Soft dynamics of the QCD critical point

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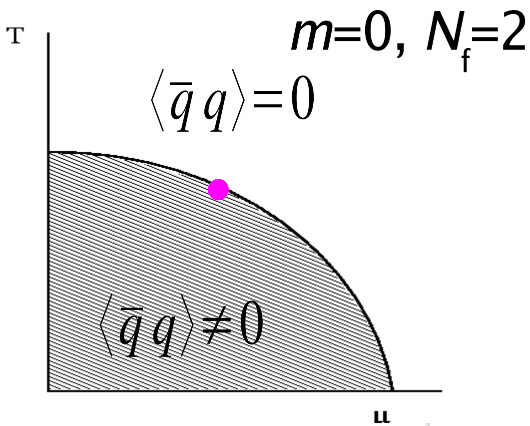
Let's discuss about the nature of the QCD_CP next 15 min!

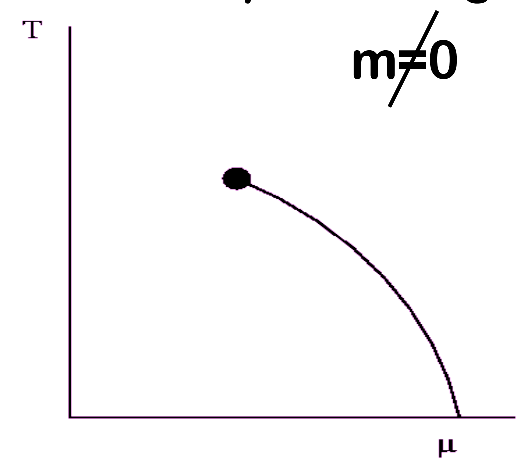
- **Elements of the QCD critical point**
- **Soft mode of the QCD_CP**
- **Summary**

Schematic phase diagram

Talk: M. Stephanov

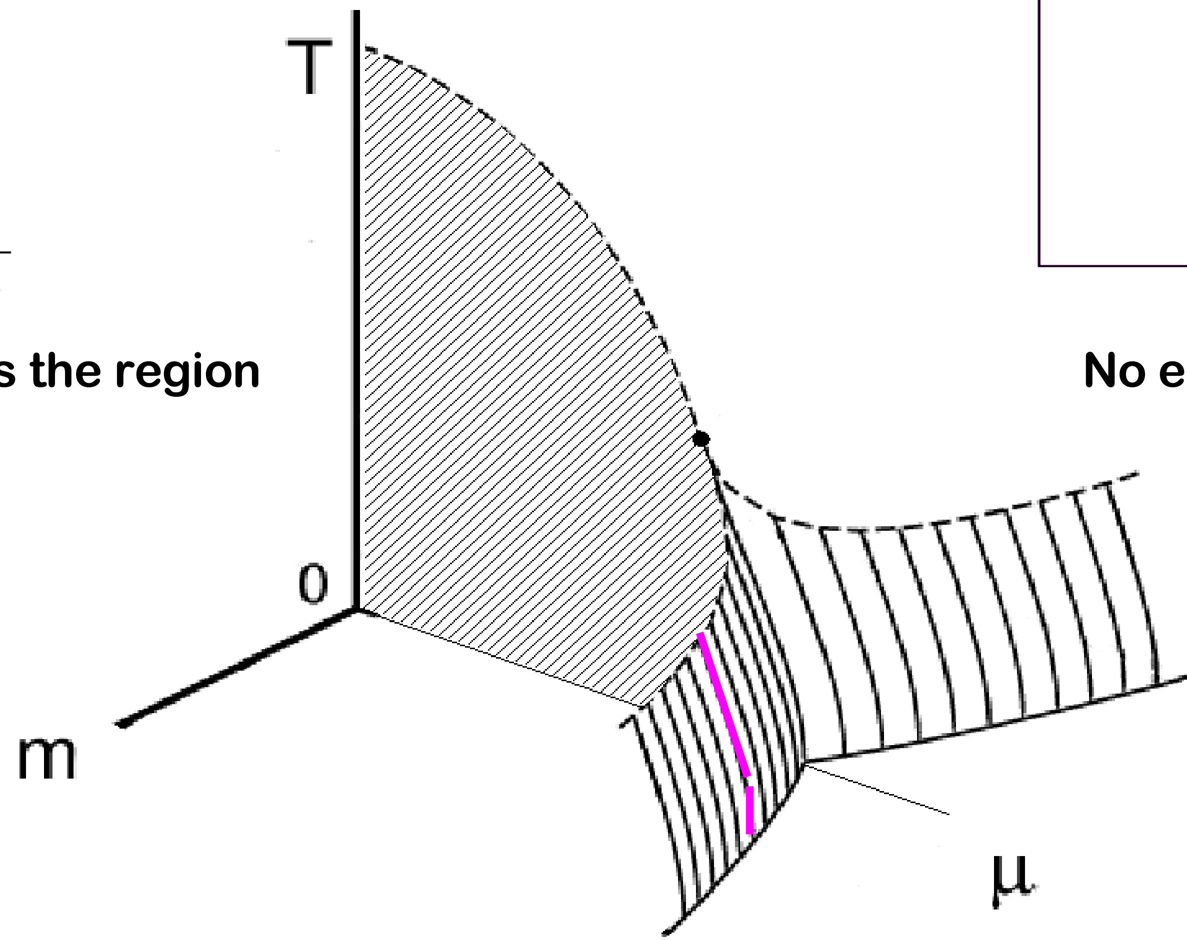
Genuine singularity on the real phase diagram





Symmetry separates the region

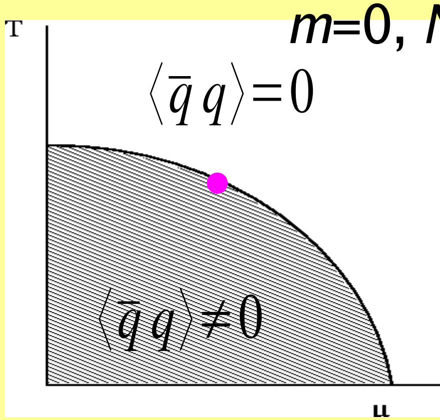
No exact symmetry

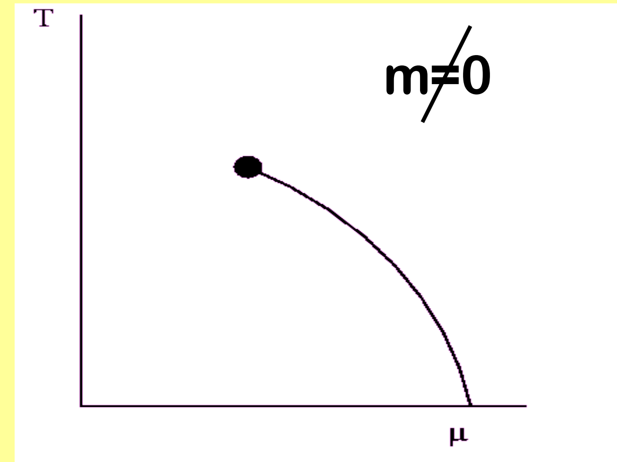


Asakawa-Yazaki, Rajagopal et al. Berdnikov-Rajagopal, Hatta-Ikeda, Asakawa-Nonaka, S. Gavin et al. , ...

Critical points at m=0 and m≠0

m=0, N_f=2





2[nd] order O(4) transition	The QCD Z_2 critical point
Along co-existence line (surface) -- Clapeyron-Clausius	
σ has a gap, n, ε continuous	σ, n, ε all have a gap
The order parameter	
σ	any of σ, n, ε
Susceptibilities at the critical point	
χ_σ divergent, χ_n, C finite	all χ_σ, χ_n, C diverge w/ exponent γ
Critical mode(s)	
softening of σ	softening of σ, ω ?, or what ?

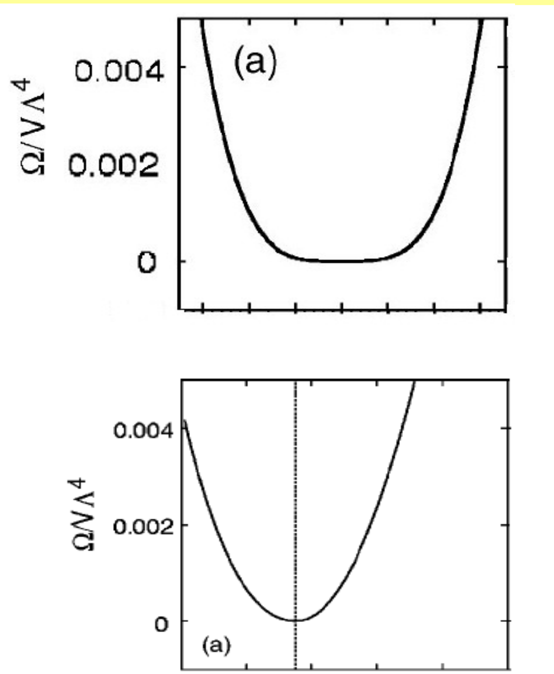
Mass

2^{nd} order O(4) transition | **The QCD Z_2 critical point**

Divergent susceptibility $\chi \sim 1/m^2$ may imply the vanishing (screening) mass or potential curvature

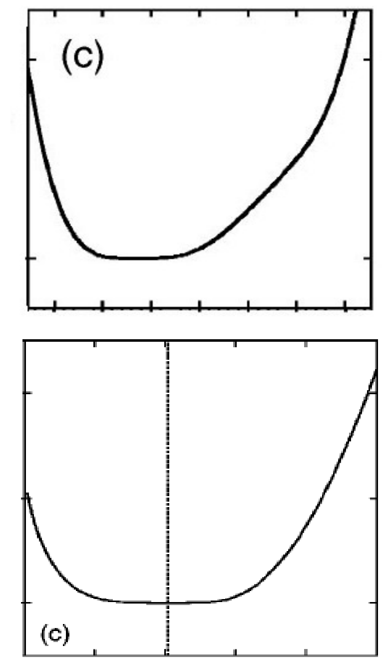
NJL calc

HF-Ohtani



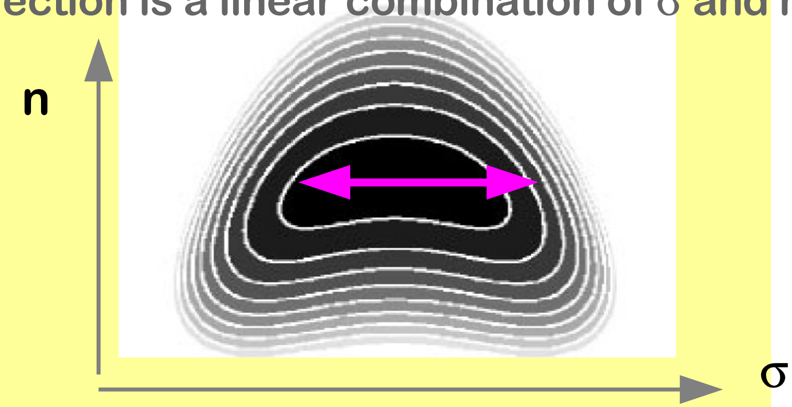
$\Omega(\sigma)$

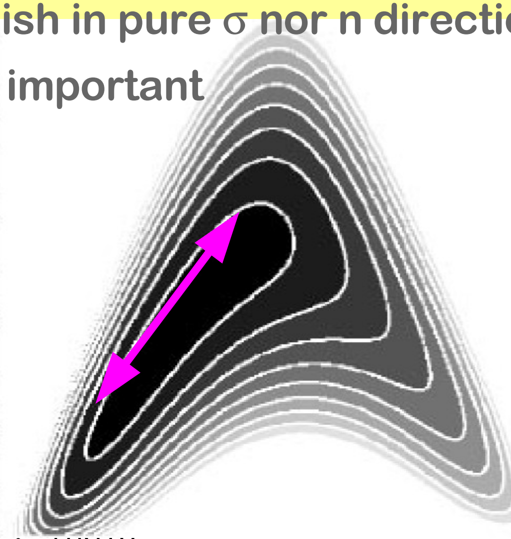
$\Omega(n)$



Taking σ and n independent, in the Z_2 case, 'mass' doesn't vanish in pure σ nor n direction

The flat direction is a linear combination of σ and n -- mixing is important





Dynamic mode near CP

HF-Ohtani
Son-Stephanov

Slow dynamics is described with hydro-modes; ω--> 0 as k --> 0

fluctuations of conserved charge densities ϕ (n, energy-momentum, ...)
NG mode in broken phase ... π
order parameters near CP .. σ

Time-dependent Landau theory:

$$L_\sigma(i\partial_t)\sigma = -\frac{\delta\Omega}{\delta\sigma}, \quad L_\varphi(i\partial_t)\varphi = -\frac{\delta\Omega}{\delta\varphi}$$

$$\Omega = \int d^3x \left(a_0\sigma^2 + b_0\sigma^4 + c\sigma^6 + \gamma\sigma^2\varphi + \frac{1}{2}\varphi^2 - h\sigma - j\varphi \right)$$

$$L_\sigma(i\partial_t) = \partial_t^2/\Gamma \qquad L_\varphi(i\partial_t) = -\partial_t/\lambda\mathbf{q}^2$$

Eigen-frequencies and vectors in linear analysis

$$-\frac{\omega_o^2}{\Gamma} = -(\chi_h^{-1} + 4\gamma^2\sigma^2)$$

pure σ

$$\frac{-i\omega_d}{\lambda\mathbf{q}^2} = -\frac{\chi_h^{-1}}{\chi_h^{-1} + 4\gamma^2\sigma^2} \equiv -\chi_j^{-1}$$

Mix of σ and ϕ

	ω_o	ω_d
O(4) CP (σ=0) :	**critical**	**soft but normal**
QCD_CP ($\sigma\neq$0) :	**fast & decoupled**	**critical**

Dynamic mode near CP

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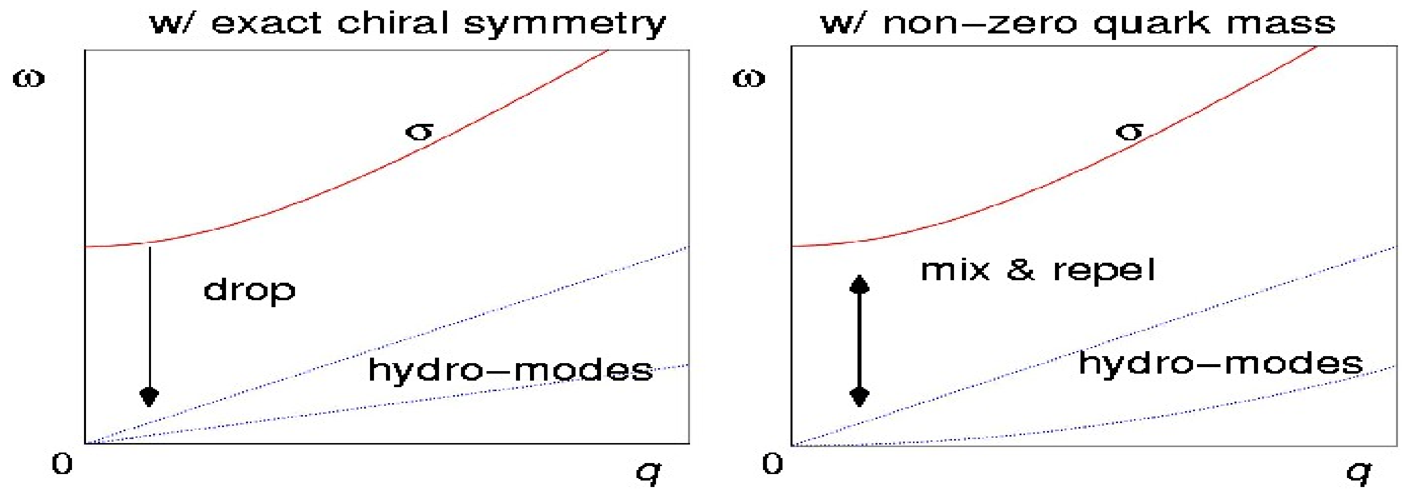
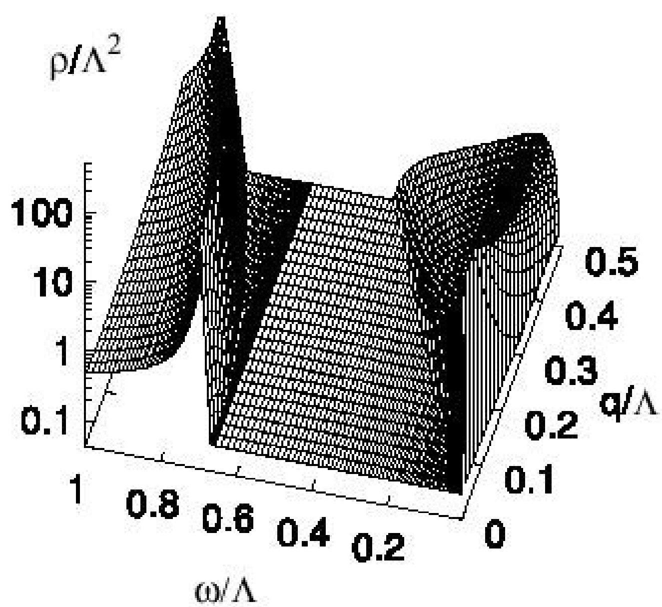
Understanding the critical behavior w/ and w/o χ symmetry :

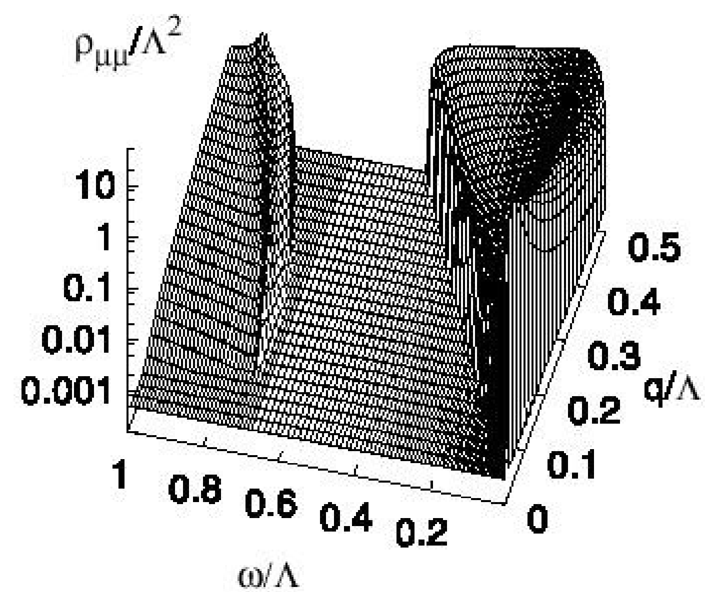


The argument is based on simple time-dependent Landau theory, but general

Model examp



Scalar spectrum



Vector spectrum

Another model: chiral σ-ω

A Model treating σ and ω in an equal footing

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - g_V \omega\!\!\!/ - g\phi)q + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - U(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$

Caveat in using simple NJL modal calculation
- scalar attraction only included
- the vector susceptibility diverges due to the scalar?
- can one expect massless ω, too ?

Model calculation including σ and ω explicitly !

Another model: chiral σ-ω

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The grand potential

$$P(T,\mu;\sigma_0,\omega_0) = d_q \int^\Lambda \frac{d^3k}{(2\pi)^3}\left[E + T\ln(1+e^{-\beta(E-\tilde{\mu})}) + T\ln(1+e^{-\beta(E+\tilde{\mu})})\right] - U(\sigma_0) + \frac{1}{2}m_\omega^2\omega_0^2$$

$$\left|\frac{\partial^2 P}{\partial\phi_i \partial\phi_j}\right|_{\phi=(\sigma,\omega)} = \left(g^2\frac{\partial\rho_s}{\partial M} - \frac{\partial^2 U(\sigma)}{\partial\sigma^2}\right)\left(m_\omega^2 - g_V^2\frac{\partial\rho_V}{\partial\mu}\right) - \left(gg_V\frac{\partial\rho_V}{\partial M}\right)^2 = 0$$

$$\frac{\partial\rho_V}{\partial M} = -d_q\int_{\boldsymbol{k}}\frac{M}{E}\left[n(1-n) - \bar{n}(1-\bar{n})\right]$$

- **M, $\mu \neq 0$ allows mixing of scalar-vector through quark-polarization**
- **n(1-n) type – typical for particle-hole contribution**
- **Exact chiral symmetry forbids the mixing --> factorization**

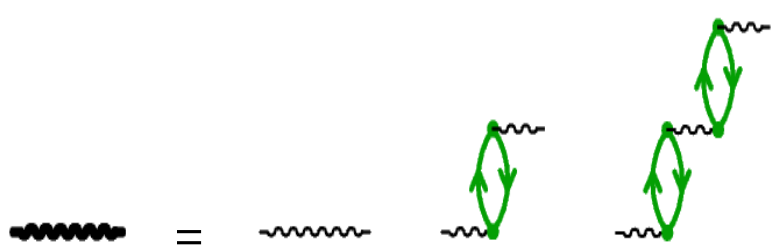
Another model: chiral σ-ω

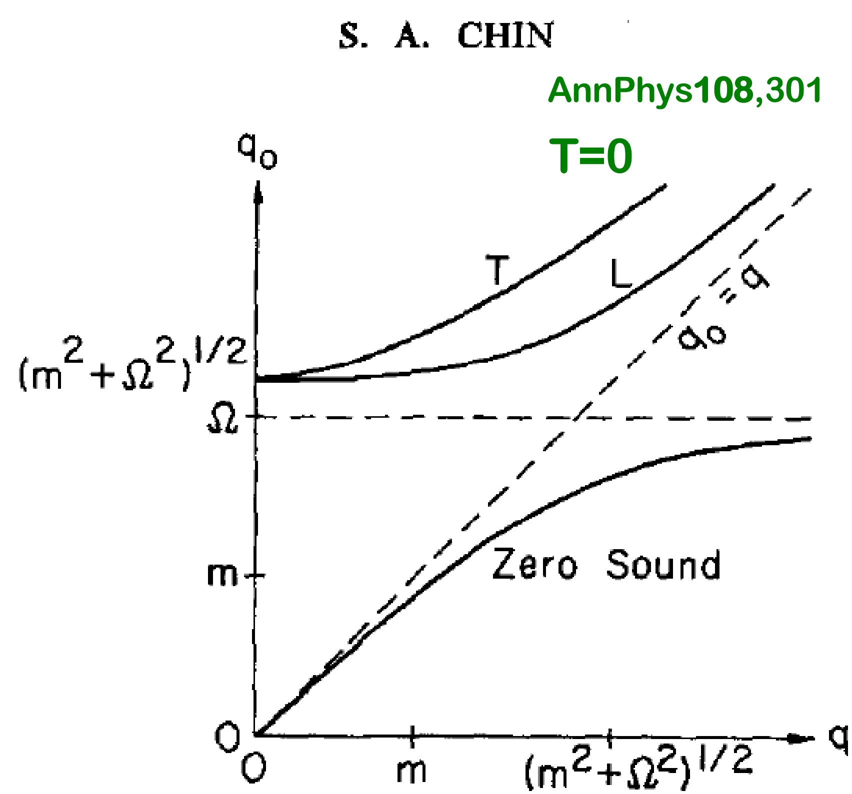
HF-Tanji

Explicit calculation is in progress -

Preliminary consideration
- sound mode exists
- '3-0' mixes when ω mode in matter
- '3-0' decoupled at q=0

$$\Pi_{VV} = \begin{pmatrix} \Pi^{11} & 0 & 0 & 0 \\ 0 & \Pi^{22} & 0 & 0 \\ 0 & 0 & \Pi^{33} & \Pi^{30} \\ 0 & 0 & \Pi^{03} & \Pi^{00} \end{pmatrix}$$





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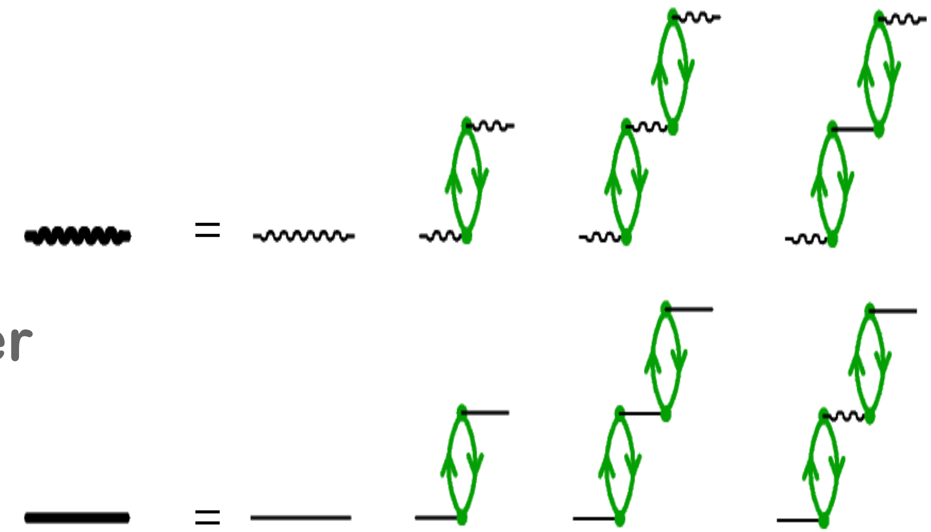
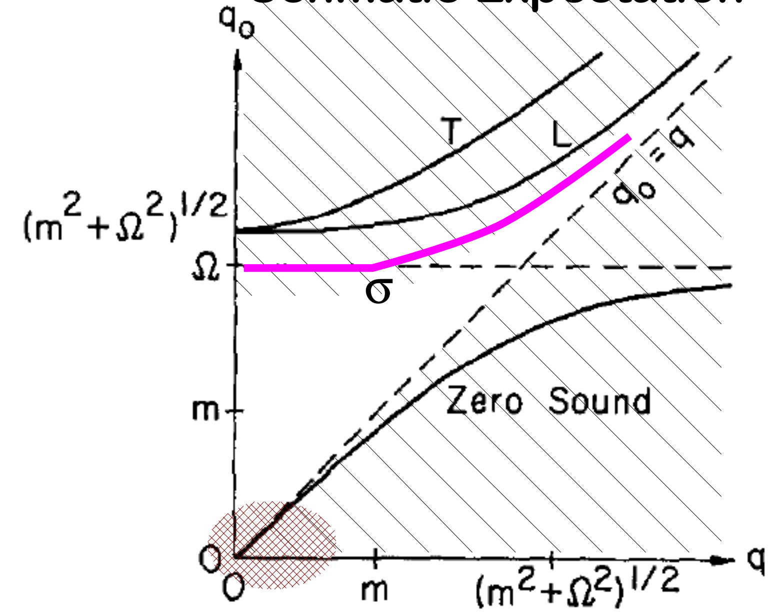
- **when σ added, only σ-ω_{long} mix**



$$(p^2 - m_\sigma^2 + \Pi^{ss})(-p^2 + m_\omega^2 - \Pi^L) - (\Pi^{sL})^2 = 0$$

$$\Pi^{ab} = \begin{pmatrix} \Pi^{11} & 0 & 0 & 0 & 0 \\ 0 & \Pi^{22} & 0 & 0 & 0 \\ 0 & 0 & \Pi^{33} & \Pi^{30} & \Pi^{3s} \\ 0 & 0 & \Pi^{03} & \Pi^{00} & \Pi^{0s} \\ 0 & 0 & \Pi^{s3} & \Pi^{s0} & \Pi^{ss} \end{pmatrix}$$

Schmatic Expectation



Summary and Outlook

Critical mode of QCD CP has qualitatively a different character from σ's behavior at the chiral O(4) CP:

- Pure σ is decoupled and diffusive mode becomes critical
- Dynamic universality class is the same as the liquid-gas CP   HF, Son-Stephanov

NJL model calculation shows a consistent feature

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Any experimental implications of the critical modes?

Critical slowing-down — Berdnikov-Rajagopal, Asakawa-Nonaka, ...

Baryon number diffusion — S. Gavin et al., ...

viscosity ... — ....

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- viscosity ... — ....

Any hint from AdS/CFT to the QCD_CP?