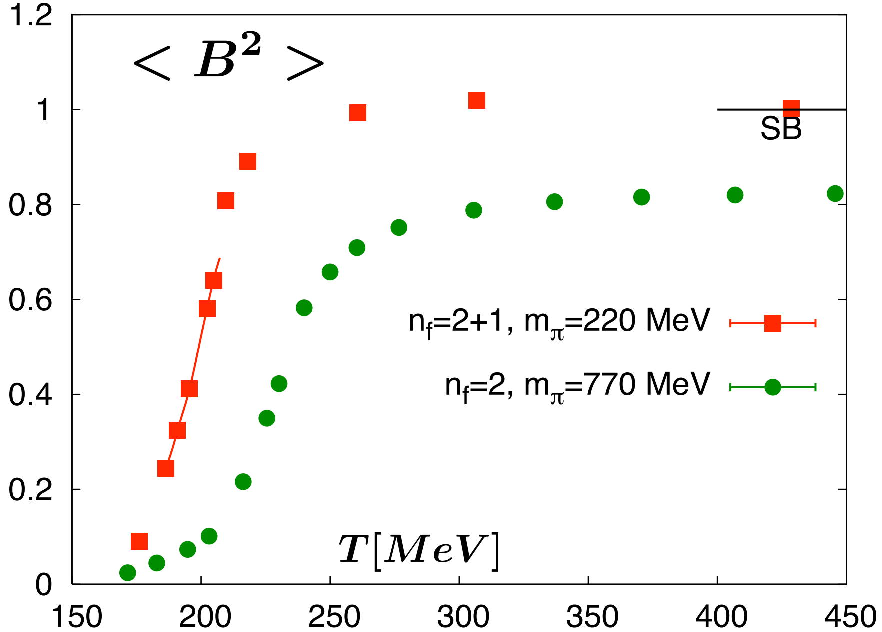
$\longrightarrow$ **small cut-off effects**
(similar to cut-off effects in other bulk thermodynamic quantities)

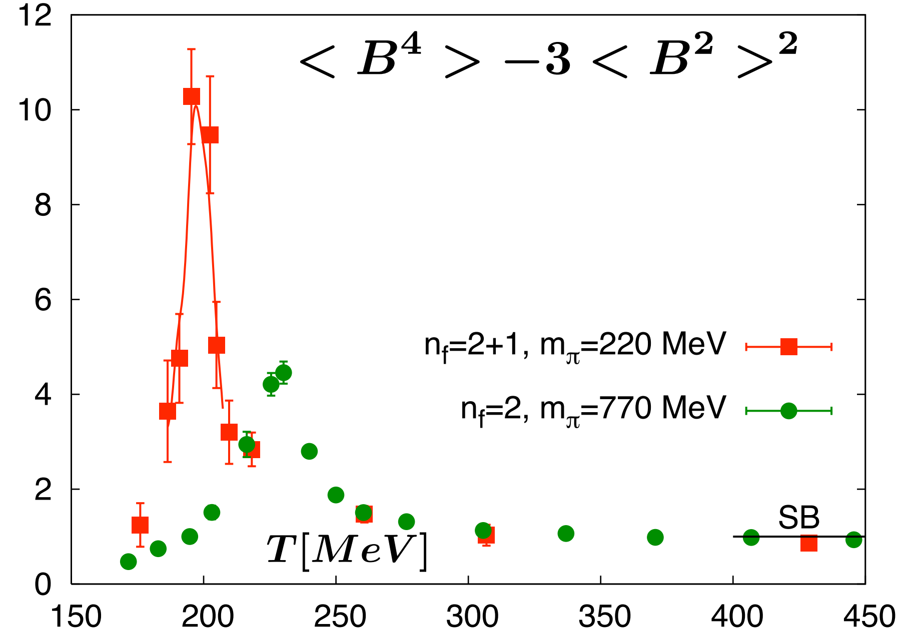
RBC-Bielefeld, 2+1 flavor

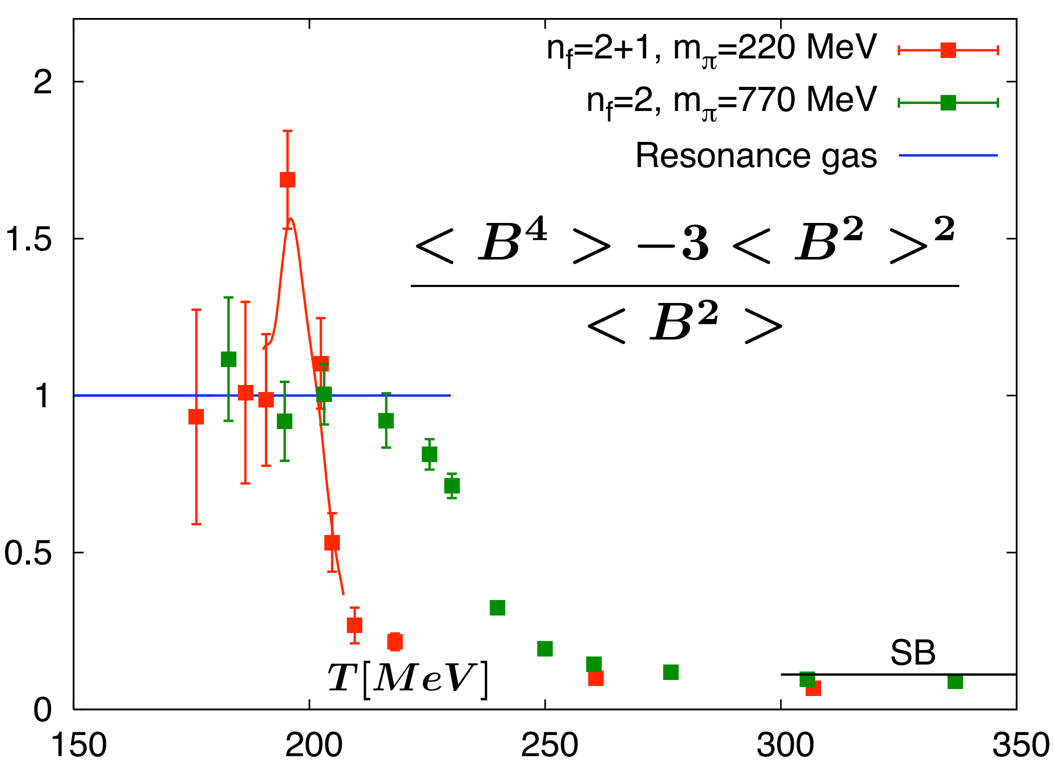
$m_\pi \approx 220 MeV$

preliminary

Baryon number fluctuations $(\mu_B = 0)$





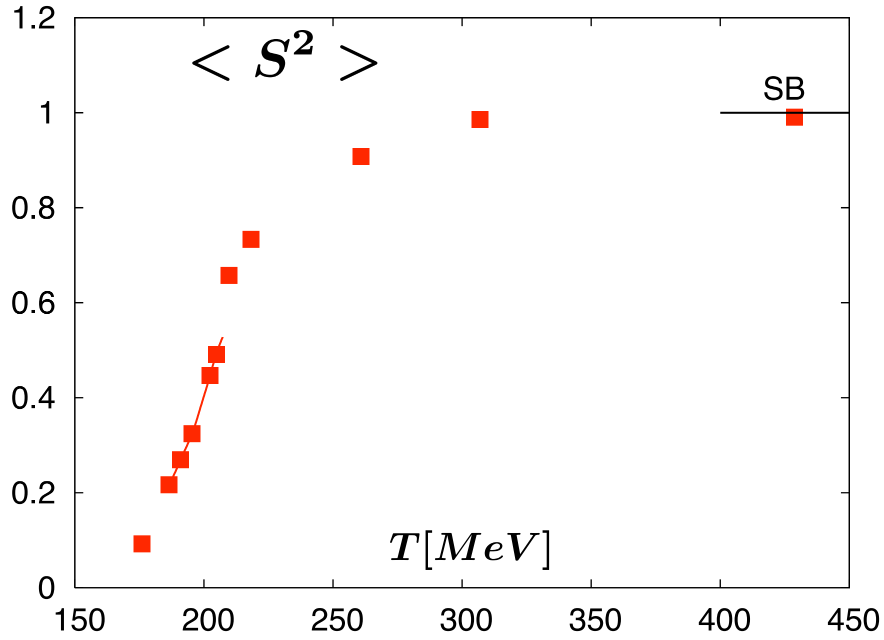


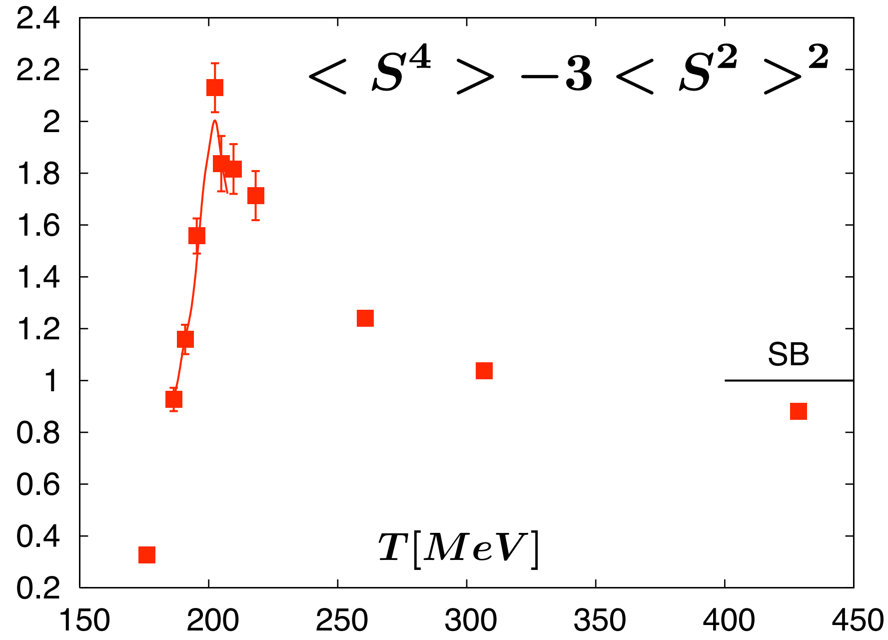
- ⟶ fluctuations increase with decreasing mass
- ⟶ fluctuations increase over the resonance gas value

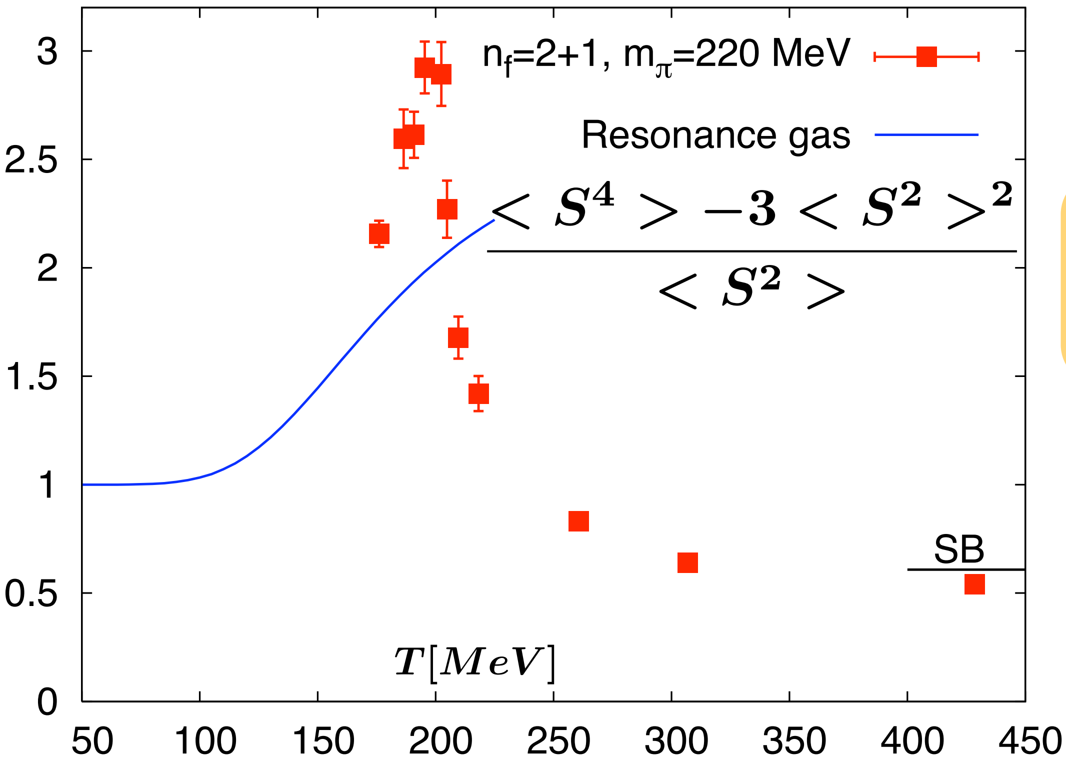
red: RBC-Bielefeld, preliminary

green: Allton et all., Phys. Rev. D71 (2005) 054508

Strangeness fluctuations $(\mu_B = 0)$







$\longrightarrow$ fluctuations increase over the resonance gas value

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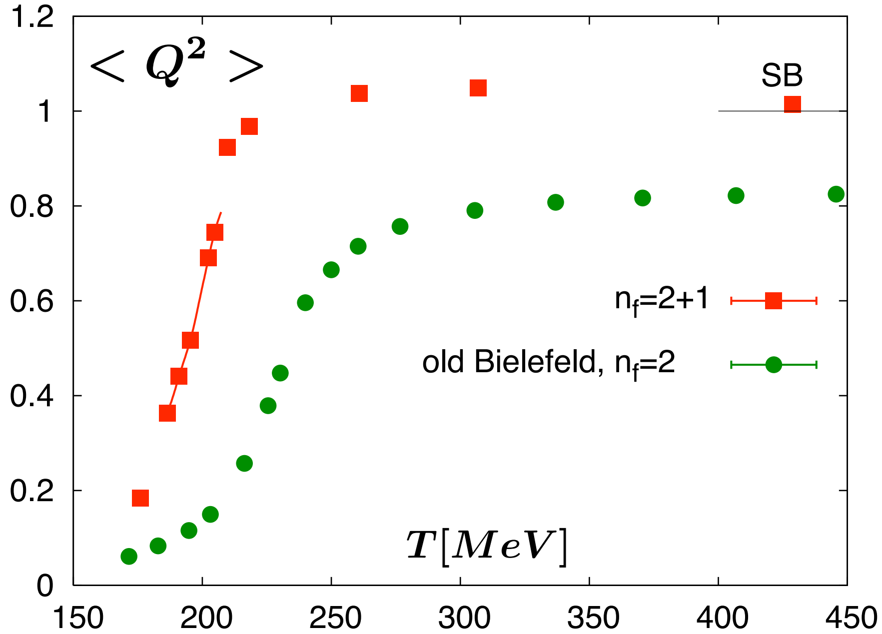
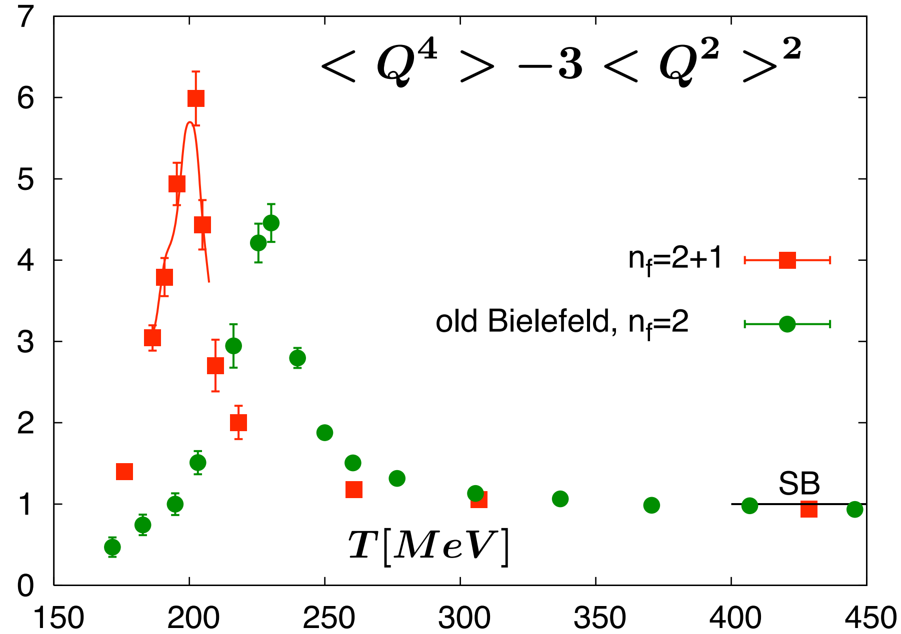
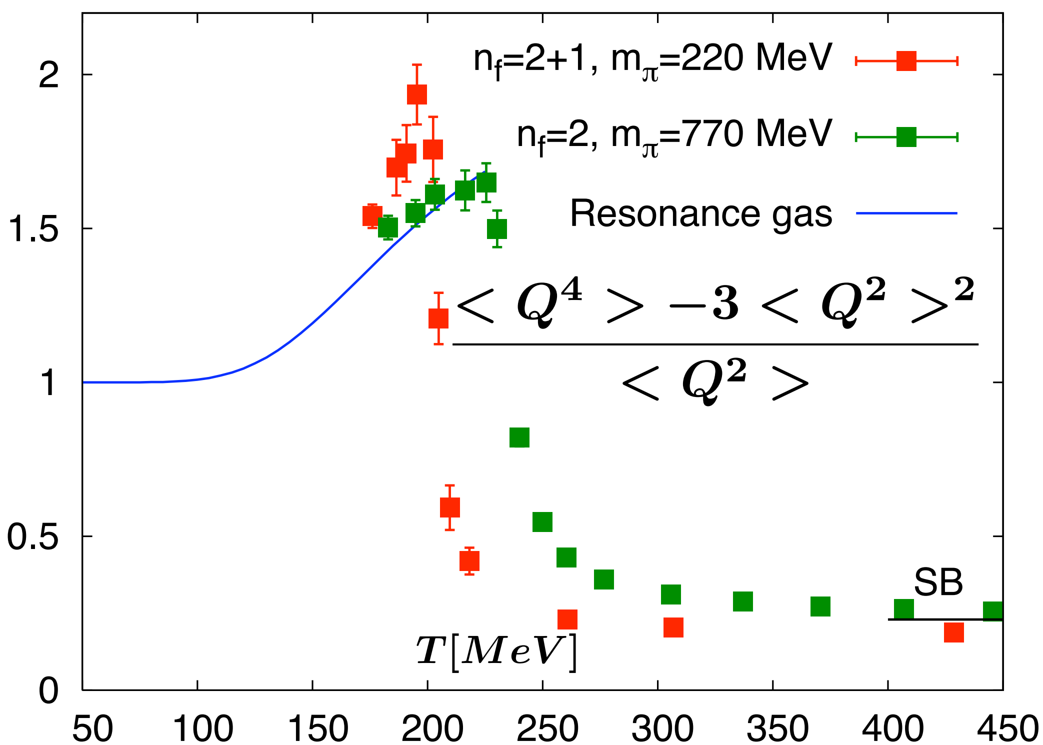
Electric charge fluctuations $(\mu_B = 0)$







$\longrightarrow$ fluctuations increase with decreasing mass

$\longrightarrow$ fluctuations increase over the resonance gas value

red: RBC-Bielefeld, preliminary

green: Ejiri, Karsch, Redlich, Phys.Lett.B633 (2006) 275.

Consequences for the phase diagram: the radius of convergence

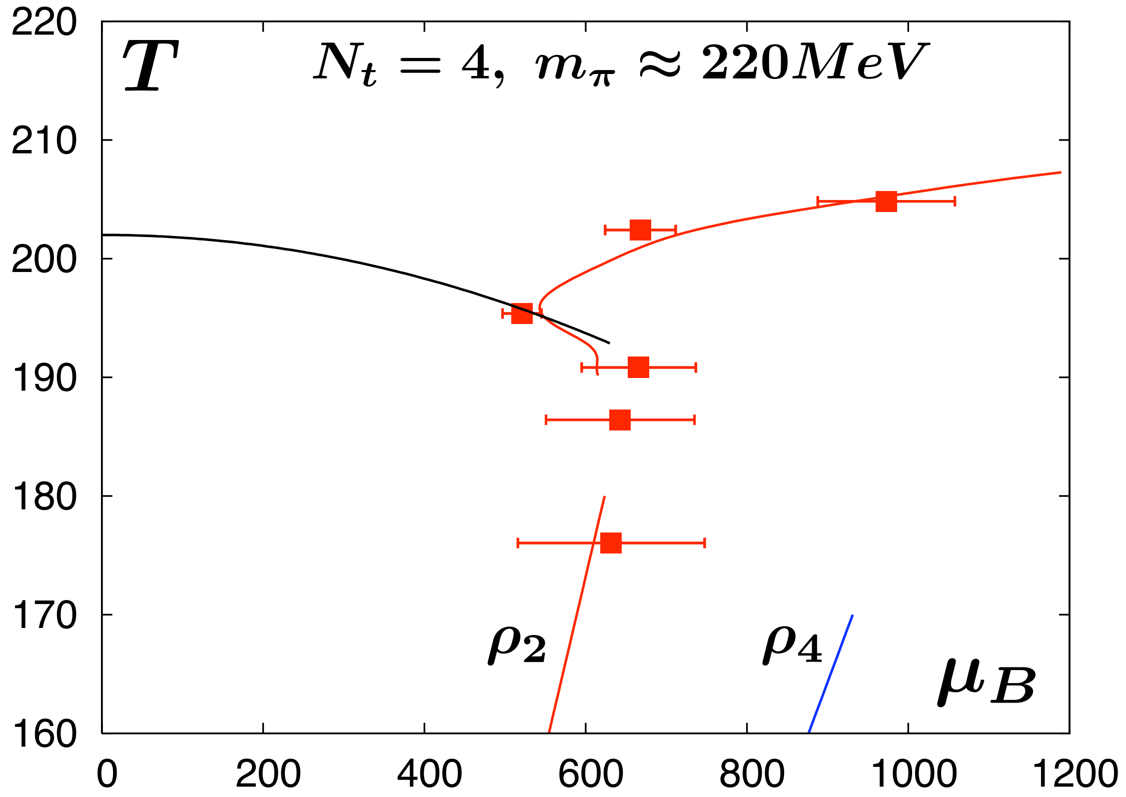
- the radius of convergence can be estimated from the Taylor coefficients of the pressure:

$$\rho = \lim_{n\to\infty} \rho_n$$

with

$$\rho_n = \sqrt{\frac{c_n^B}{c_{n+2}^B}}$$

- for $T > T_c, \quad \rho_n \to \infty$
- for $T < T_c, \ \rho_n$ is bound by the transition line



- non monotonic behavior of the convergence radius

$\longrightarrow$ **first hint for a critical region ?**

- higher order approximations are needed to locate the critical point

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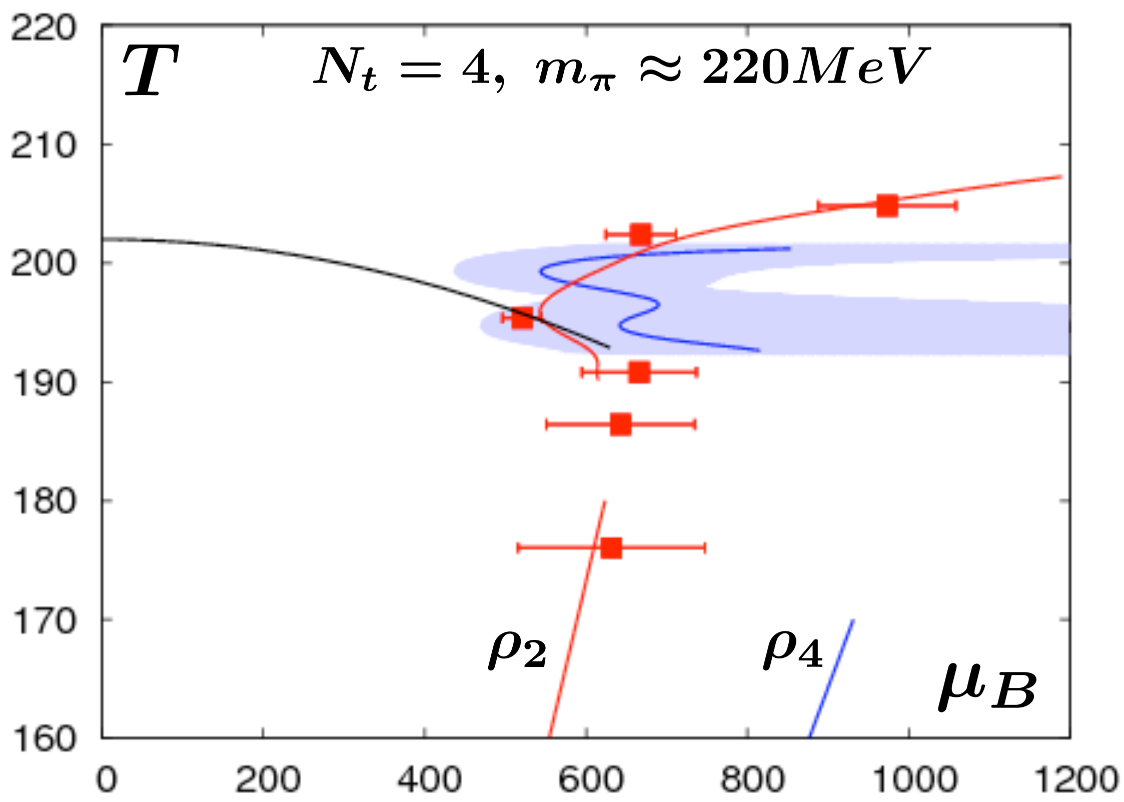


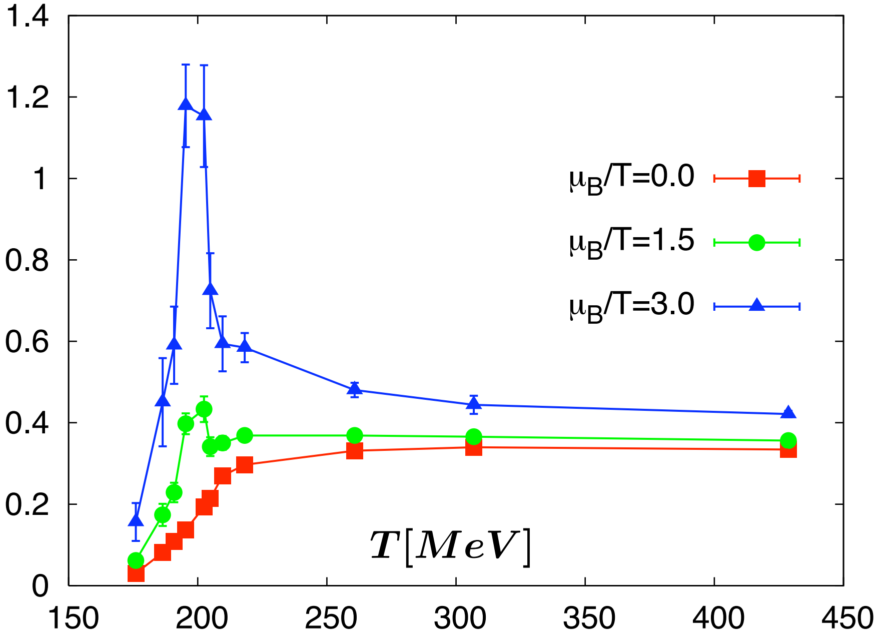
- non monotonic behavior of the convergence radius

  $\longrightarrow$ **first hint for a critical region ?**

- higher order approximations are needed to locate the critical point

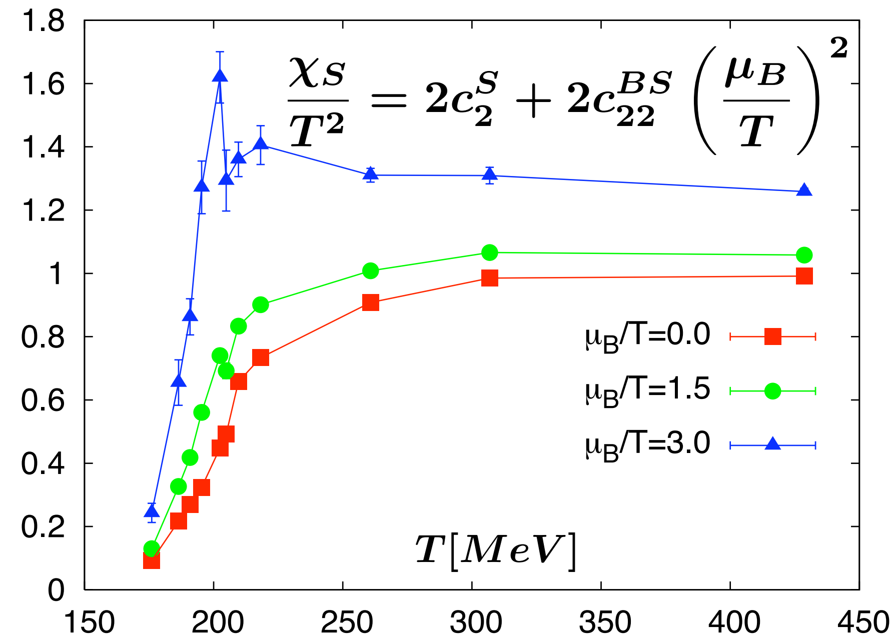
Hadronic fluctuations $\mu_B > 0 \; (\mu_S = \mu_Q = 0)$

$$\frac{\chi_B}{T^2} = 2c_2^B + 12c_4^B\left(\frac{\mu_B}{T}\right)^2$$

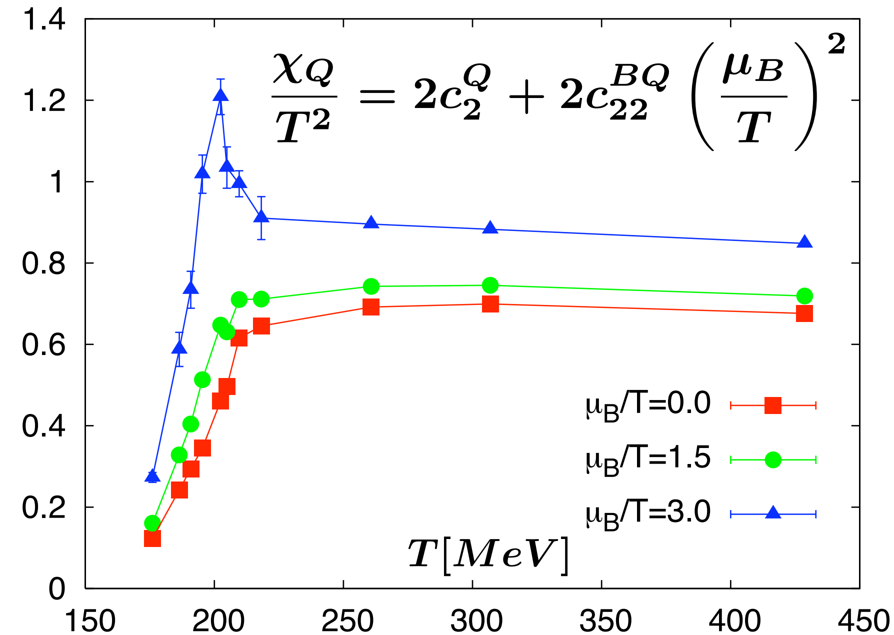


- ⟶ large baryon number fluctuations
- ⟶ enhanced strangeness and electric charge fluctuations (~ factor 3 at Tc)

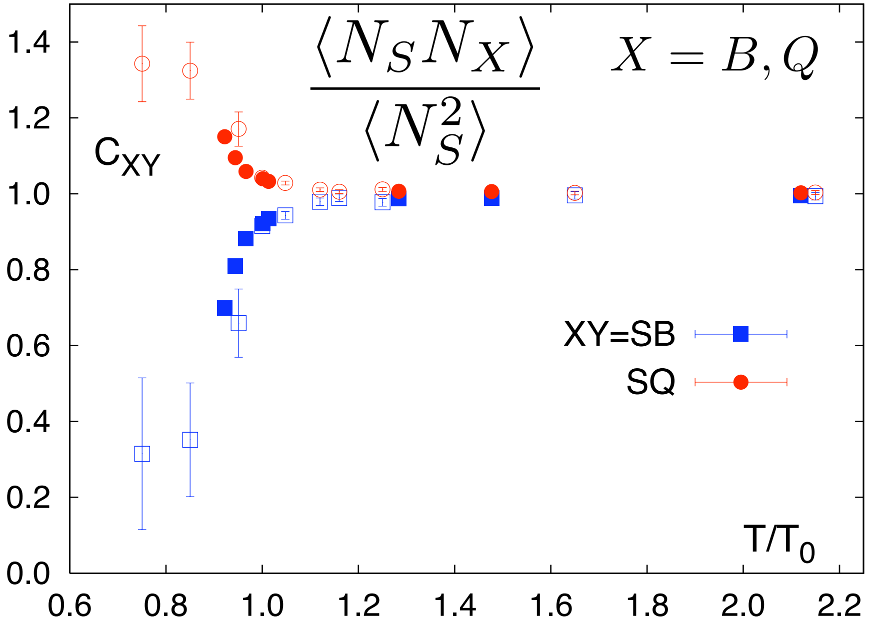
$$\frac{\chi_S}{T^2} = 2c_2^S + 2c_{22}^{BS}\left(\frac{\mu_B}{T}\right)^2$$



$$\frac{\chi_Q}{T^2} = 2c_2^Q + 2c_{22}^{BQ}\left(\frac{\mu_B}{T}\right)^2$$



Correlations of S,B and S,Q



full: RBC-Bielefeld, nf=2+1, preliminary

open: Gavai, Gupta, nf=2, partially quenched

Gavai, Gupta, Phys.Rev.D73 (2006) 014004.

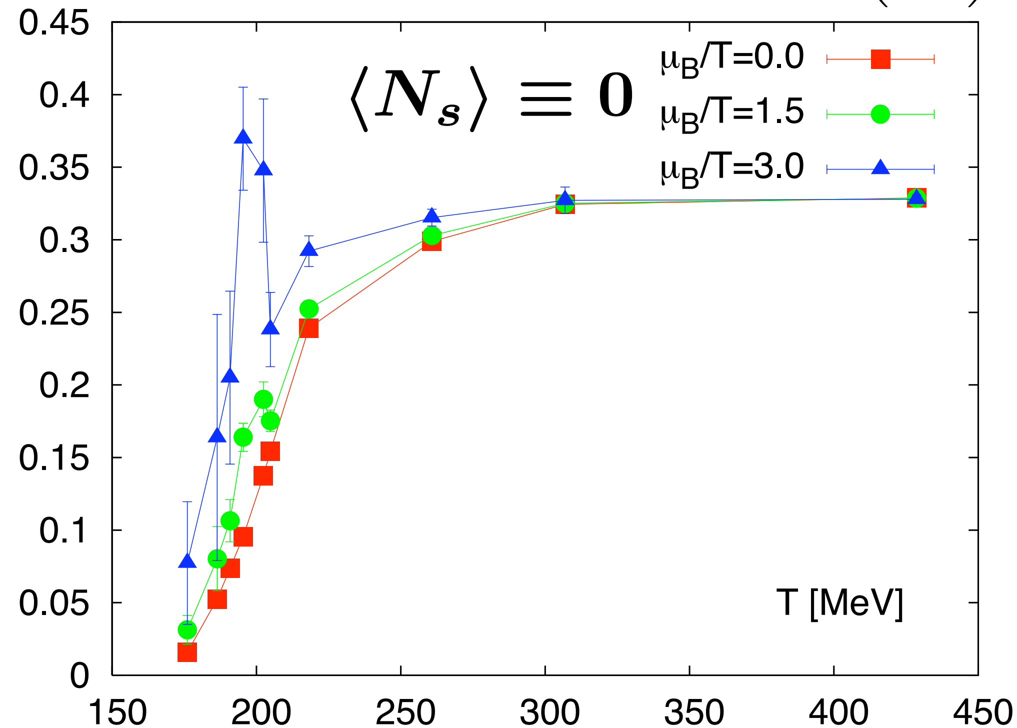
Koch, Majumder, Randrup, Phys.Rev.Lett.95 (2005)182301.

- Small quenching effect

  ⟶ Normalization effect ?

$$\frac{\langle N_S N_X \rangle}{\langle N_S^2 \rangle} = \frac{f_X c_{11}^{qs} + 2c_s^s}{2c_2^s}$$

$$\frac{1}{T}\left(\langle N_B N_S \rangle - \langle N_B \rangle \langle N_S \rangle\right) = \hat{c}_{11}^{BS} + 3\hat{c}_{31}^{BS}\left(\frac{\mu_B}{T}\right)^2$$



⟶ Correlations increase for $\mu_B > 0$

Conclusions / Summary

- Taylor expansion method at 2+1 flavor provides lots of input for HIC phenomenology.
- All results on fluctuations and correlations develop a peak with increasing chemical potential and are consistent with a gas of quasi-free quarks, already at 1.2-1.5 Tc.
- Ratios of cumulants are robust quantities and should be well suited for comparison with the experiment.
- The fluctuations increase with decreasing mass and rise over the resonance gas value close to Tc.
  (This is a necessary condition for the determination of the critical point with means of the radius of convergence.)
- Strangeness and electric charge chemical potentials have to be tuned to meet the conditions of HIC.