VISH2+1: Causal relativistic hydrodynamics for viscous fluids*

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References:

U. Heinz, H. Song, A. K. Chaudhuri, Phys. Rev. C **73** (2006) 034904

H. Song and U. Heinz, Phys. Lett. B **658** (2008) 279

H. Song and U. Heinz, arXiv:0712.3715 [nucl-th]

Viscous relativistic hydrodynamics (Israel & Stewart 1979)

Include shear viscosity η, neglect bulk viscosity (massless partons) and heat conduction ($\mu_B \approx 0$); solve

$$\partial_\mu T^{\mu\nu} = 0$$

with modified energy momentum tensor

$$T^{\mu\nu}(x) = T_0^{\mu\nu}(x) + \pi^{\mu\nu} = \big(e(x)+p(x)\big) u^\mu(x) u^\nu(x) - g^{\mu\nu} p(x) + \pi^{\mu\nu}.$$

$\pi^{\mu\nu}$ = traceless viscous pressure tensor which relaxes locally to 2η times the shear tensor $\sigma^{\mu\nu} \equiv \nabla^{\langle\mu} u^{\nu\rangle}$ on a microscopic kinetic time scale τ_π:

$$D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta \nabla^{\langle\mu} u^{\nu\rangle}\right) - \left(u^\mu \pi^{\nu\lambda} + u^\nu \pi^{\mu\lambda}\right) D u_\lambda$$

where $D \equiv u^\mu \partial_\mu$ is the time derivative in the local rest frame.

Kinetic theory relates η and τ_π, but for a strongly coupled QGP neither η nor this relation are known $\Longrightarrow$ treat η and τ_π as independent phenomenological parameters. For consistency: $\tau_\pi \theta \ll 1$ ($\theta = \partial^\mu u_\mu =$ local expansion rate).

(1+1)-d viscous hydrodynamic equations

(Muronga & Rischke 2004, Chaudhuri & Heinz 2005)

[For (2+1)-d viscous hydrodynamic equations see Heinz, Song & Chaudhuri, PRC 73 (2006) 034904]

Azimuthally symmetric transverse dynamics with long. boost invariance:
Use (τ, r, ϕ, η) coordinates and solve

- hydrodynamic equations for $T^{\tau\tau} = (e+\mathcal{P})\gamma_r^2 - \mathcal{P}$, $T^{\tau r} = (e+\mathcal{P})\gamma_r^2 v_r$ (with "effective pressure" $\mathcal{P} = p - r^2\pi^{\phi\phi} - \tau^2\pi^{\eta\eta}$) together with
- kinetic relaxation equations for $\pi^{\phi\phi}$, $\pi^{\eta\eta}$:

$$\frac{1}{\tau}\partial_\tau\Big(\tau T^{\tau\tau}\Big) + \frac{1}{r}\partial_r\Big(r(T^{\tau\tau}+\mathcal{P})v_r\Big) = -\frac{p+\tau^2\pi^{\eta\eta}}{\tau},$$

$$\frac{1}{\tau}\partial_\tau\Big(\tau T^{\tau r}\Big) + \frac{1}{r}\partial_r\Big(r(T^{\tau r}v_r+\mathcal{P})\Big) = +\frac{p+r^2\pi^{\phi\phi}}{r},$$

$$\Big(\partial_\tau + v_r\partial_r\Big)\pi^{\eta\eta} = -\frac{1}{\gamma_r\tau_\pi}\left[\pi^{\eta\eta} - \frac{2\eta}{\tau^2}\left(\frac{\theta}{3} - \frac{\gamma_r}{\tau}\right)\right],$$

$$\Big(\partial_\tau + v_r\partial_r\Big)\pi^{\phi\phi} = -\frac{1}{\gamma_r\tau_\pi}\left[\pi^{\phi\phi} - \frac{2\eta}{r^2}\left(\frac{\theta}{3} - \frac{\gamma_r v_r}{r}\right)\right].$$

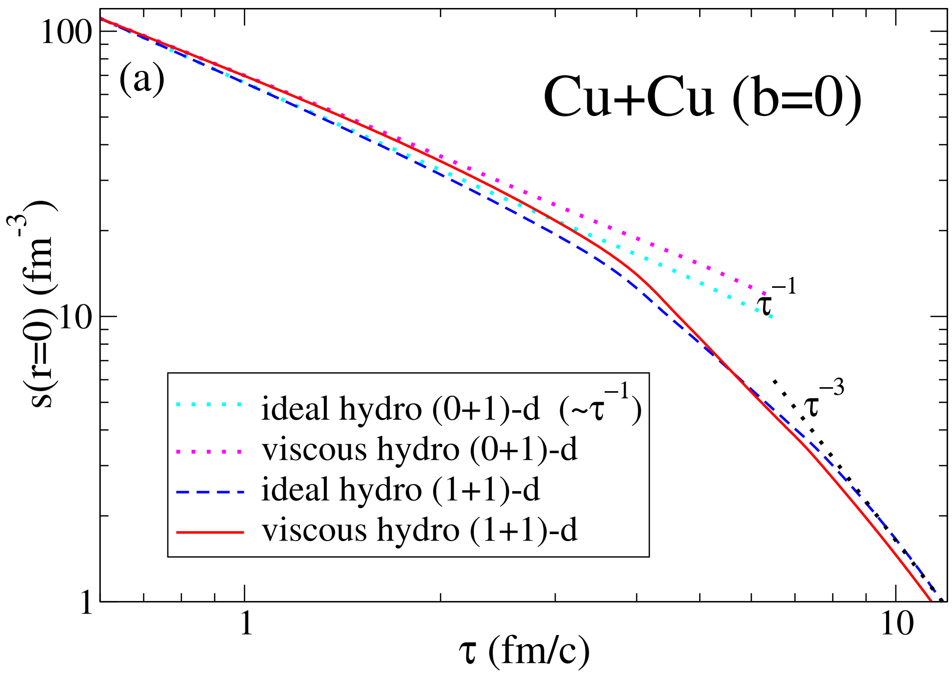
Close equations with EOS $p(e)$ where $e = T^{\tau\tau} - v_r T^{\tau r}$ and $v_r = T^{\tau r}/(T^{\tau\tau}+\mathcal{P})$.

(2+1)-d viscous hydro: less longitudinal work, more radial flow

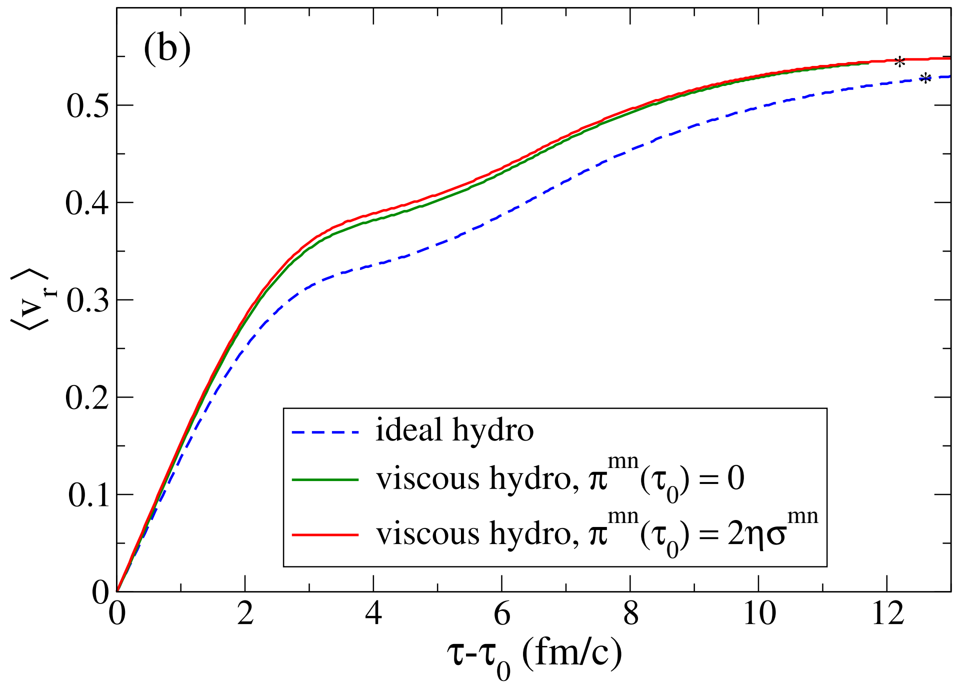
Cu+Cu @ $b = 0$, EOS Q

$\tau_0 = 0.6\,\frac{\text{fm}}{c}$, $e_0 = 30\,\frac{\text{GeV}}{\text{fm}^3}$, $\frac{\eta}{s} = \frac{1}{4\pi}$, $\tau_\pi = 0.24\left(\frac{200\,\text{MeV}}{T}\right)\frac{\text{fm}}{c}$, $T_{\text{dec}} = 130\,\text{MeV}$

Entropy density vs. time



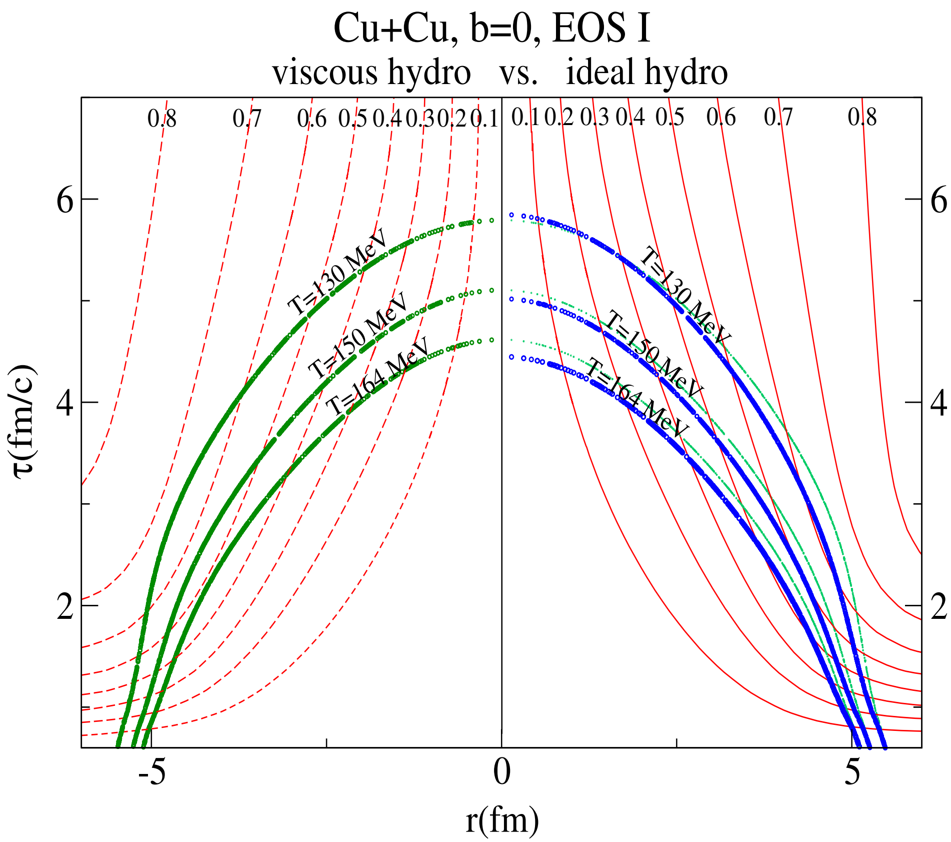
avg. radial flow vs. time

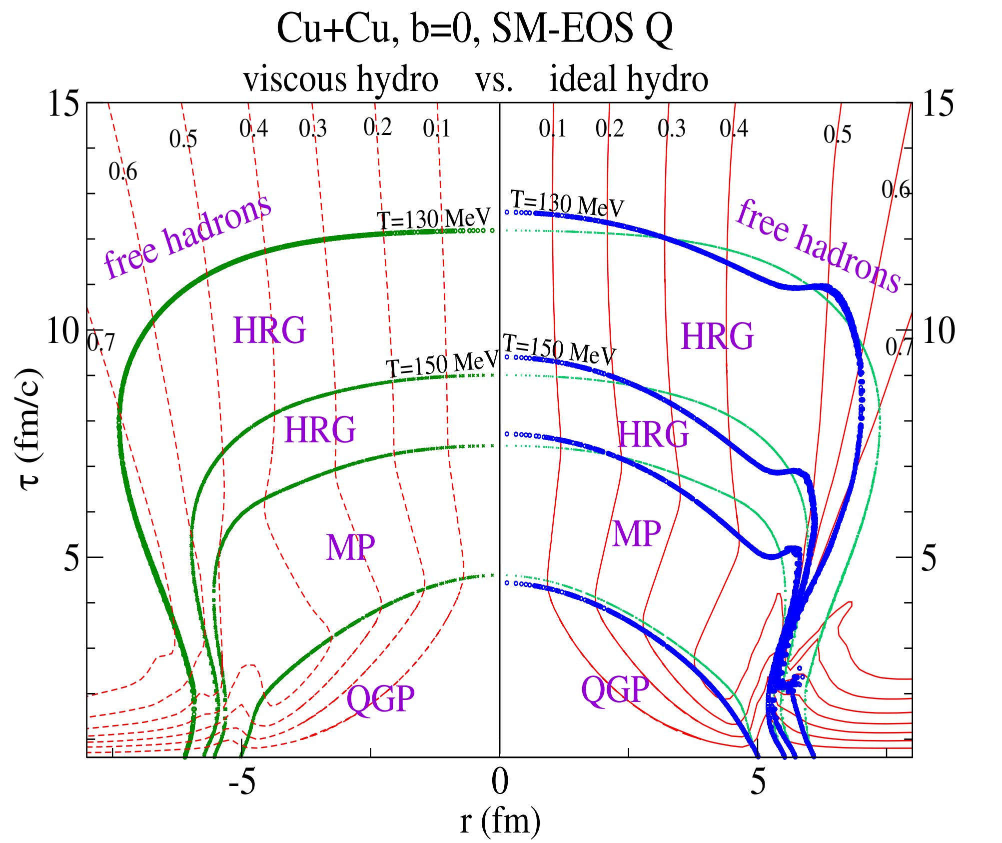


- Radial flow develops much faster, expansion turns 3-dimensional more abruptly
- Shear viscosity initially reduces the cooling due to longitudinal work, but then leads to faster cooling in the fireball center than for ideal fluid later, due to stronger radial flow
  (seen also by Teaney 2004, Chaudhuri 2006,2007; Romatschke et al. 2006,2007)

Central Cu+Cu (b=0): ideal vs. viscous hydro

$\tau_0 = 0.6\,\frac{\text{fm}}{c},\ e_0 = 30\,\frac{\text{GeV}}{\text{fm}^3},\ \frac{\eta}{s} = \frac{1}{4\pi},\ \tau_\pi = 0.24\left(\frac{200\,\text{MeV}}{T}\right)\frac{\text{fm}}{c},\ T_{\text{dec}} = 130\,\text{MeV}$



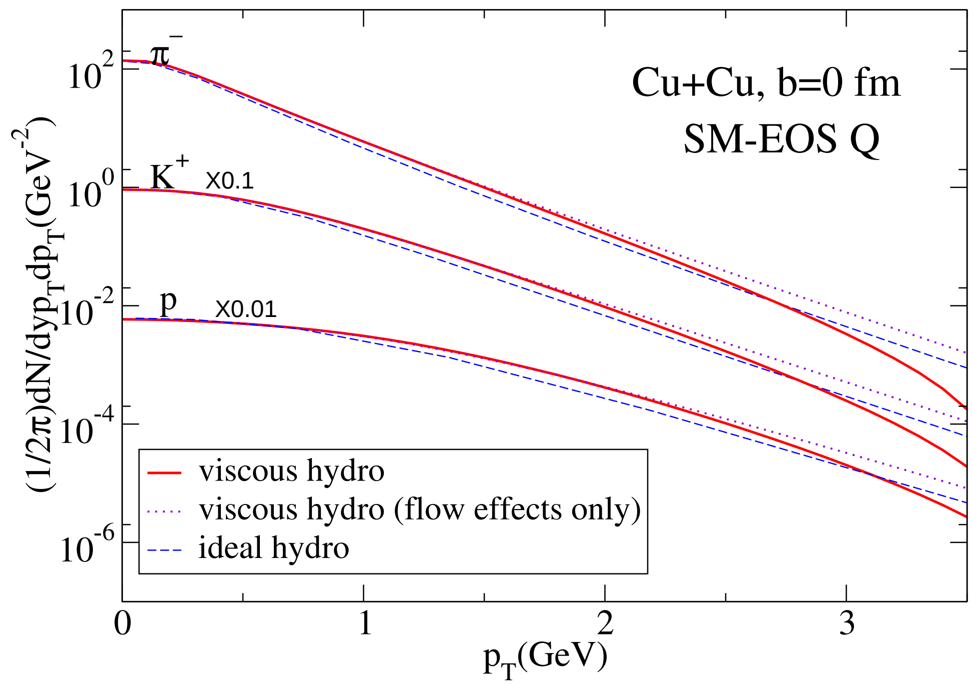


- Viscous hydro smoothes out phase transition structures
- Viscous hydro cools more slowly than ideal hydro, except for the center where cooling is accelerated at late times by faster radial expansion in the viscous case
- Viscous effects increase QGP lifetime, but viscous pressure gradients in the mixed phase shorten the mixed phase lifetime

(2+1)-d viscous hydro: more radial flow $\Longrightarrow$ flatter spectra

hadron p_T-spectra:

$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p\cdot d^3\sigma(x)}{(2\pi)^3}\Big[f_{\rm eq}(x,p) + \delta f(x,p)\Big] = \int_\Sigma \frac{p\cdot d^3\sigma(x)}{(2\pi)^3}\, f_{\rm eq}(x,p)\left(1 + \frac{1}{2}\frac{p^\alpha p^\beta}{T^2(x)}\frac{\pi_{\alpha\beta}(x)}{(e{+}p)(x)}\right)$$



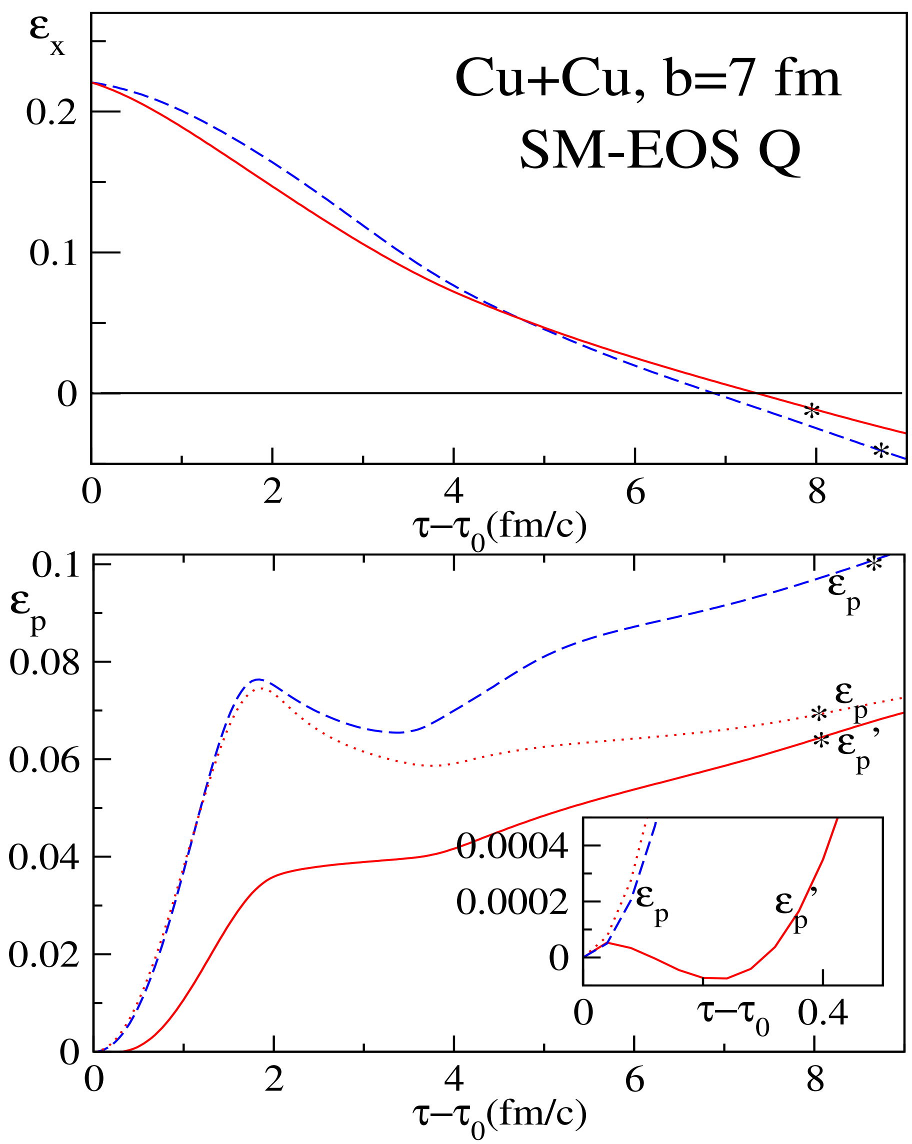
$$\tau_0 = 0.6\,\frac{\rm fm}{c},\ e_0 = 30\,\frac{\rm GeV}{\rm fm^3},\ \frac{\eta}{s} = \frac{1}{4\pi},\ \tau_\pi = 0.24\left(\frac{200\,\rm MeV}{T}\right)\frac{\rm fm}{c},\ T_{\rm dec} = 130\,\rm MeV$$

- For identical initial and freeze-out conditions, viscous evolution yields more radial flow and flatter spectra (as previously seen by Chaudhuri 2006,2007; Romatschke 2007)
- Effect on $b=0$ spectra can be largely absorbed by starting viscous hydro later with lower initial density (Romatschke et al., 2006,2007)

(2+1)-d viscous hydro: less momentum anisotropy

Cu+Cu @ $b = 7$ fm, EOS Q, same initial and final conditions

spatial eccentricity and momentum anisotropy



- - - ideal hydro, ——, · · · viscous hydro

- Source eccentricity $\epsilon_x = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$ decays initially faster, but later more slowly;

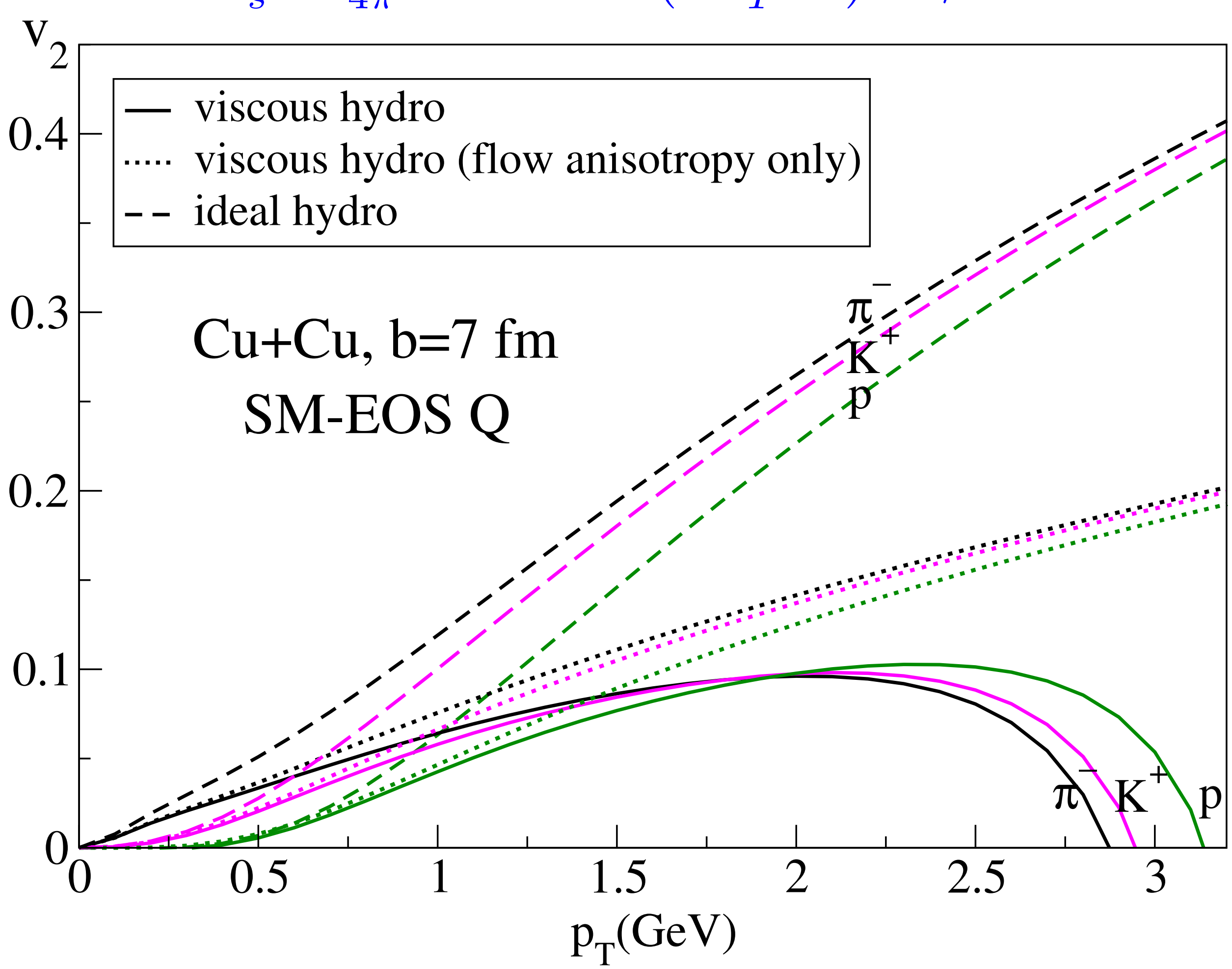
- Flow anisotropy $\epsilon_p = \frac{\langle T_0^{xx} - T_0^{yy} \rangle}{\langle T_0^{xx} + T_0^{yy} \rangle}$ develops faster initially, but soon drops significantly below ideal fluid values;

- during the first 3-4 fm$/c$ viscous pressure components $\pi^{\mu\nu}$ contribute strong out-of-plane (i.e. negative) momentum anisotropy in the local fluid rest frame; inhibit build-up of flow anisotropy

- Total momentum anisotropy $\epsilon'_p = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$ is reduced by almost 50% relative to ideal fluid.

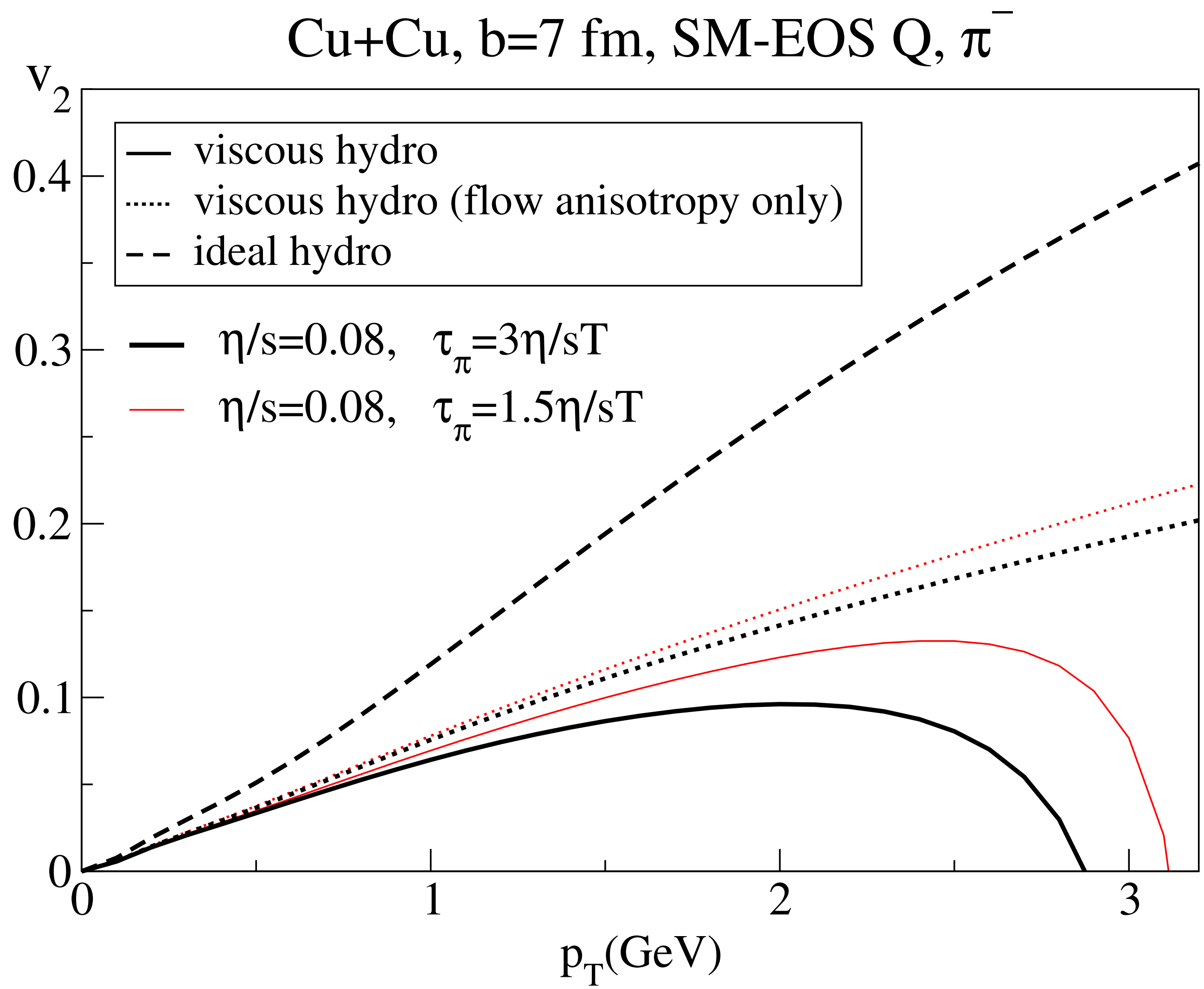
Comparison between VISH2+1 Elliptic flow from (2+1)-dim. viscous hydrodynamics (I)

$\frac{\eta}{s} = \frac{1}{4\pi},\ \tau_\pi = 0.24\left(\frac{200\,\mathrm{MeV}}{T}\right)\,\mathrm{fm}/c$



- **Elliptic flow very sensitive to even minimal shear viscosity!**
- Viscous corrections to equilibrium distribution fct. have significant effect on v_2 (Teaney 2003), but at low p_T the effects from the reduced hydrodynamic flow anisotropy are larger

Elliptic flow from (2+1)-dim. viscous hydrodynamics (II)

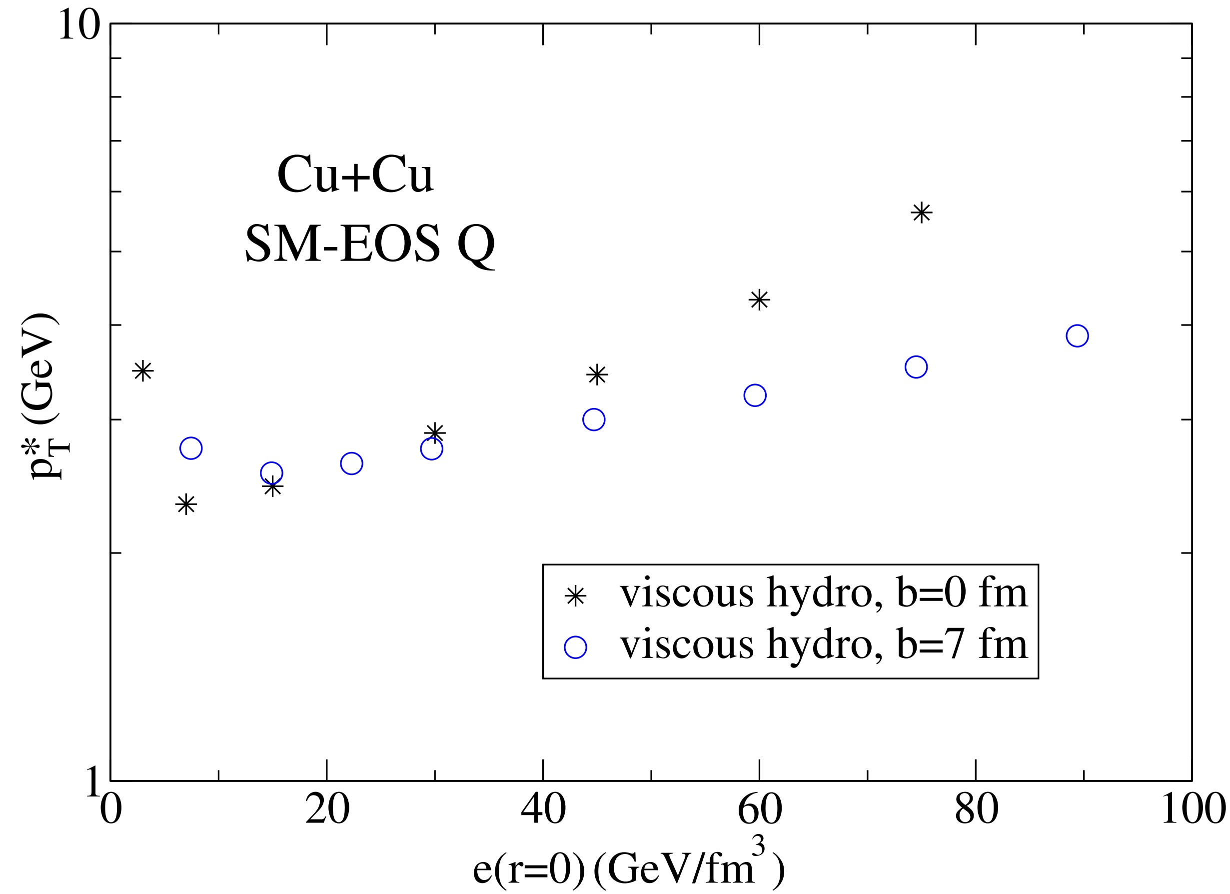


- **Faster kinetic relaxation at fixed η/s reduces viscous effects $\longrightarrow$ Janik 2007**

The limits of viscous hydrodynamics

At sufficiently large p_T, viscous corrections become large even if η/s is small.

$|\delta N(p)| > \frac{1}{2}|N_0(p)|$ indicates breakdown of the assumptions:



- **For larger initial energy densities, p_T-range increases where viscous hydro can be applied to describe hadron spectra.**

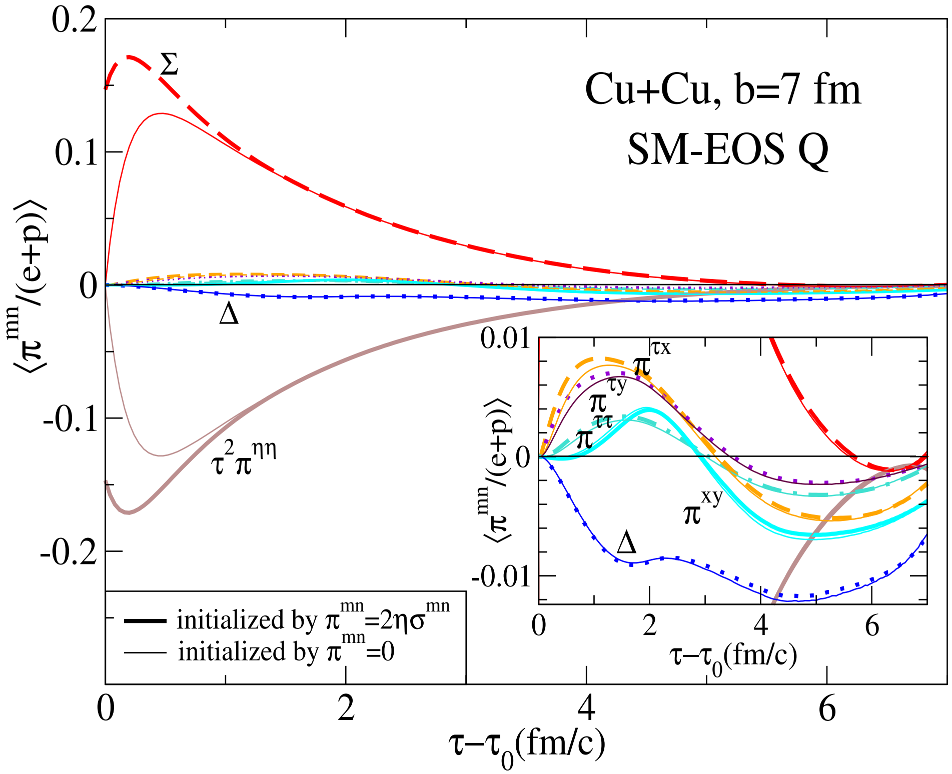
Sensitivity to initial values for viscous pressure tensor

Thin lines: $\pi_0^{mn} = 0$; Thick lines: $\pi_0^{mn} = 2\eta\sigma^{mn} \equiv 2\eta\nabla^{\langle m}u^{n\rangle}$.

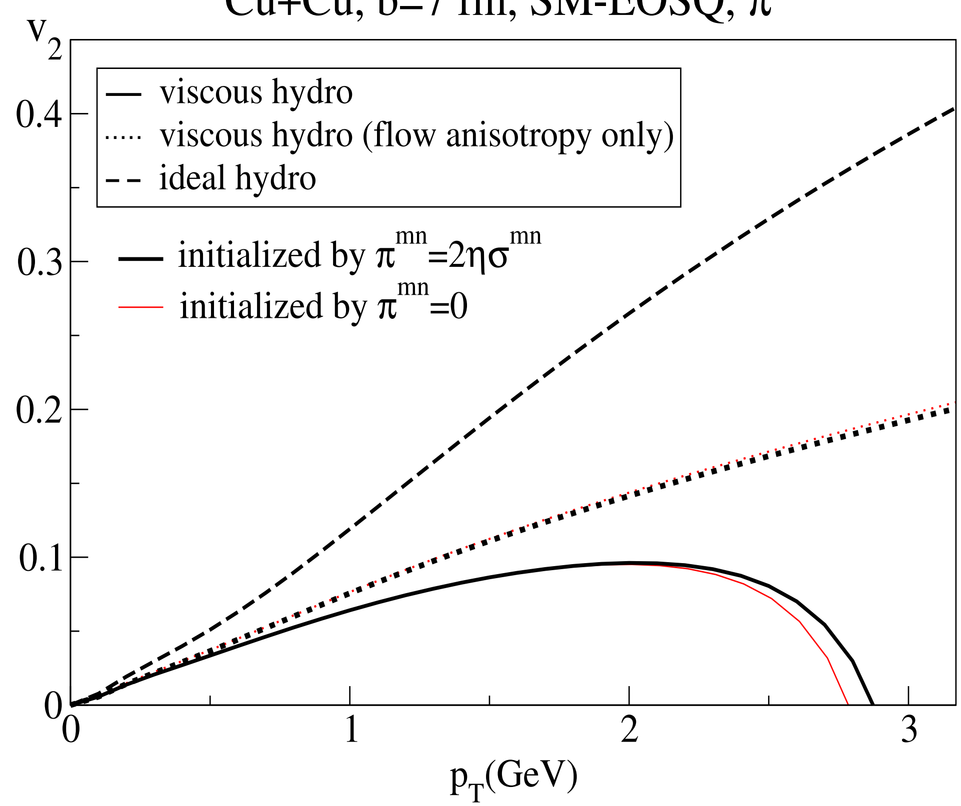
$\tau_0 = 0.6\,\frac{\text{fm}}{c},\ e_0 = 30\,\frac{\text{GeV}}{\text{fm}^3},\ \frac{\eta}{s} = \frac{1}{4\pi},\ \tau_\pi = 0.24\left(\frac{200\,\text{MeV}}{T}\right)\frac{\text{fm}}{c},\ T_{\text{dec}} = 130\,\text{MeV}$

largest viscous pressure components vs. time

$\Sigma = \pi^{xx}+\pi^{yy},\quad \Delta = \pi^{xx}-\pi^{yy}$



pion elliptic flow



- For fixed η/s, viscous pressure components become small at late times $\longrightarrow$ ideal hydro
- After $\tau \sim 1\,\text{fm}/c \sim 5\tau_\pi$, viscous pressure tensor has lost all memory of initial conditions!
- Effects of initial π^{mn} on final v_2 are small

Tests of the viscous hydro code VISH2+1

- $\eta \to 0 \quad \longrightarrow$ ideal fluid code AZHYDRO (test hydro evolution algorithm)

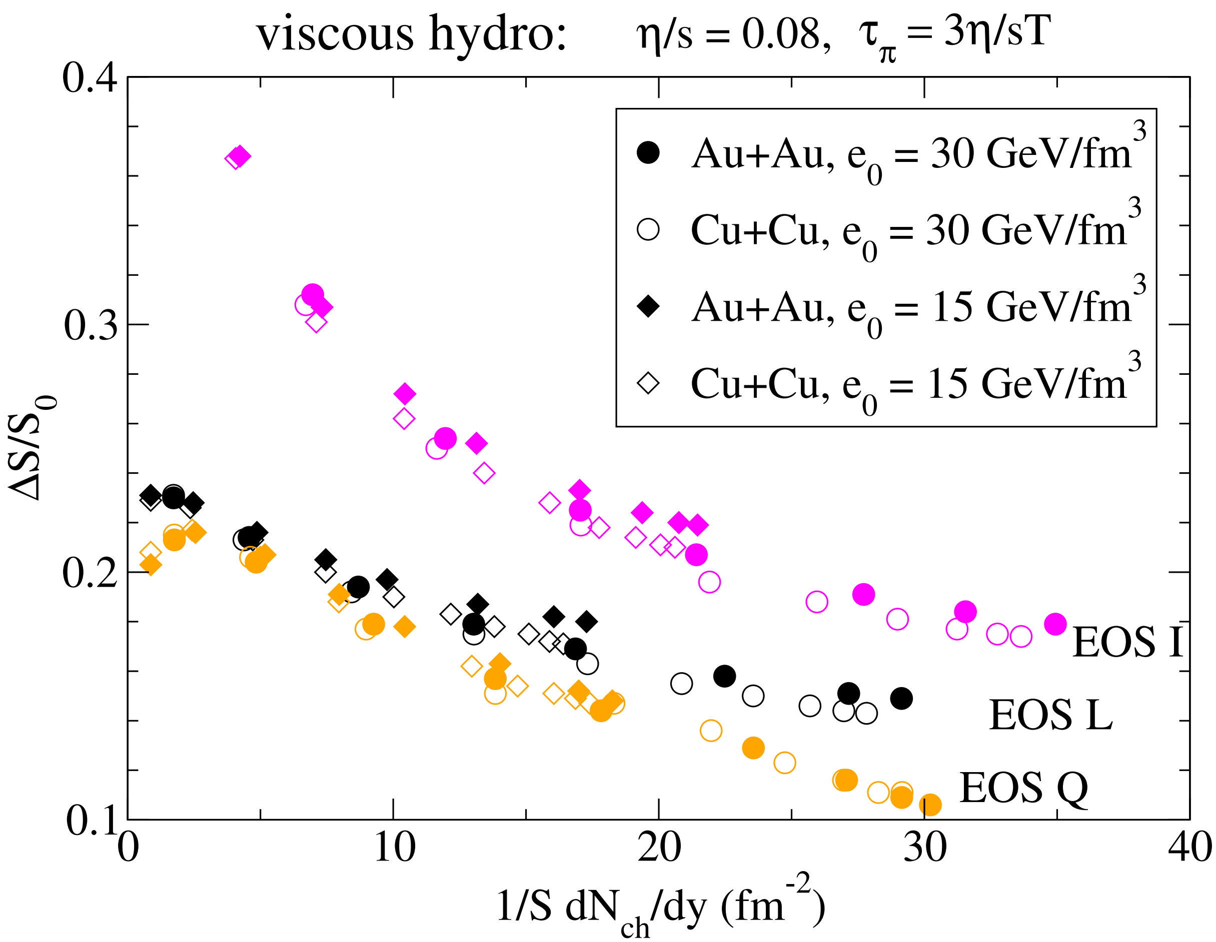
- $\nabla_\perp p = 0$, $\tau_\pi \to 0 \quad \Longrightarrow$ reproduce analytic soln. of boost-invariant Navier-Stokes

- η, τ_π small $\Longrightarrow$ Israel-Stewart $\mapsto$ Navier-Stokes (tests kinetic evolution algorithm for $\pi^{\mu\nu}$)

- $\pi^\mu_\mu = 0$, $u_\mu \pi^{\mu\nu} = 0$ to better than 2%

- Evolution of e, u^μ, $\pi^{\mu\nu}$ by VISH2+1 tested against Romatschkes' code:
  - excellent agreement for identical initial conditions, EOS, kinetic evolution equations
  - large difference in published $v_2(p_T)$ due to extra terms in $D\pi^{\mu\nu} = \ldots$ used by the Romatschkes

Viscous entropy production

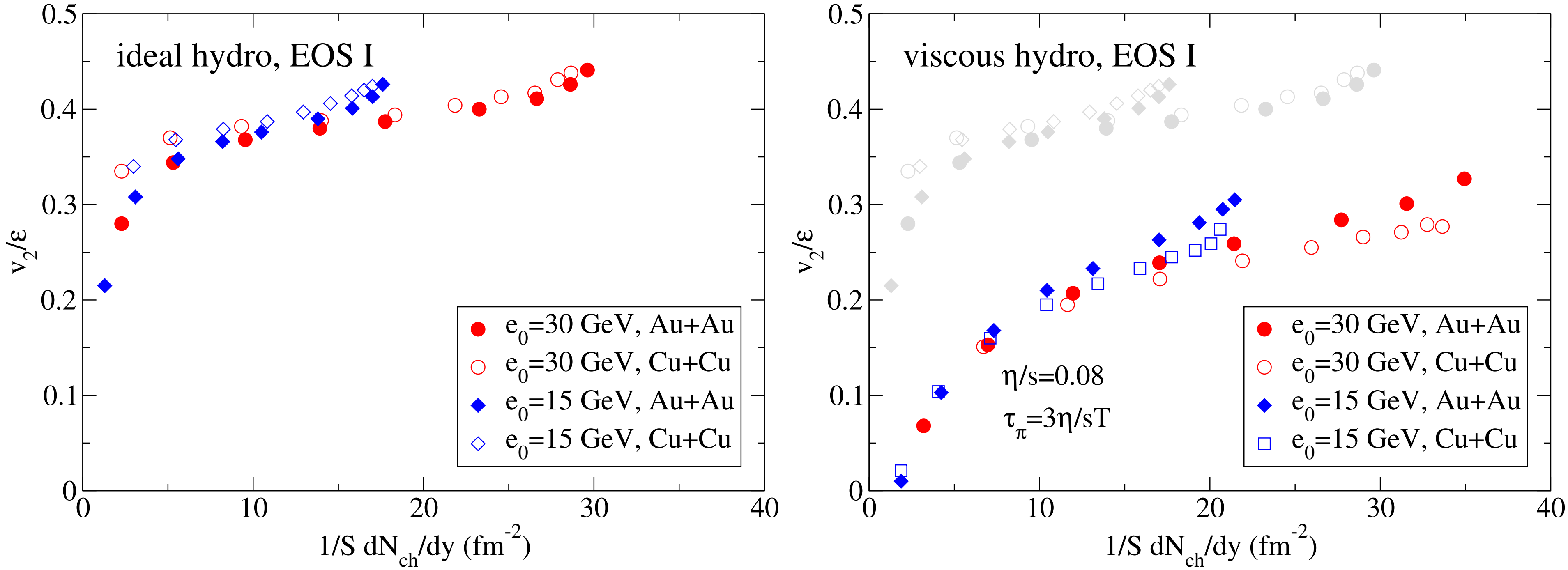


EOS I: ideal gas of massless partons
EOS Q: 1st order QGP-HRG phase transition
EOS L: smooth crossover from lattice QCD data above T_c to HRG below T_c.

- Viscous entropy production larger for faster-expanding fireballs
- Entropy production scales approximately with charged multiplicity density per unit area, $\frac{1}{S}\frac{dN_{ch}}{dy}$
- Entropy production fraction is larger for smaller $\frac{1}{S}\frac{dN_{ch}}{dy}$ (lower-energy and more peripheral collisions)
- At the same $\frac{1}{S}\frac{dN_{ch}}{dy}$, collisions between larger nuclei take longer to freeze out, generating slightly more entropy

Multiplicity scaling of the normalized elliptic flow v_2/ϵ_x (I)

Preliminary!

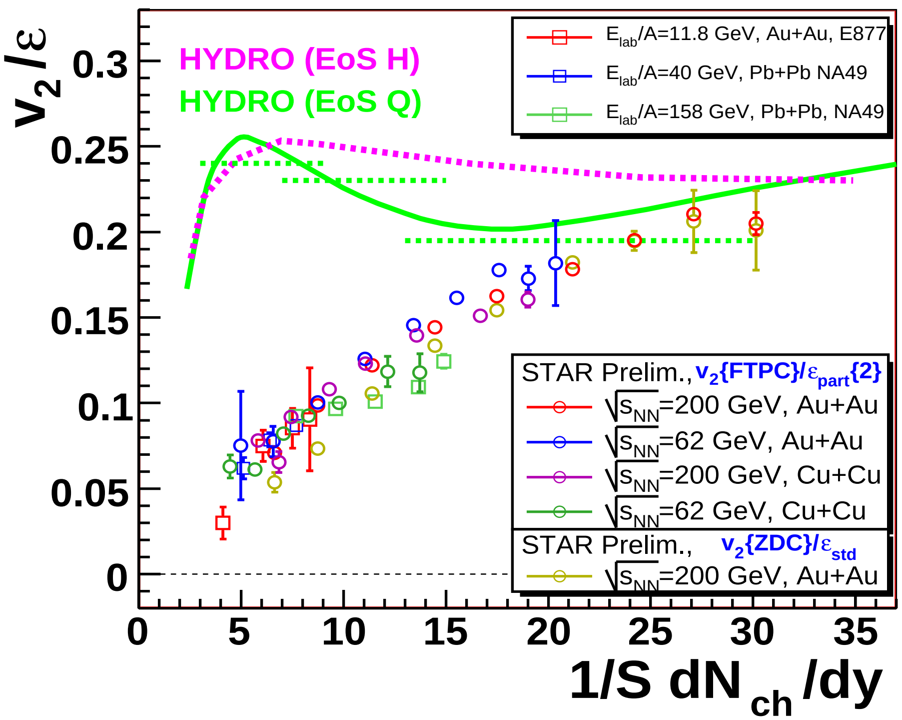


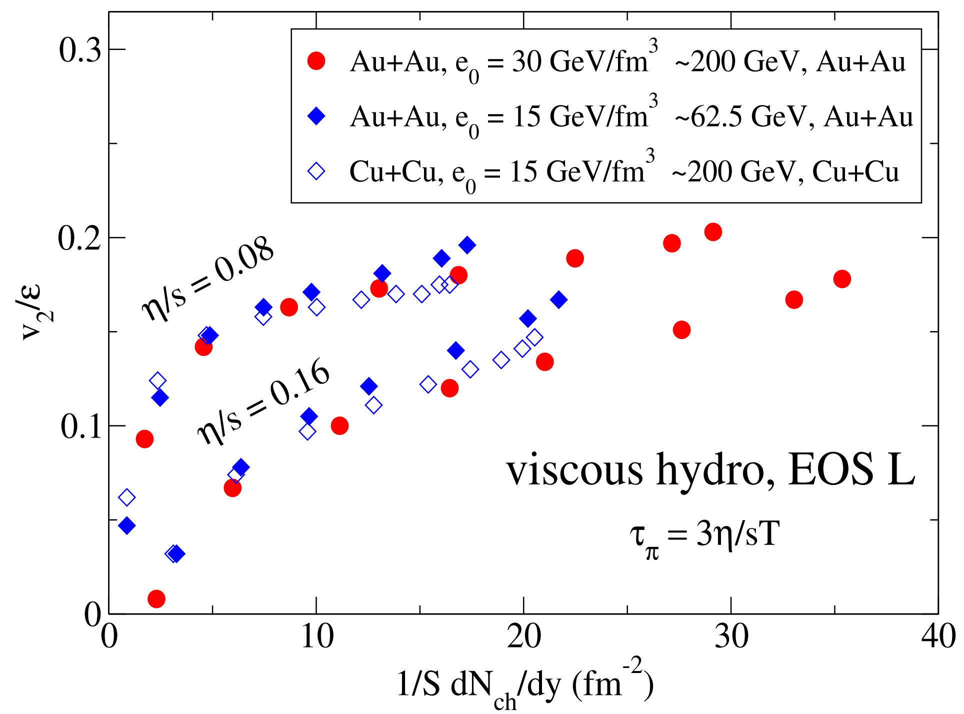
- Freeze-out at constant $e_{\rm dec}$ introduces time scale, breaking the scale invariance of ideal hydro and cutting short the build-up of elliptic flow before it saturates
- At the same $\frac{1}{S}\frac{dN_{\rm ch}}{dy}$, collisions between smaller nuclei and more peripheral collisions freeze out earlier, with less elliptic flow v_2/ϵ_x
- This breaks the multiplicity scaling with $\frac{1}{S}\frac{dN_{\rm ch}}{dy}$ even for ideal hydro
- This scaling is broken even more strongly in viscous hydro!

  At fixed $\frac{1}{S}\frac{dN_{\rm ch}}{dy}$, smaller collision systems and more peripheral collisions show more viscous suppression of v_2/ϵ_x than more central collisions or collisions of larger nuclei

Multiplicity scaling of the normalized elliptic flow v_2/ϵ_x (II)

A case study with fixed specific viscosity η/s:





- General tendency of experimental data consistent with viscous effects
- Data require more than minimal shear viscosity (due to highly viscous late hadron gas stage!)
- Search for scale-breaking effects requires more accurate data
- Realistic modeling must account for T-dependence of shear and bulk viscosity, especially near T_c

Summary

- Shear viscosity reduces the longitudinal pressure but increases the transverse pressure in heavy ion collision
  $\Longrightarrow$ slower cooling by longitudinal work initially, but faster cooling by stronger transverse expansion later

- While viscous pressure effects on angle-averaged p_T-spectra (radial flow) can be largely absorbed by changing the initial conditions (starting the transverse expansion later and with lower initial energy density), this increases the destructive effects of shear viscosity on the buildup of elliptic flow.

- The effects of shear viscosity on elliptic flow are large; even Son's minimal viscosity $\eta/s = 1/4\pi$ seems incompatible with RHIC data $\Longrightarrow$ needs more checking (resolve ambiguities in Israel-Stewart approach!).

- Shorter kinetic relaxation times for the viscous pressure tensor reduce the effects from shear viscosity; only weak sensitivity to initial values for $\pi^{\mu\nu}$.

- Viscous entropy production roughly scales with multiplicity per transverse area; larger viscous effects for smaller collision systems and larger impact parameters

- Multiplicity scaling of normalized elliptic flow v_2/ϵ_x weakly broken by freeze-out in ideal hydro and slightly more strongly broken by shear viscosity in viscous hydro

Supplements

(2+1)-d viscous hydrodynamic equations

Heinz, Song & Chaudhuri, PRC 73 (2006) 034904

Transverse dynamics w/o azimuthal symmetry, but with long. boost invariance: Use (τ, x, y, η) coordinates and solve

- hydrodynamic equations for $T^{\tau\tau} = (e+p)\gamma_r^2 - p + \pi^{\tau\tau}$, $T^{\tau x} = (e+p)\gamma_\perp^2 v_x + \pi^{\tau x}$, $T^{\tau y} = (e+p)\gamma_\perp^2 v_y + \pi^{\tau y}$:

$$\frac{1}{\tau}\partial_\tau\left(\tau T^{\tau\tau}\right) + \partial_x\left(v_x T^{\tau\tau}\right) + \partial_y\left(v_y T^{\tau\tau}\right) = \mathcal{S}^{\tau\tau}[v_x, v_y, \pi^{\eta\eta}, \pi^{\tau\tau}, \pi^{\tau x}, \pi^{\tau y}]$$

$$\frac{1}{\tau}\partial_\tau\left(\tau T^{\tau x}\right) + \partial_x\left(v_x T^{\tau x}\right) + \partial_y\left(v_y T^{\tau x}\right) = \mathcal{S}^{\tau x}[v_x, v_y, \pi^{xx}, \pi^{xy}, \pi^{\tau x}]$$

$$\frac{1}{\tau}\partial_\tau\left(\tau T^{\tau y}\right) + \partial_x\left(v_x T^{\tau y}\right) + \partial_y\left(v_y T^{\tau y}\right) = \mathcal{S}^{\tau y}[v_x, v_y, \pi^{yy}, \pi^{xy}, \pi^{\tau y}]$$

- kinetic relaxation equations for $\pi^{\tau\tau}$, $\pi^{\tau x}$, $\pi^{\tau y}$, and $\pi^{\eta\eta}$ (4, not 3!).

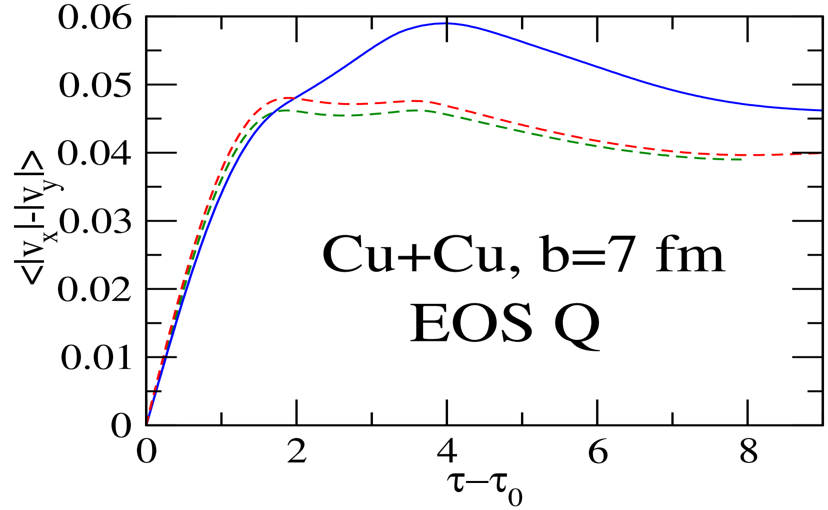
Close equations with EOS $p(e)$ where $e = M_0 - v_\perp M$ and $v_\perp = M/(M_0 + p(e))$ (again one implicit scalar equation!), with the definitions $(M_0, M_x, M_y) \equiv (T^{\tau\tau} - \pi^{\tau\tau},\ T^{\tau x} - \pi^{\tau x},\ T^{\tau y} - \pi^{\tau y})$ and $M = \sqrt{M_x^2 + M_y^2}$, and the relations $v_x = M_x/M$, $v_y = M_y/M$.

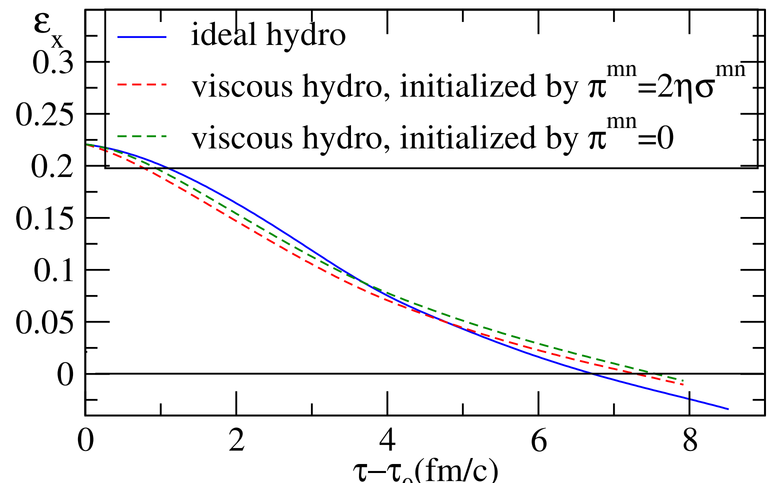
Sensitivity to initial values for viscous pressure tensor

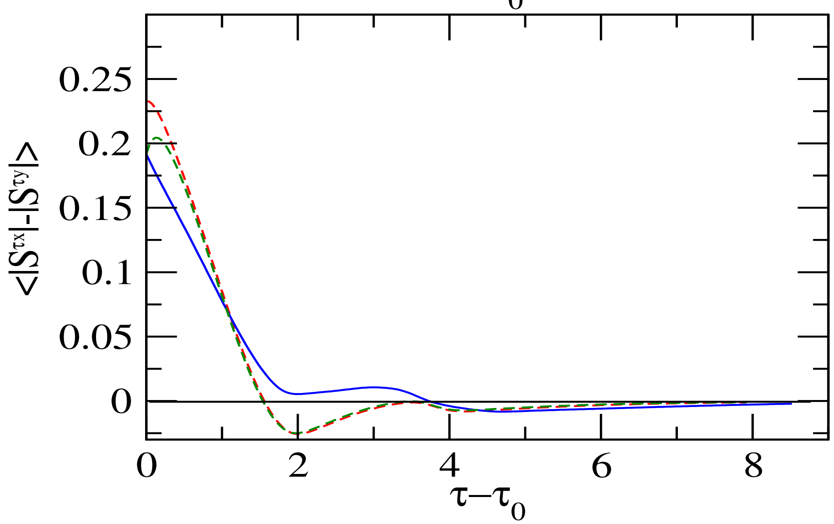
Romatschke & Romatschke 2007 seem to find much smaller viscous effects than we do. But they initialize their evolution with $\pi^{mn} = 0$. Could this be the origin of the discrepancy? **No!**

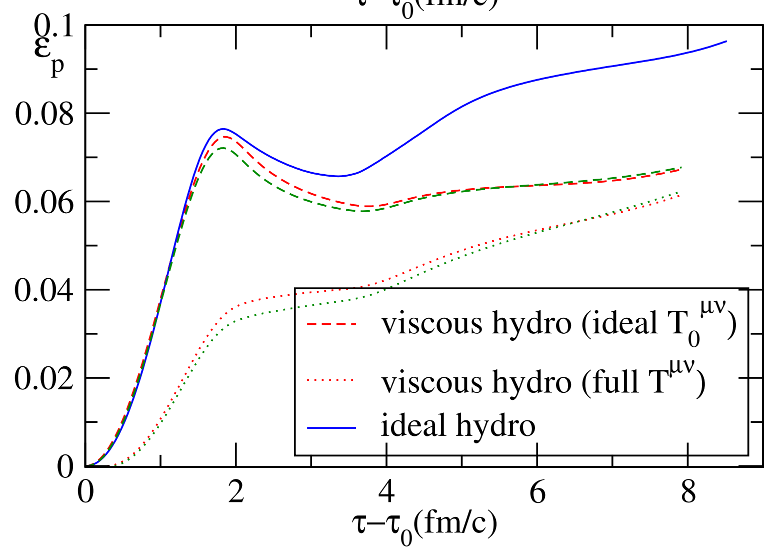
flow anisotropy vs. time

spatial eccentricity and momentum anisotropy





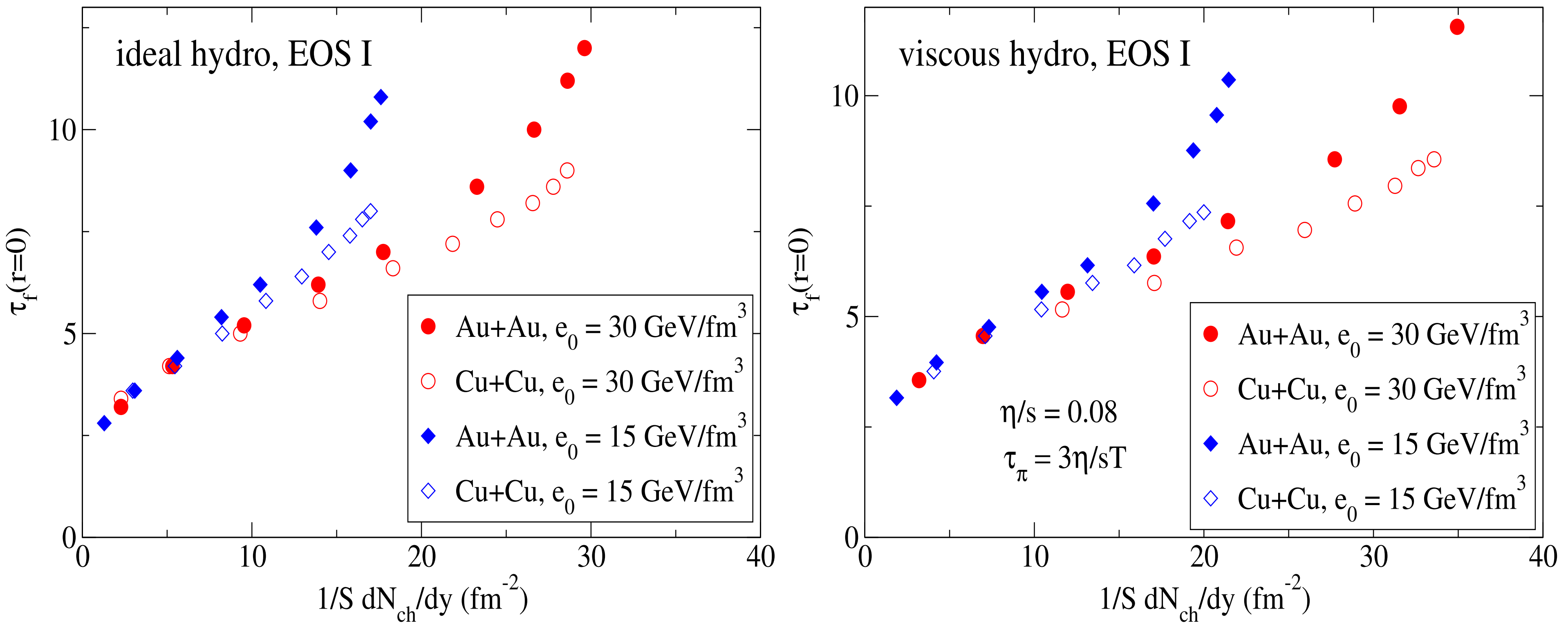




Green lines show results for $\pi_0^{mn} = 0$, with otherwise identical parameters

$\Longrightarrow$ weak sensitivity to initial conditions for viscous pressure tensor.

Central freeze-out times for different collisions systems and centralities



- At the same $\frac{1}{S}\frac{dN_{\rm ch}}{dy}$, collisions between larger nuclei and more central collisions take longer to freeze out

Comparison between VISH2+1 and Romatschkes' code

Evolution of total momentum anisotropy ϵ'_p, Au+Au with EOS I