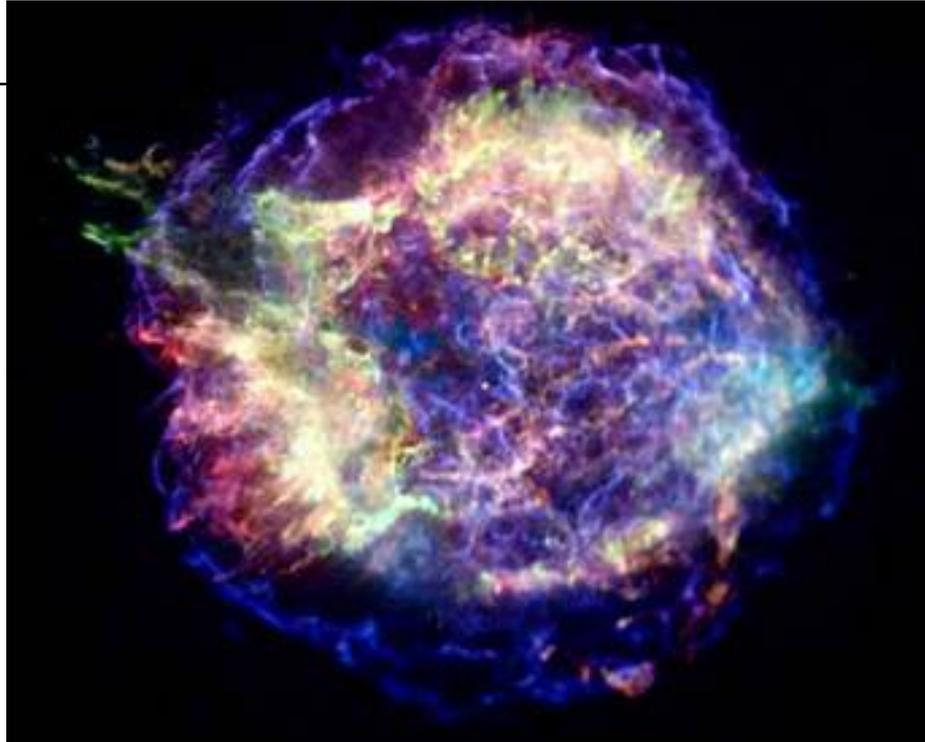


# Landau Model and Viscosity



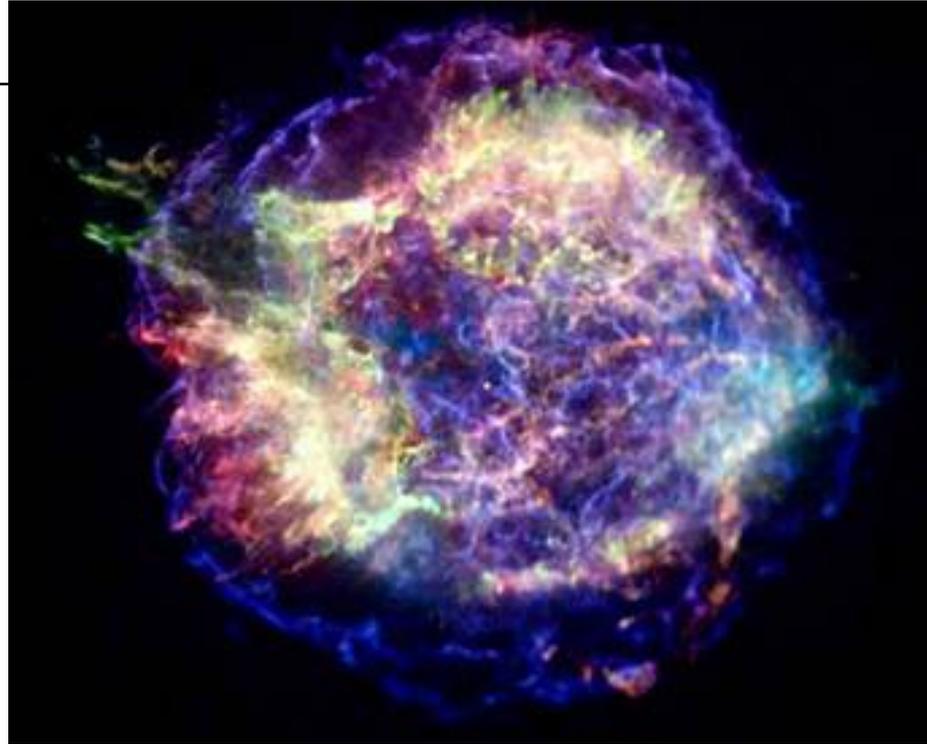
Takeshi Kodama



Instituto de Física - Universidade Federal do Rio de Janeiro



# Landau Model and Viscosity



Takeshi Kodama

Gabriel S. Denicol  
Tomoi Koide  
Philippe de A. Mota



# Success of Hydrodynamical Modeling in RHIC

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- ○ Formation of Hot Dense Matter which flows as if an (almost) Ideal Fluid
- Expectation of extracting the EoS of the Quark Gluon Plasma



# Basic Ideas of Hydrodynamical Modeling

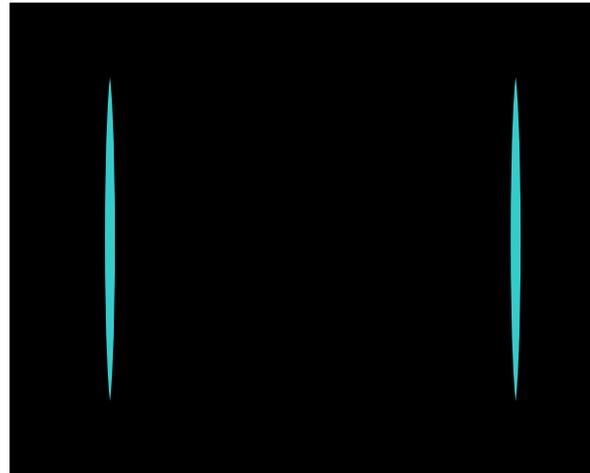
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- ➔ ○ Formation of matter in Local Thermal Equilibrium
- Initial condition given by some other models

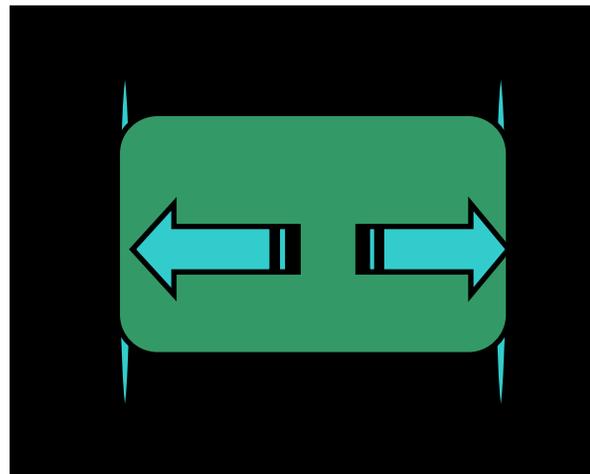


# Normal procedures for hydro calculations

---

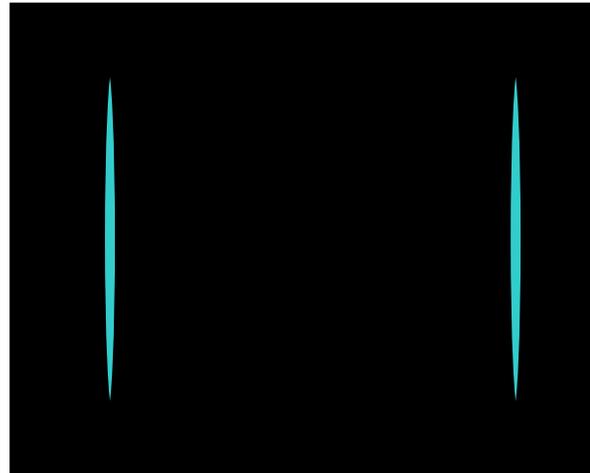


Prepare some initial distribution of energy density and flow field.

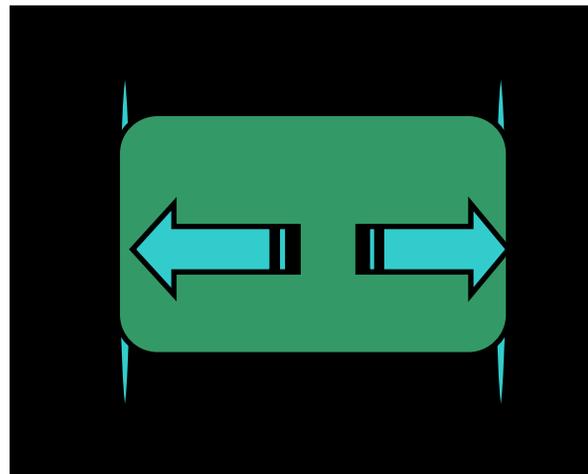


# Normal procedures for hydro calculations

---



Prepare some initial distribution of energy density and flow field.



Very important to know how much depends on initial conditions..  
glasma? (Raju's talk)



## Two Extreme Cases:

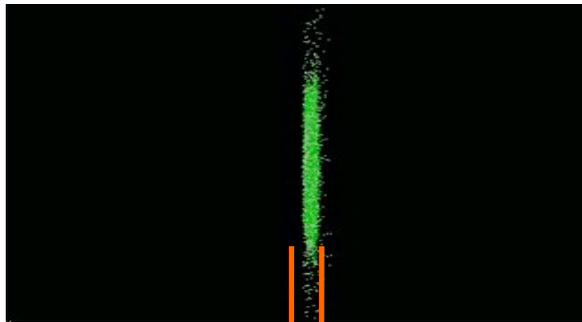
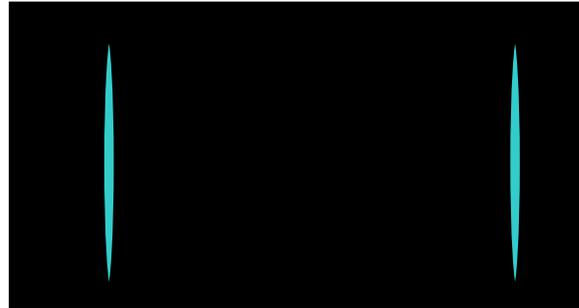
# Landau

(full stopping)

# vs.

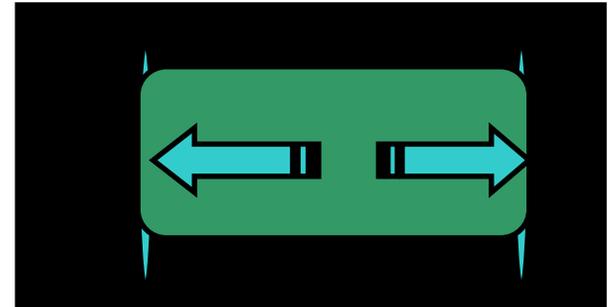
# Bjorken

(boost invariant -almost)



$$\Delta z \sim 1/\gamma,$$

$$\varepsilon \sim E_{tot} / \left( \pi R^3 / \gamma \right) \sim \gamma^2,$$



$$\eta = z/t,$$

$$\varepsilon \sim \varepsilon_0 \Theta(\eta^2_{inc} - \eta^2),$$



# Longitudinal Dynamics

---

L. M. Satarov, I. N. Mishustin, A. V. Merdeev and H. Stoecker

PHYS. REV. C **75**, 024903 (2007)

**Studies on the longitudinal fluid dynamics for ultrarelativistic heavy-ion collisions, changing the initial condition and EoS (1D)**

**Simple Landau initial condition doesn't work.**

**But...**



# Landau Model behavior of Data

---

- Many data on global aspects show Landau behavior (NA49, PHOBOS, BRAHMS), R. Debbe's talk of this conference and also see for example, M. Murray, J. Phys. G30, s6667 (QM2004), G. Roland, P. Steinberg, nucl-ex/0702019, and references therein.



# “Landau Model Behavior”

---

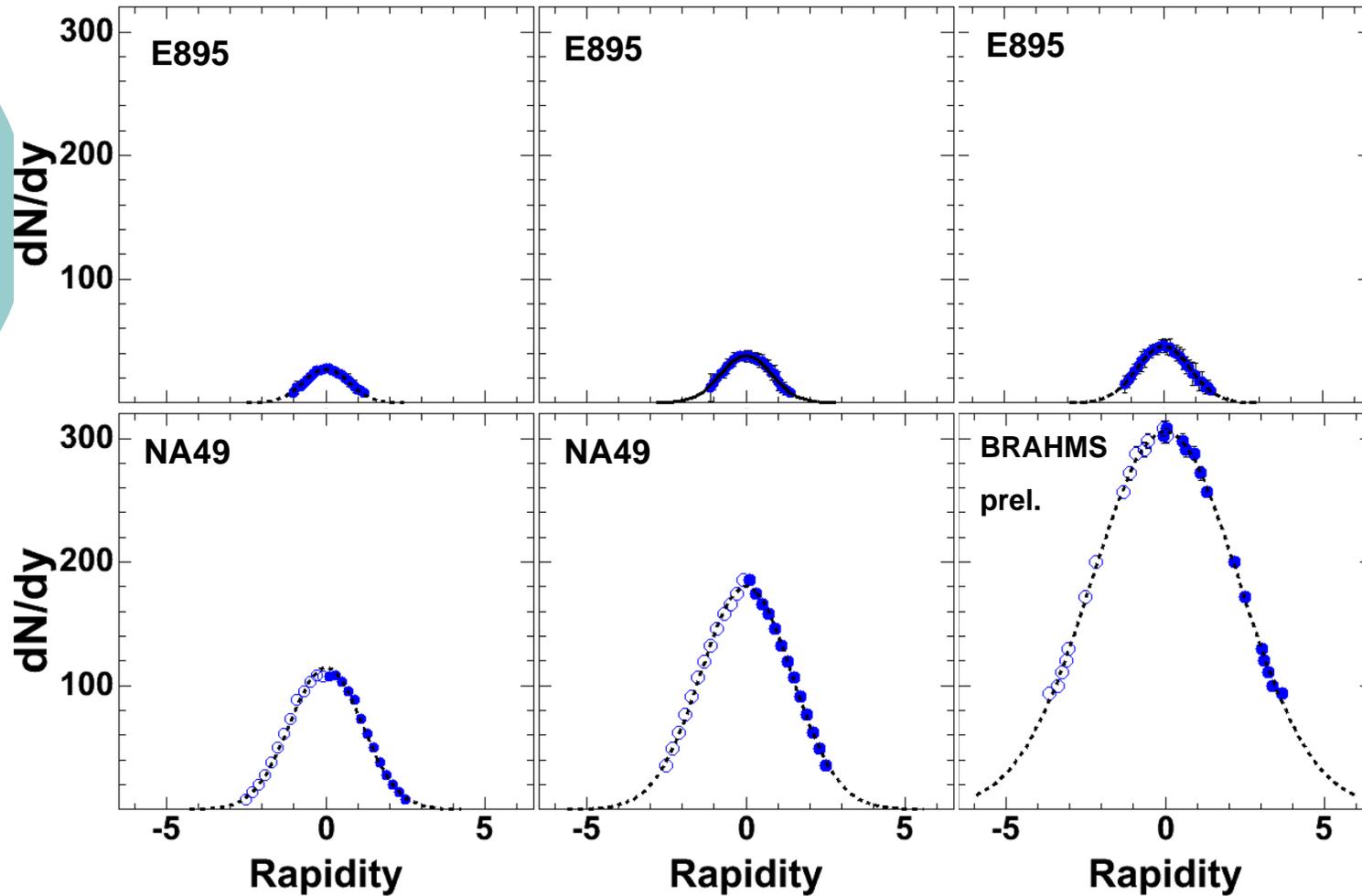
$$N_{ch} \sim K \sqrt{s}^{1/2}$$

$$\frac{dN}{dy} = N_0 \frac{e^{-y^2/2\sigma_y^2}}{\sqrt{2\pi\sigma_y^2}}$$

$$\sigma_y^2 = \frac{8}{3} \frac{c_s^2}{1-c_s^4} \ln \left( \frac{\sqrt{s}}{2m_p} \right)$$



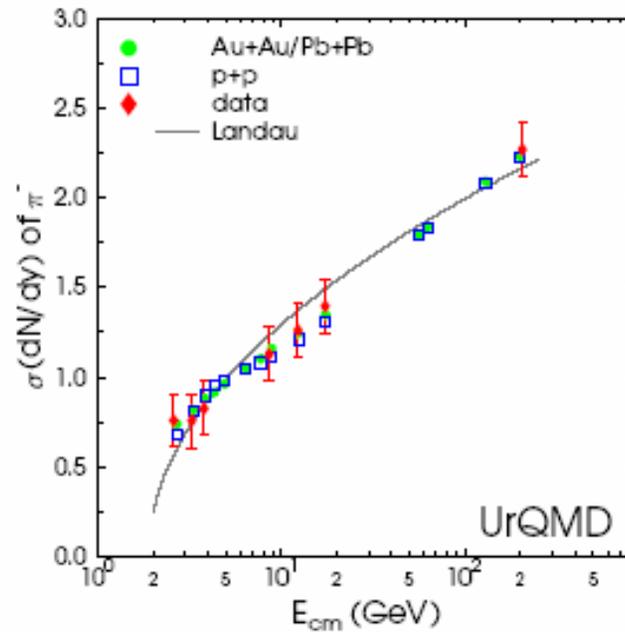
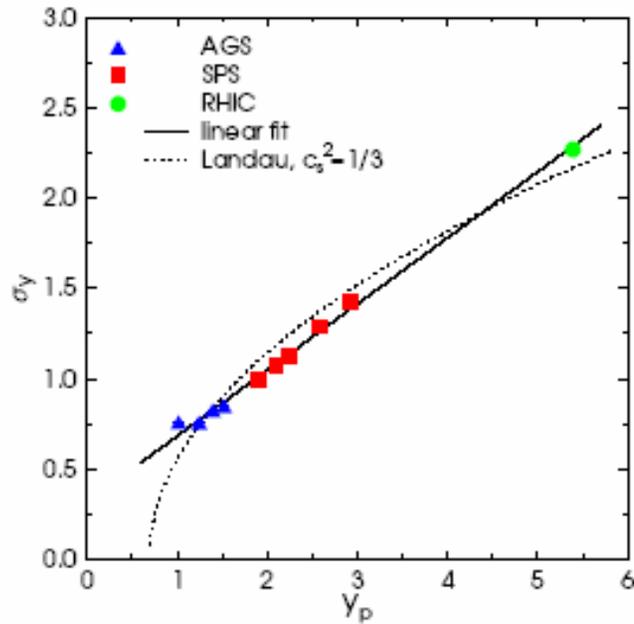
### $\pi^+$ dN/dy spectra



Single Gaussian fits from 2 to 200 GeV



# Width of Rapidity distributions compared to a microscopic model

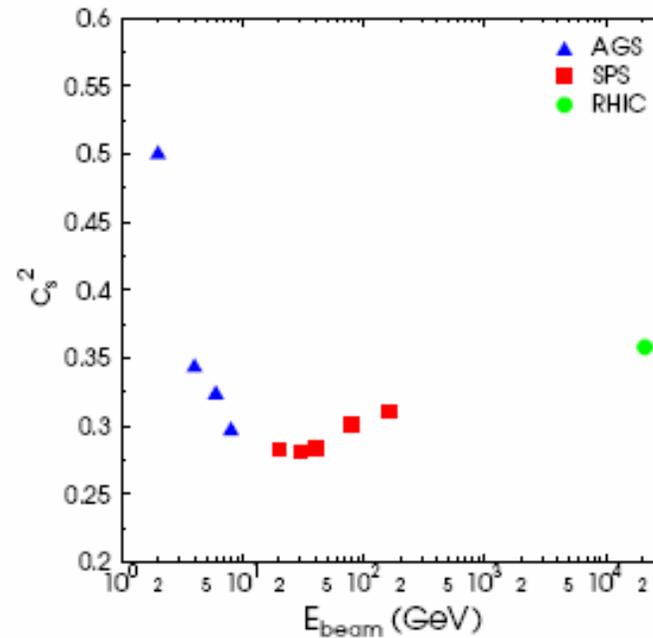


Taken from H.Petersen and M. Bleicher, PoS (CPOD2006) 025



## Sound of velocity extracted from “Landau” type model

---



Taken from H.Petersen and M. Bleicher, PoS (CPOD2006) 025



# So, what is happening here?

---

## Questions addressed in this talk

- Local Thermal Equilibrium?
- Role of Viscosity?
- 3D effects?
- If Landau is OK, then what is the significance?
- Why the same model works for p+p and N+N?



# 1D Hydrodynamics and Landau Model

---

$$\partial_{\mu} T^{\mu\nu} = 0$$

Define the velocity field by:

$$T^{\mu\nu} u_{\nu} = \varepsilon u^{\mu}$$

Then in a local rest frame  $u^{\mu} \rightarrow (1, 0)$

$$T^{\mu\nu} \rightarrow \begin{pmatrix} \varepsilon & 0 \\ 0 & \pi \end{pmatrix}$$



# 1D Hydrodynamics and Landau Model

---

For an ideal gas (not necessarily in equilibrium):

$$T^{\mu\nu} = \int \frac{d^3 p}{p^0} p^\mu p^\nu f(p)$$

Massless particles

$$T^{\mu\nu} \rightarrow \begin{pmatrix} \varepsilon & 0 \\ 0 & -p \end{pmatrix} \quad \text{and} \quad \varepsilon = 3p$$

if

$$f(p) = f(|p|)$$

(local isotropy)



# 1D Hydrodynamics and Landau Model

---

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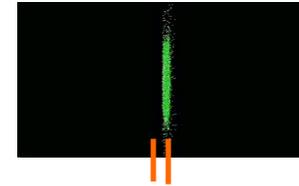
(local isotropy) **But no need for thermal equilibrium**



# 1D Hydrodynamics and Landau Model

Initial Energy density

$$\mathcal{E}_0 \propto \sqrt{s}^2$$



$$\Delta z \sim 1/\gamma,$$

“entropy density (no. of accessible states)”  $S/V \propto \mathcal{E}_0^{3/4}$

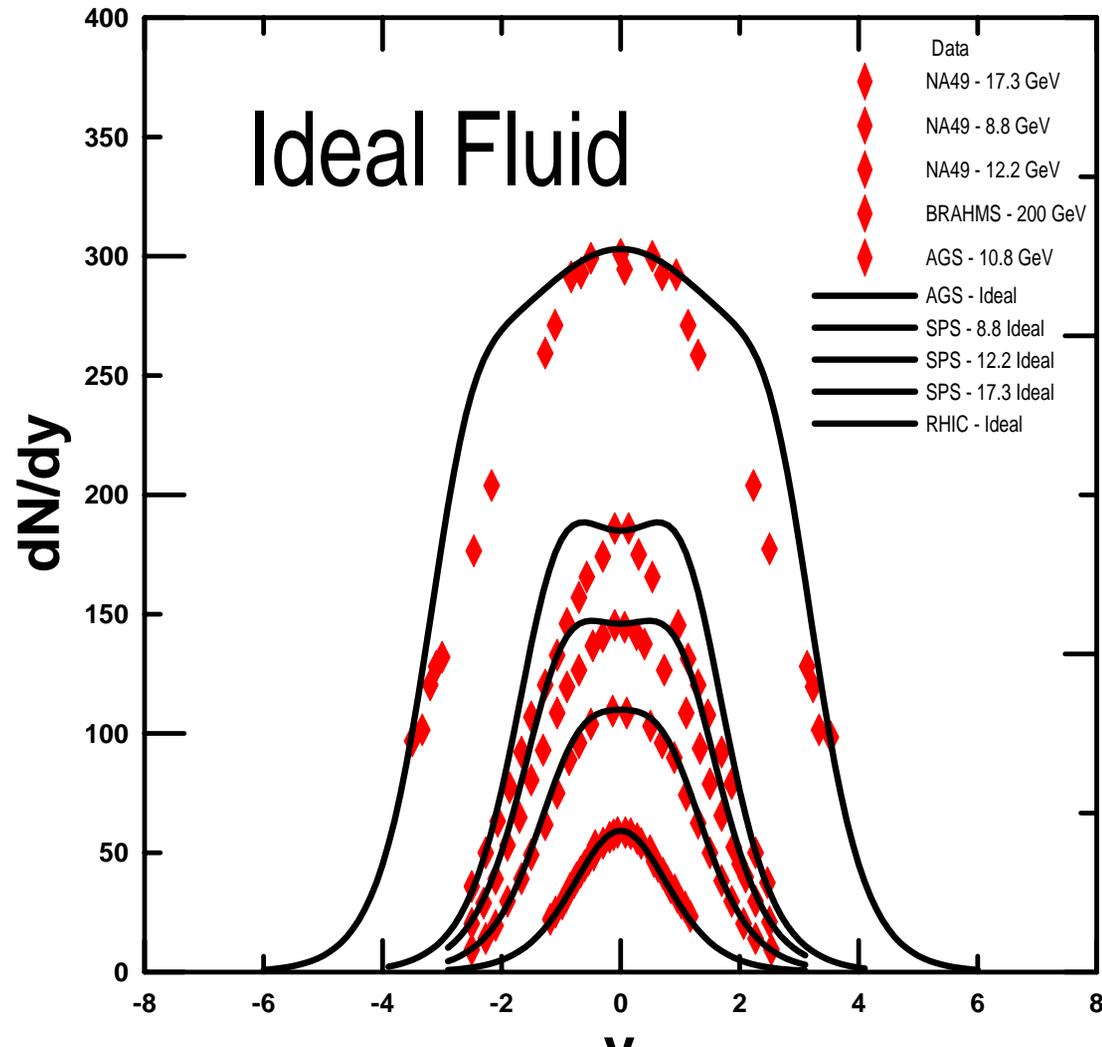
So that 
$$S \propto (\sqrt{s})^{1/2}$$

If the number of particles is proportional to the no. of states,

$$N \propto \sqrt{s}^{1/2}$$



# Landau Initial Condition (full stopping) 1D + thermal freezeout at $T=170$ MeV



Ideal Massless  
EoS



# Viscous fluid dynamics

---

$$\partial_{\mu} T^{\mu\nu} = 0$$

Conservation of energy and momentum

$$T^{\mu\nu} u_{\nu} = \varepsilon u^{\mu}$$

Definition of velocity field

$$T^{\mu\nu} \rightarrow \begin{pmatrix} \varepsilon & 0 \\ 0 & \pi \end{pmatrix}$$

in a local rest frame

If not ideal fluid, then  $\varepsilon - 3\pi \equiv -3\Pi \neq 0$

We need

$$\frac{d\Pi}{d\tau} = ?$$



# Minimum Ansatz within a linear response theory with causality:

---

$$\Pi(\tau) = -\int_{-\infty}^{\tau} d\tau' G(\tau, \tau') \zeta \partial_{\mu} u^{\mu},$$

with

$$G(t, t') = \frac{1}{\tau_R} e^{-\frac{t-t'}{\tau_R}}$$

then

$$\frac{d\Pi}{d\tau} = -\frac{1}{\tau_R} \Pi - \frac{\zeta}{\tau_R} \partial_{\mu} u^{\mu}$$

(2<sup>nd</sup> order causal viscous theory)



# Minimum Ansatz within a linear response theory with causality:

---

$$\Pi(\tau) = -\int_{-\infty}^{\tau} d\tau' G(\tau, \tau') \zeta \partial_{\mu} u^{\mu},$$

with 
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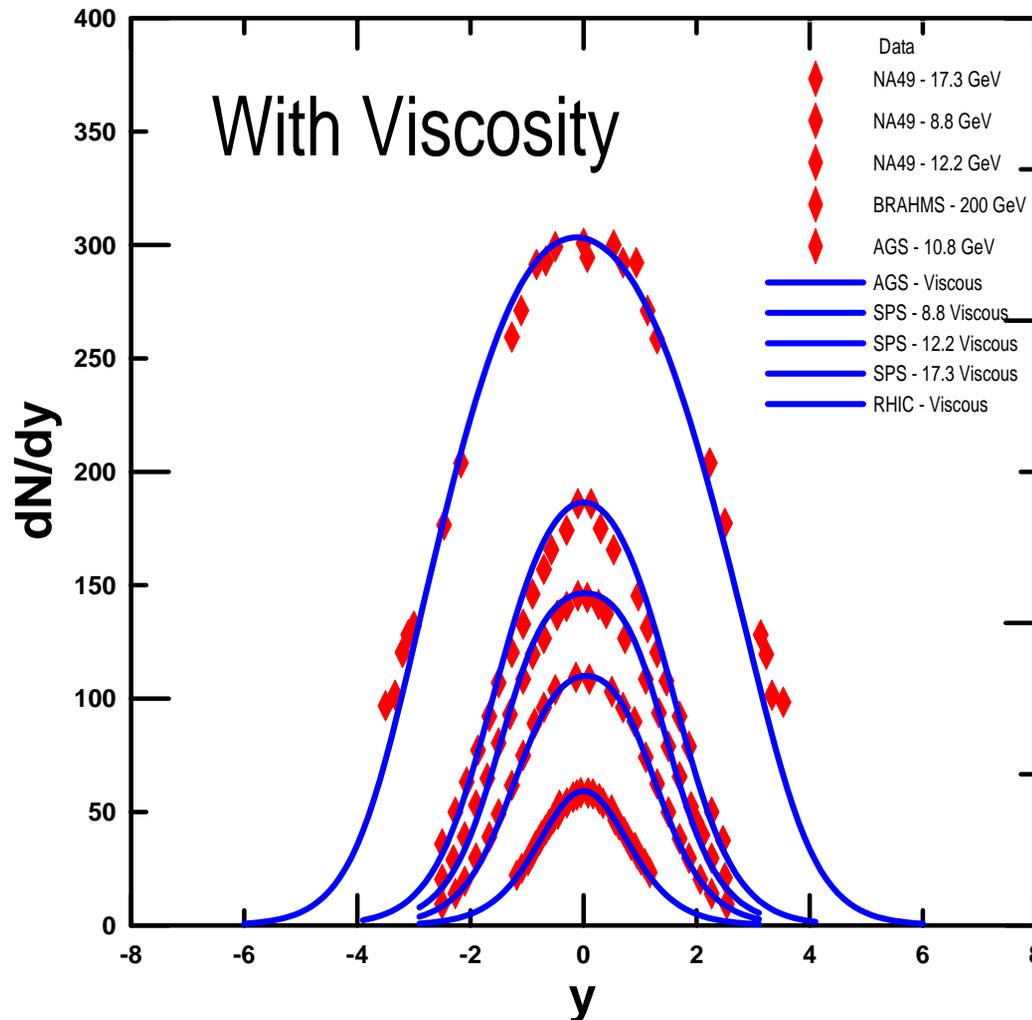
then

$$\frac{d\Pi}{d\tau} = -\frac{1}{\tau_R} \Pi - \frac{\zeta}{\tau_R} \partial_{\mu} u^{\mu} \quad \zeta = \zeta_{Bulk} + \frac{3}{2} \eta_{shear}$$

(2<sup>nd</sup> order causal viscous theory)



# Landau Initial Condition (full stopping) 1D + thermal freezeout at $T=170$ MeV

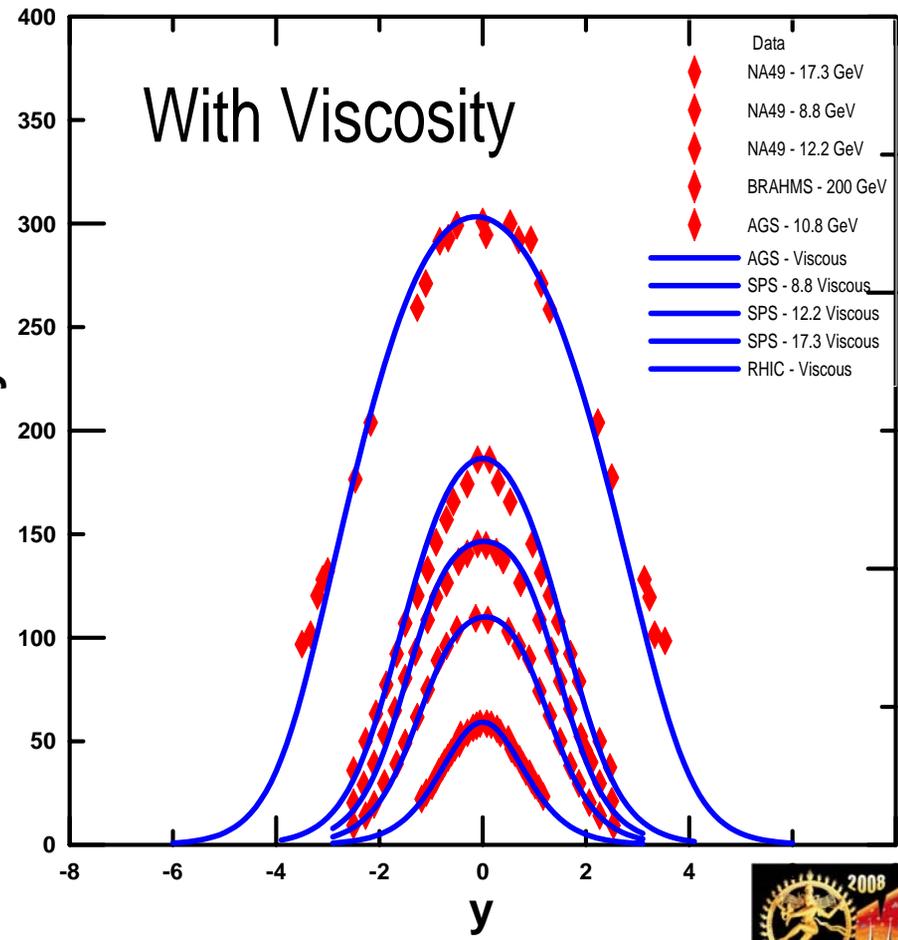
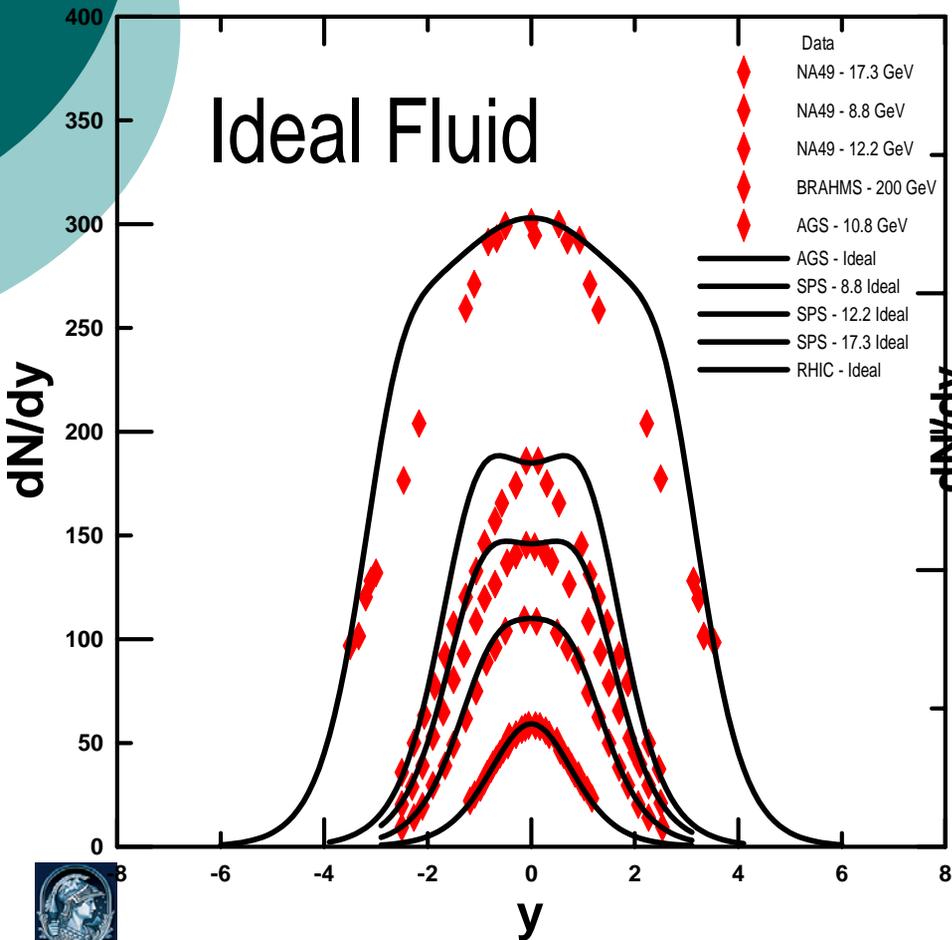


Not a best fit

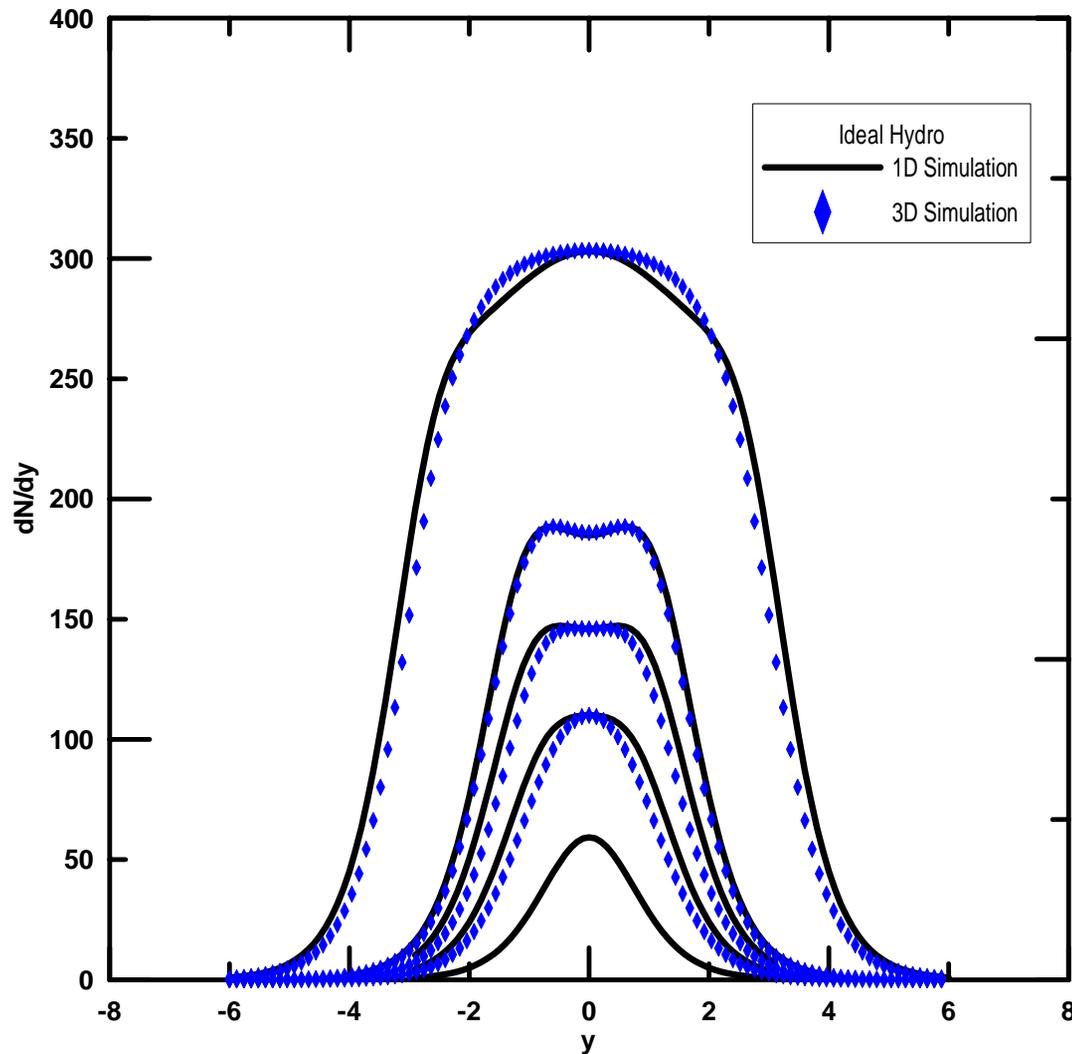
$$\zeta = 0.4 s$$



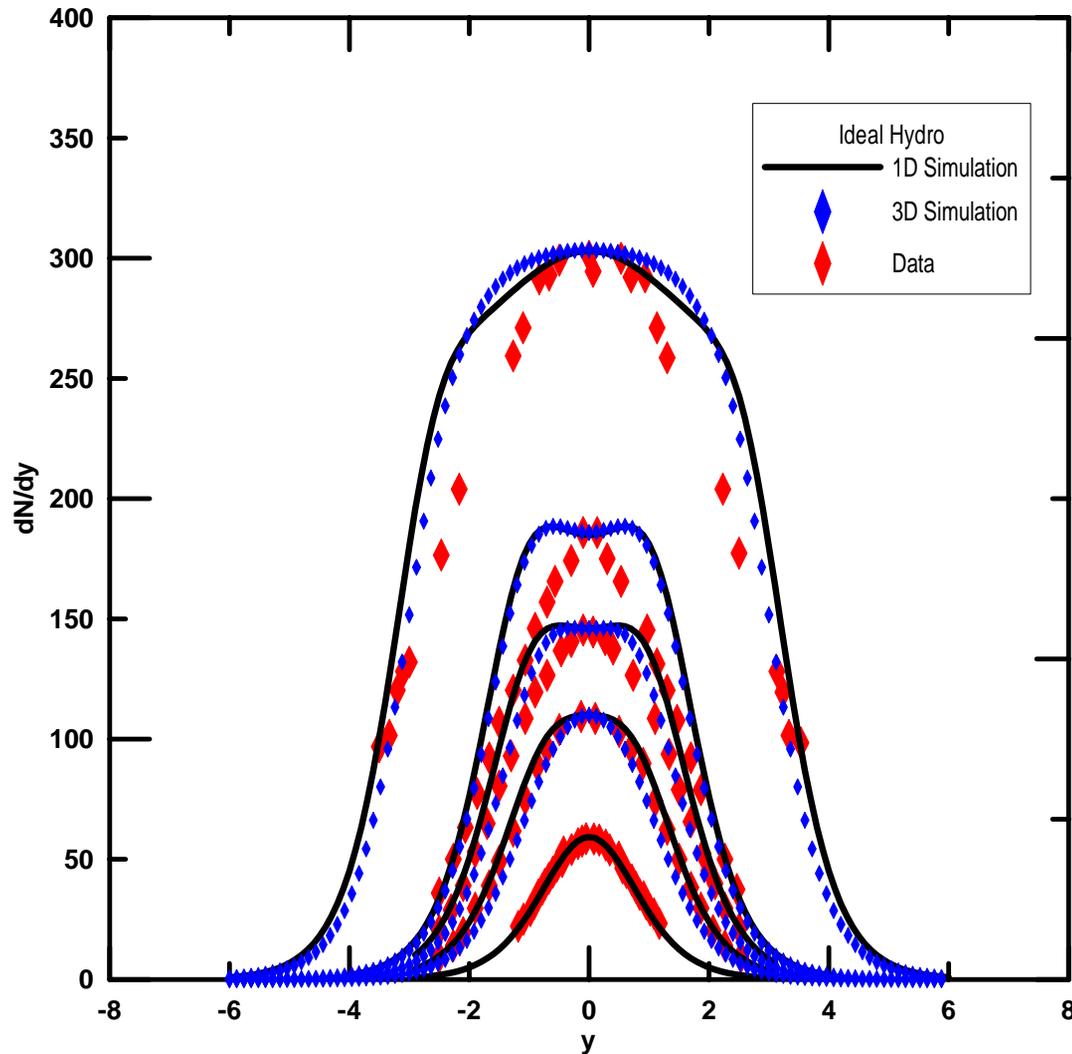
# Landau Initial Condition (full stopping) 1D + thermal freezeout at $T=170$ MeV



# Rapidity Distribution 3D vs. 1D (Ideal)

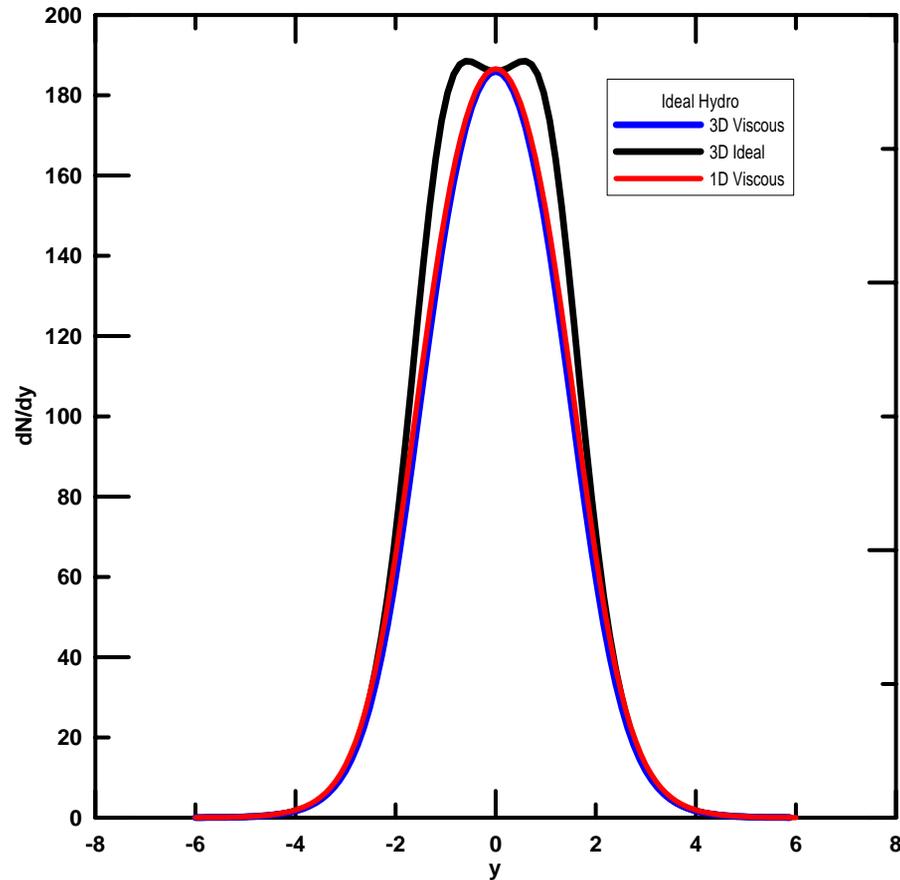


# Rapidity Distribution 3D vs. 1D (Ideal)



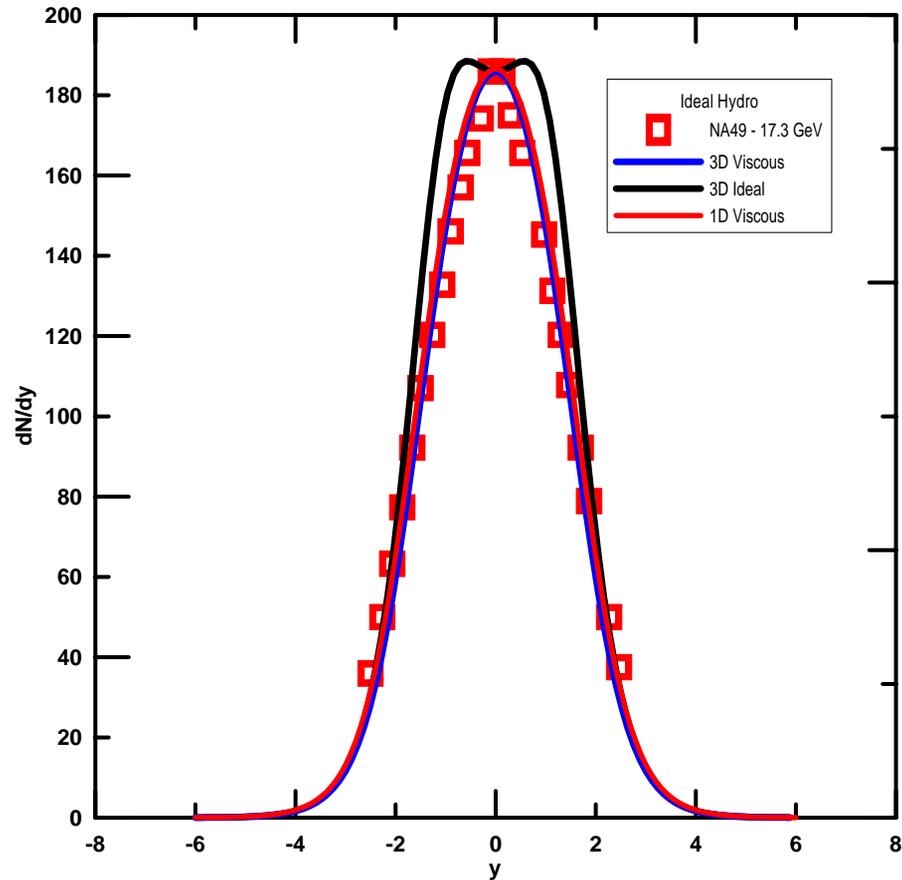
# Rapidity with viscosity (3D)

SPS 17.3GeV

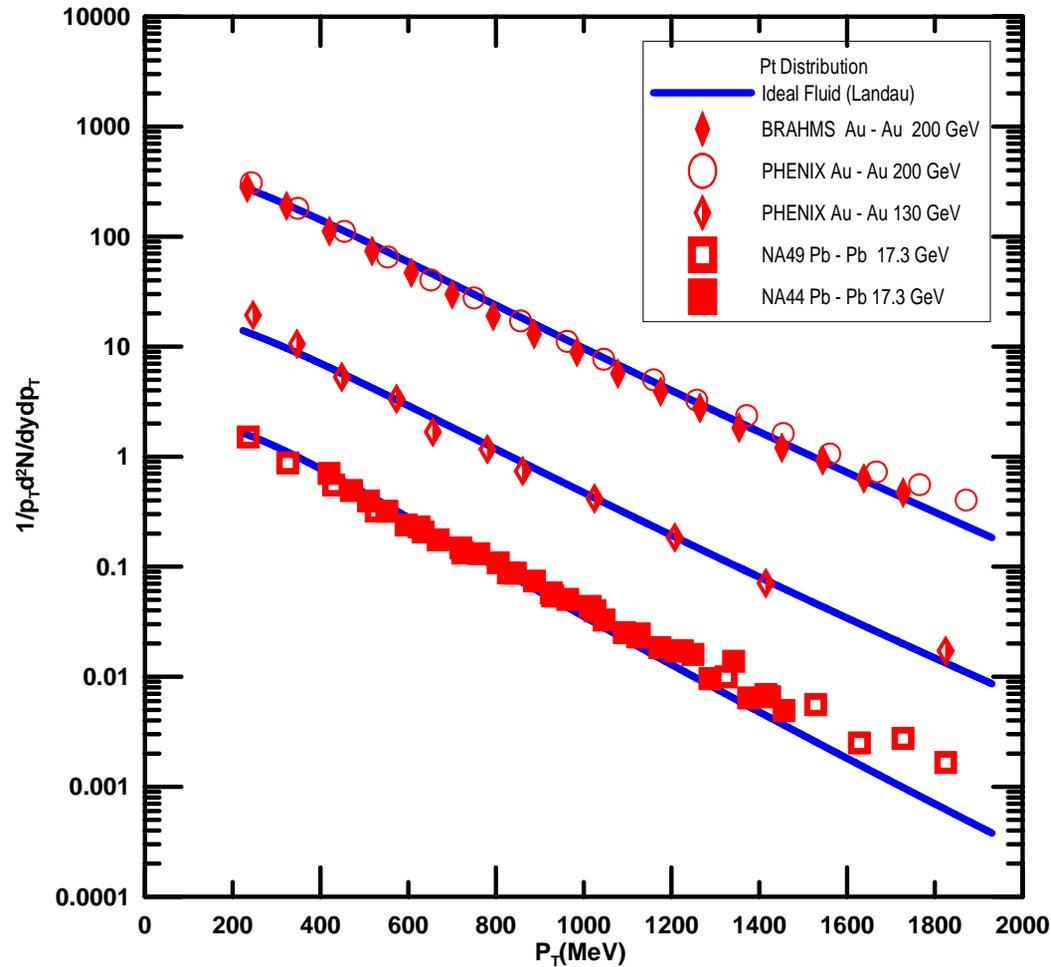


# Rapidity with viscosity (3D)

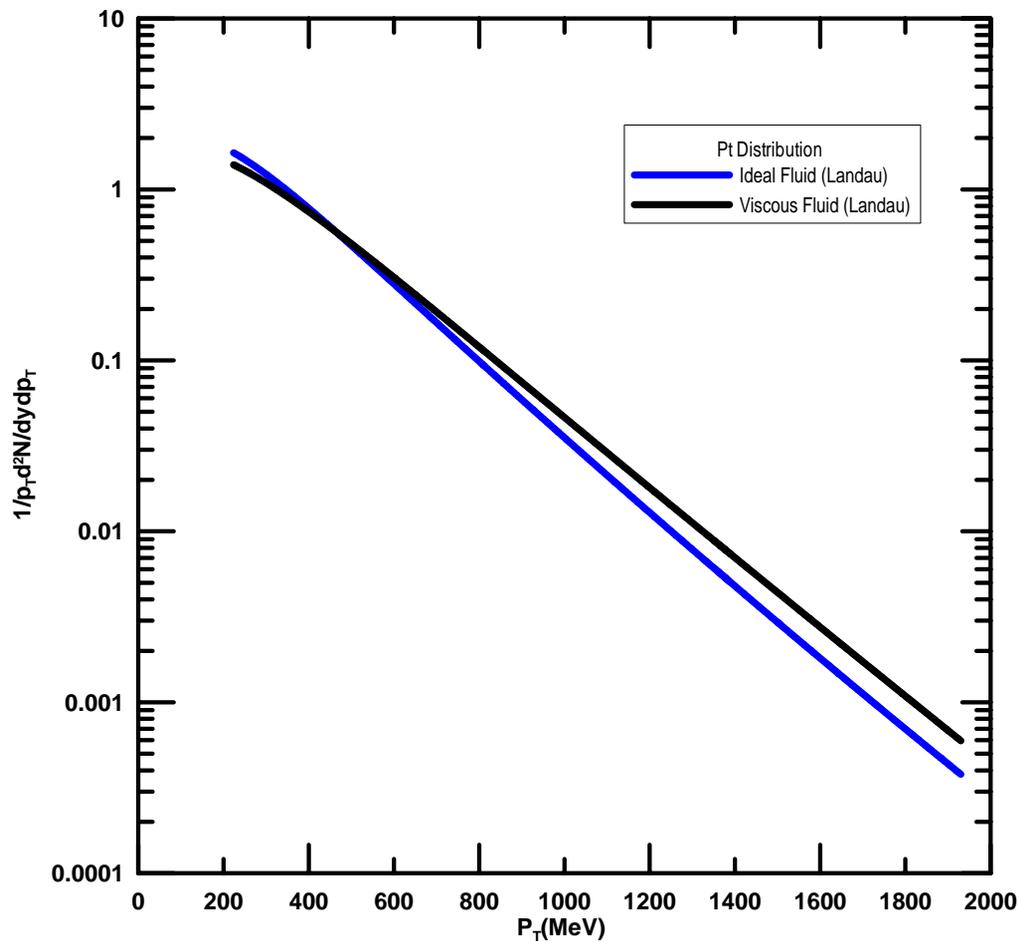
SPS 17.3GeV



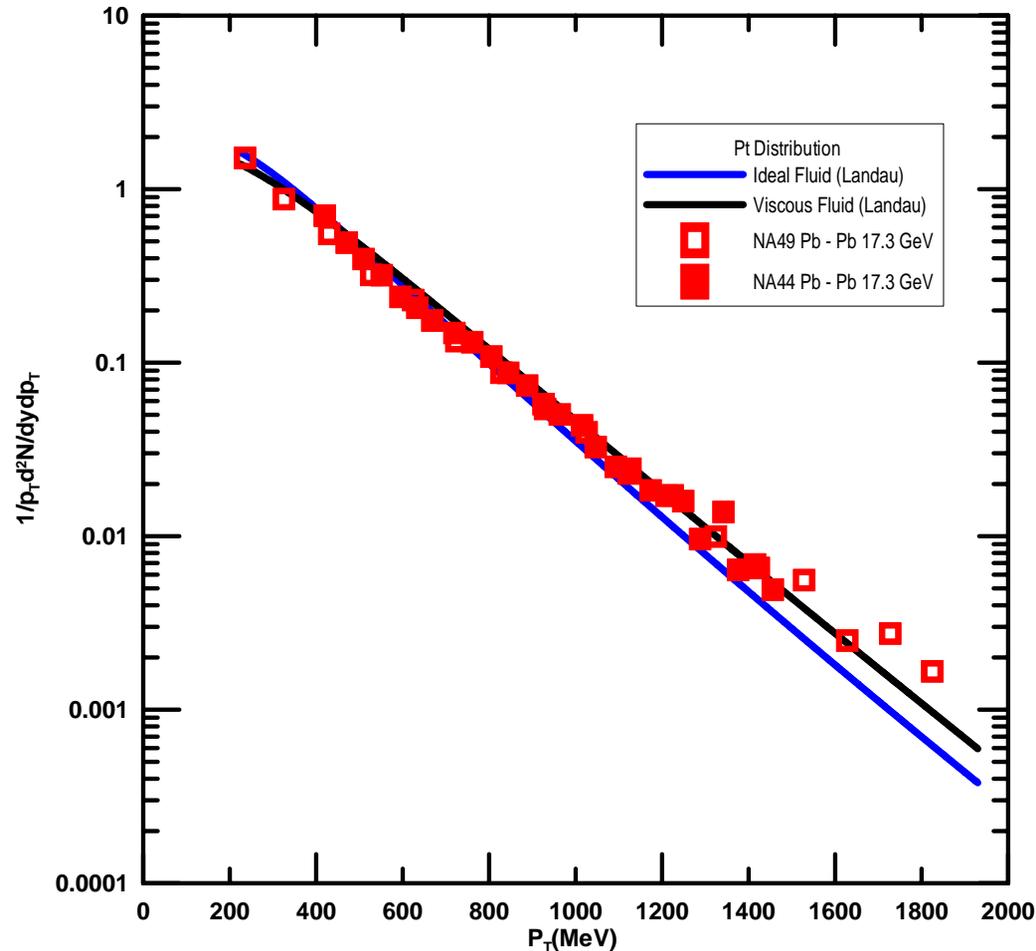
# $\pi$ -Transverse Momentum Distribution Ideal 3D Landau



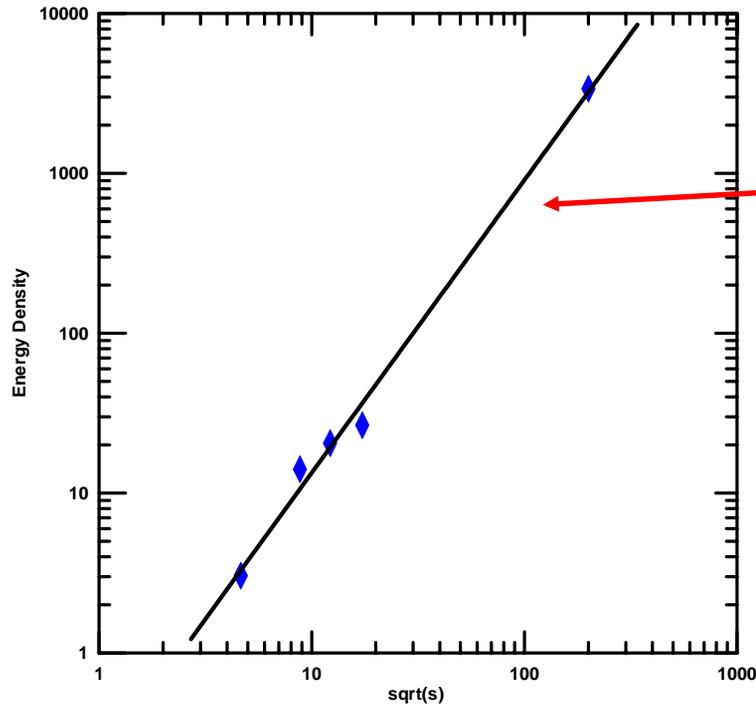
# $\pi$ -Transverse Momentum Distribution Ideal 3D Landau vs. Viscous



# $\pi$ -Transverse Momentum Distribution NA49 17.3GeV case



# Energy density vs. incident energy



$$\mathcal{E}_0 \propto (\sqrt{s})^2$$

and

$$S \propto (\sqrt{s})^{1/2}$$



# 3D Landau model with viscosity (preliminary)

---

Experimental data (rapidity and  $p_t$ ) can be fitted by Landau (full stopping) Initial condition with viscosity

Can we take this as seriously as the usual hydro interpretation ??

Dilemmas:

- Initial temperature and entropy too high. Can not be interpreted “thermally equilibrated energy density” as usual QGP degrees of freedom.

$T_0=300\text{MeV}$  at SPS and  $1.5\text{GeV}$  at RHIC !!!

- Why the same mechanism works similarly to p+p case? This is certainly out of equilibrium.



# What we have used for Ideal case?

---

$$\partial_{\mu} T^{\mu\nu} = 0$$

Conservation of energy and momentum

$$T^{\mu\nu} u_{\nu} = \varepsilon u^{\mu}$$

Definition of velocity field

$$T^{\mu\nu} = A u^{\mu} u^{\nu} + B g^{\mu\nu}$$

Isotropy in local  $u$  - frame

$$T_r(T^{\mu}_{\nu}) = 0$$

Scale invariance

Then  $\longrightarrow$   $A = \frac{4}{3} \varepsilon, \quad B = -\frac{1}{3} \varepsilon$

and  $\partial_{\mu} (\varepsilon^{3/4} u^{\mu}) = 0$



# Conclusion:

---

For a system where the longitudinal dynamics is dominant, everything works as if a hydrodynamical system, but this has nothing to do with the **local thermal equilibrium**. Here, any “temperature” and “entropy”,

$$"T" = K^{-1} \varepsilon^{1/4}, \quad "s" = \frac{4}{3} K \varepsilon^{3/4}$$

with any K. And also, the above argument valid for p+p if we substitute

$$T^{\mu\nu} \Rightarrow \langle T^{\mu\nu} \rangle$$



# To be understood:

---

Does this reflect some 'glasma' dynamics from the vacuum?  
What is the conserved quantity corresponding to 'entropy', for  $\text{Tr}(T)=0$ ,

$$T = K^{-1} \varepsilon^{1/4}, \quad S = \frac{4}{3} K \varepsilon^{3/4}$$

- Further studies such as  $v_2$  and HBT observables should be done, changing IC and EoS (see L. M. Satarov, I. N. Mishustin, A. V. Merdeev and H. Stoecker, PHYS. REV. C **75**, 024903 (2007)). Also investigate the shear effect.

Interesting question: Study the Event-by-Event fluctuations of rapidity distribution varying the system size. See the role of  $\tau$  (fluctuation-dissipation)

How to deal with the dynamics of baryon number?

How will be in LHC energies?



