3D Hydro + UrQMD Model

with

QCD Critical Point

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February 8, 2008 @QM2008, Jaipur, India
The QCD Critical Point in HIC

The QCD critical point search from phenomenology


3D Hydro + UrQMD
Realistic dynamical model for Heavy Ion Collisions
Relativistic Heavy Ion Collision

- Schematic sketch

Collision → Thermalization → Hydrodynamical Expansion → Hadronization → Freeze-out

Full 3-d Hydrodynamics

EoS: 1st order phase transition
QGP + excluded volume model

Hadronization

Cooper-Frye formula
Monte Carlo

UrQMD

Final state interactions

$T_C$: critical temperature
$T_{SW}$: Hydro $\rightarrow$ UrQMD

Nonaka and Bass PRC75:014902(2007)
Highlight of 3D Hydro+UrQMD

Nonaka and Bass PRC75:014902(2007)
Realistic Equation of States

- **3D Hydro + UrQMD**
  - Full 3-d Hydrodynamics
    - EoS: 1st order phase transition
    - QGP + excluded volume model
    - Cooper-Frye formula
    - Cooper-Frye formula
    - Monte Carlo
  - Hadronization
  - Final state interactions
  - UrQMD
  - \(T_C\): critical temperature
  - \(T_{SW}\): Hydro → UrQMD
  - Initial conditions
    - Parametrization
  - Equation of states
    - Bag model
  - Freezeout process
    - Viscosity effect of hadron phase
    - Final state interactions

Realistic EOS with QCD critical point

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**EOS with QCD Critical Point**

Nonaka and Asakawa, PRC71,044904(2005)

**Singular part near QCD critical point + Non-singular part**

- Non-singular part
  - QGP phase and hadron phase
- Singular part

**3d Ising Model** ➔ **QCD**

\[ r = \frac{T - T_c}{T_c} \]

\[ h: \text{extermal magnetic field} \]

**Same Universality Class**

\[ (r, h) \leftrightarrow (T, \mu_B) \]

- Mapping \((r, h) \rightarrow (T, \mu_B)\)
- Matching with known QGP and hadronic entropy density
- Thermodynamical quantities

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Equation of State

\[ T_E = 154.7 \, [\text{MeV}], \quad \mu_E = 367.7 \, [\text{MeV}] \]

QCD critical point

entropy density

baryon number density
Isentropic Trajectories

Isentropic trajectories: $n_B/s =$ const. line

$T_E = 154.7 \text{ MeV}, \mu_E = 367.7 \text{ MeV}$

$T_E = 143.7 \text{ MeV}, \mu_E = 652.0 \text{ MeV}$

- Behavior of focusing depends on the location of QCP on $T$-$\mu_B$ plane.
- The focusing effect appears stronger in the case $(T, \mu_B) = (143, 652)$.
- The location of QCP is a parameter.

Experiments
Comparison with Bag Model

- Isentropic trajectories on $T$-$\mu_B$ plane

With QCD critical point

Bag Model + Excluded Volume Approximation (No Critical Point)

= Usual Hydro Calculation

Focused

Not Focused
Initial Conditions

- Energy density
  \[ \varepsilon(x,y,\eta) = \varepsilon_{\text{max}} W(x,y;b) H(\eta) \]

- Baryon number density
  \[ n_B(x,y,\eta) = n_{B\text{max}} W(x,y;b) H(\eta) \]

- Parameters
  \[ \begin{align*}
  \tau_0 &= 0.6 \text{ fm/c} \\
  \eta_0 &= 0.5 \quad \sigma_\eta = 1.5
  \end{align*} \]

- Flow
  \[ \nu_L = \eta \text{ (Bjorken’s solution)}; \nu_T = 0 \]

**EOS:** QCP, Bag Model

**Switching temperature**

\[ T_{SW} = 150 \text{ [MeV]} \]
Isentropic Trajectories in Hydro

- $T$ and $\mu_B$ in one volume element close to center

QCP: $T_E = 143.7$ MeV, $\mu_E = 652.0$ MeV

Hydro with QCD critical point

Hydro with Bag Model

- $T_f = 110$ MeV
- Behavior of isentropic trajectories in hydro with QCP is different from one in hydro with bag model.
- Focusing effect appears in hydro with QCD critical point.
$P_T$ Spectra in 3D Hydro

- $T_f = 110$ MeV
- $P_T$ slope is almost the same.
3D Hydro + UrQMD

Hydro with QCD critical point

Hydro with Bag Model

Switching temperature: 150 MeV
\( P_T \) Spectra

- \( P_T \) slope is almost the same.
- Larger difference of different initial conditions appears in the case of QCP.
Hadron Ratios

QCD critical point

Because of focusing effect

At $T_{SW}$ $\langle \mu_B \rangle_{QCP} > \langle \mu_B \rangle_{BG} \Rightarrow \frac{p}{\pi_{QCP}} > \frac{p}{\pi_{BG}}$
Summary

- 3D Hydro + UrQMD Model with the QCD critical point
  - Isentropic trajectories
  - $P_T$ spectra, hadron ratio

- The QCD critical point search
  - Energy scan
  - Parameter sets of $\varepsilon$ and $n_B$ in initial conditions
  - Switching temperature dependence
  - Location of QCD critical point

- Physical observables
  - Fluctuations
  - Balance function
BACKUP
Mean $P_T$ Fluctuation

$P_T < 1$ GeV

$\pi$  $p$  $K$
1  $0.567E-02$  $0.266E-01$  $0.156E-01$
2  $0.416E-02$  $0.242E-01$  $0.115E-01$

*In this calculation, definition of mean Pt fluctuation is different from CERES.*
Consequences

- Slowing out of equilibrium
- Large fluctuation
- Freeze out temperature at RHIC
- Fluctuation

Existence and location of CEP in phase diagram: Collision energy dependent experiments are indispensable.

Work in Progress

- Realistic hydro calculation with critical point
Sound Velocity

EoS    Hydrodynamic Expansion

\[ C_s^2 = \frac{\partial P}{\partial \varepsilon} \bigg|_{n_B/s} \]

\[ C_s^2 = \frac{\partial P}{\partial \varepsilon} \bigg|_{n_B/s} \]

Ex. Rischke et al. nucl-th/9504021

Effect of mixed phase?
Shear Viscosity

\[ \frac{\eta}{\eta_0} = \left( \frac{\xi_{eq}}{\xi_{eq,0}} \right) \left[ \frac{1}{19} \xi + O(\xi^2) \right] \]
Multi-parameter reweighting technique

- Fodor and Katz (JHEP 0203 (2002) 014)
  hep-lat/0402006
- Allton et al. (Bielefeld-Swansea)

Overlap problem:
The lattice size is small.

Maezawa san’s talk
Phenomenological Consequence?

Divergence of Fluctuation
Correlation Length

Still we need to study
- Hadronic Observables: NOT directly reflect properties at CEP
  - Fluctuation, Collective Flow
- EOS
  - Focusing in $n_B/s$ trajectories
- Dynamics (Time Evolution)

M. Stephanov, K. Rajagopal, and E. Shuryak, PRL81 (1998) 4816

If expansion is adiabatic.
Time Evolution

- Berdnikov and Rajagopal’s Schematic Argument

B. Berdnikov and K. Rajagopal,

- Correlation Length longer than \( L_{eq} \)

- Realistic Hydro Calculation with Realistic EOS

What’s missing:

- Realistic Hydro Calculation with Realistic EOS
3-d Hydrodynamic Model

Hydrodynamic equation

\[ T^{\mu \nu} : T^{\mu \nu} = \epsilon u^{\mu} u^{\nu} - p (g^{\mu \nu} - u^{\mu} u^{\nu}) \]

- Baryon number density conservation

Coordinates

\[ x^{\mu} = (t, \vec{x}) \]

Lagrangian hydrodynamics

- Tracing the adiabatic path of each volume element
- Effects of phase transition of observables

Algorithm

- Focusing on conservation law
Trajectories on the phase diagram

Lagrangian hydrodynamics

Effect of phase transition

Temperature and chemical potential of volume element of fluid

Jets in medium

Jet quenching mechanisms

Ex. Nuclear modification factor in 3D hydro

- parton path
- pQCD
- Mixed and hadron phase
- AMY
- gluon bremsstrahlung

nucl-th/0611027 with Renk and Ruppert
nucl-th/0703019 with Majumder
arXiv:0705.2575(hep-ph) with Qin, Ruppert, Turbide Gale

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**Soft + Hard**

**Soft**
- Full 3-d Hydrodynamic Model
- QGP formation, EoS

**Hard**
- Hard scattering & jet production
- Propagation of jet in medium, energy loss

**Interaction between Soft and Hard**

**Hasronization**

**Final Interactions**

- Improved Cooper-Frye formula (Reco)
- UrQMD

- First schematic attempt
  Hirano & Nara
  PRC66:041901,2002,
  PRL91:082301,2003

- Dynamical effect on jets
- Jet correlations

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Non-Singular Part

**Hadron Phase**
- Excluded volume model
  \[ P(T, \mu_B) = \sum_i P_i^{\text{ideal}}(T, \mu_{B_i} - V_0 P(T, \mu_{B_i})) = \sum_i P_i^{\text{ideal}}(T, \tilde{\mu}_{B_i}) \]

**QGP Phase**
\[ P(T, \mu_B) = \frac{(32 + 21N_f)\pi^2}{180} T^4 + \frac{N_f}{2} \left( \frac{\mu_B}{3} \right) T^2 + \frac{N_f}{4\pi^2} \left( \frac{\mu_B}{3} \right)^4 - B \]

- \( N_f=2 \)
- \( B: \) Bag constant \((220 \text{ MeV})^4\)
Isentropic Trajectories in Hydro

- Conservation law in ideal fluid

\[ \n \frac{\partial}{\partial t} \left( \n \frac{n_{B}}{s} \right) + \n \frac{\partial}{\partial s} \left( \n \frac{n_{B}}{s} \right) = 0 \]

- Hydrodynamic expansion: \( n_{B}/s = \text{const.} \)

- \( n_{B}/s = \text{const.} \) lines on \( T-\mu_{B} \) plane: behavior of expansion
Trajectories on the phase diagram

Lagrangian hydrodynamics

- Temperature and chemical potential of volume element of fluid
- Effect of phase transition

Hadron ratios near CEP

- Chemical Freeze-out

• Hadron ratio (ex. $p/\pi$) is not sensitive to collision energy near CEP.

• Near chemical freeze-out temperature the contribution from recombination, resonances is small.

Heinz : hep-ph/0109006

- Statistical Model
  Free resonance gas model

- At chemical freeze-out
  Quasi-particle state
EOS of 3-d Ising Model

Parametric Representation of EOS

\[
M = M_0 R^\beta \theta \\
h = h_0 R^\beta \tilde{h}(\theta) = h R^\beta \theta^0 (\theta - 0.76201 \theta^3 + 0.00804 \theta^5) \\
r = R(1 - \theta^2) \quad (R \geq 0, \ -1.154 \leq \theta \leq 1.154)
\]

\[
r = \frac{T - T_c}{T_c} \\
h : \text{external magnetic field}
\]

\[
\beta = 0.326 \\
\delta = 4.8
\]

Guida and Zinn-Justin NPB486(97)626

Mapping QCD

Order parameter

\[
M \leftrightarrow \langle \bar{q} q \rangle
\]

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**Singular Part of EOS**

### Gibbs Free Energy

\[ G(h,r) = F(M,r) - Mh \]

Free energy: \( F(M,r) = h_0 M_0 R^{2-\alpha} g(\theta) \)

Guida and Zinn-Justin NPB486(97)626

### Entropy Density for Singular Part

\[
S_c = -\left. \frac{\partial G}{\partial T} \right|_\mu = -\left. \frac{\partial G}{\partial h} \right|_r \frac{\partial h}{\partial T} - \left. \frac{\partial G}{\partial r} \right|_h \frac{\partial r}{\partial T}
\]

\[
\left\{ \begin{align*}
\frac{\partial G}{\partial h} \bigg|_r &= -M \\
\frac{\partial G}{\partial r} \bigg|_h &= \left. \frac{\partial F}{\partial r} \right|_h - \left. \frac{\partial M}{\partial r} \right|_h
\end{align*} \right.
\]

\( (r,h) \leftrightarrow (T,\mu_B) \)

\( \Delta \mu_{B_{\text{crit}}} \)

\( \Delta T_{\text{crit}} \)

Critical region

\( \text{CEP} \)

\( \text{mapping} \)}
Singular Part + Non-singular Part

**Entropy Density**

\[
S_{\text{real}}(T, \mu_B) = \frac{1}{2} \{ 1 - \tanh[S_c(T, \mu_B)] \} S_H(T, \mu_B) + \frac{1}{2} \{ 1 + \tanh[S_c(T, \mu_B)] \} S_Q(T, \mu_B)
\]

- \( S_H(T, \mu_B) \) Hadron Phase (excluded volume model)
- \( S_Q(T, \mu_B) \) QGP phase

**Dimensionless parameter:** \( S_c \)

\[
S_c(T, \mu_B) = s_c \sqrt{ (\Delta T_{\text{crit}})^2 + (\Delta \mu_{\text{crit}})^2 } \times D
\]

Critical domain

- **Choice of parameters:** \( \Delta T_{\text{crit}}, \Delta \mu_{\text{crit}}, D \)

Thermodynamical inequalities

\[
\left( \frac{\partial S}{\partial T} \right)_{V,N} \geq 0, \quad \left( \frac{\partial P}{\partial V} \right)_{T,N} \geq 0, \quad \left( \frac{\partial \mu}{\partial N} \right)_{T,V} \geq 0
\]
Thermodynamical Quantities

\[ S_{\text{real}}(T, \mu_B) = \frac{1}{2} \{1 - \tanh[S_c(T, \mu_B)]\} S_H(T, \mu_B) + \frac{1}{2} \{1 + \tanh[S_c(T, \mu_B)]\} S_Q(T, \mu_B) \]

- **Baryon number density**
  \[ n_B(T, \mu_B) = \frac{\partial P}{\partial \mu_B} = \int_0^T \frac{\partial S(T', \mu_B)}{\partial \mu_B} dT' + n_B(0, \mu_B) \]
  \[ + \left| \frac{\partial T_C(\mu_B)}{\partial \mu_C}(S_Q(T_C, \mu_B(T_C)) - S_H(T_C, \mu_B(T_C))) \right| \]
  1st order

- **Pressure**
  \[ P(T, \mu_B) = \int_0^T S_{\text{real}}(T', \mu_B) dT' + P(0, \mu_B) \]

- **Energy Density**
  \[ \varepsilon = TS_{\text{real}} - P - \mu_B n_B \]
Dimensionless Parameter

\[
S_c = - \left. \frac{\partial G}{\partial T} \right|_\mu = - \left. \frac{\partial G}{\partial h} \right|_r \left( \frac{\partial h}{\partial T} \right)_r - \left. \frac{\partial G}{\partial r} \right|_h \left( \frac{\partial r}{\partial T} \right)_h
\]

\[
S_c(T, \mu_B) = s_c \sqrt{\left( \Delta T_{\text{crit}} \right)^2 + \left( \Delta \mu_{\text{crit}} \right)^2} \times D
\]

Phase transition region
If the critical region is large enough

The fine-tuning of the collision energy is not necessary not only on the high energy side but also on the low energy side.
Focusing

- $n_B/s$ trajectories on $T-\mu$ plane \iff Hydrosdyamic expansion

Scavenius et al. PRC64(2001)045202

entropy, baryon number conservation

- CEP is not an attractor for $n_B/s$ trajectories.
- Critical phenomena near CEP?
Sound Velocity

EoS       Hydrodynamic Expansion

\[ C_s^2 = \left. \frac{\partial P}{\partial \varepsilon} \right|_{n_B/s} \]

\[ C_s^2 = \left. \frac{\partial P}{\partial \varepsilon} \right|_{n_B/s} \]

hadron   Mixed   QGP

\[ \varepsilon_H \quad \varepsilon_Q \quad \varepsilon \]

Ex. Rischke et al. nucl-th/9504021

Effect of mixed phase?
Sound Velocity in EoS with CEP

Effect on Time Evolution
Collective flow ↔ EOS

\[ C_s^2 = \frac{\partial P}{\partial \varepsilon}_{n_B/S} \]

- The system may not expand uniformly.

Collective flow & HBT

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Correlation length longer than $\xi_{eq}$ along $r = \text{const.}$ line

Effect of Focusing on correlation length?

Time evolution: Bjorken’s scaling solution along $n_B/s$

$\tau_0 = 1 \text{ fm, } T_0 = 200 \text{ MeV}$
Correlation Length in Equilibrium

\[ \xi_{\text{eq}}(r, M) = f^2 M^{-2v/\beta} g \left( \frac{|r|}{|M|^{1/\beta}} \right) \]

- Max. \( \xi_{\text{eq}} \) depends on \( n_B/s \).
- Trajectories pass through the region where \( \xi_{\text{eq}} \) is large. (focusing)
- \( n_B/s \) — non-critical component of the EOS
Evolution of Correlation Length

- $\xi$ : time evolution (1-d)
  \[
  \frac{d}{d\tau} m_\sigma(\tau) = -\Gamma[m_\sigma(\tau)] \left( m_\sigma(\tau) - \frac{1}{\xi_{eq}(\tau)} \right)
  \]
  \[
  \Gamma(m_\sigma) = \frac{a}{\xi_0} (m_\sigma \xi_0)^z, \quad m_\sigma(\tau) = \frac{1}{\xi(\tau)}
  \]
  
  $z = 3.0$  
  Model H (Halperin RMP49(77)435)

- $\xi$ is larger than $\xi_{eq}$ at $T_f$.  
- Critical slowing down
- Differences among $\xi_s$ on $n_{\bar{p}/s}$ are small.
- In 3-d, the difference between $\xi_{eq}$ and $\xi$ becomes large due to transverse expansion.
The cube of the equilibrium correlation length indicates that there is the possibility that the fluctuation shows some enhancement.
Fluctuations (I)

Fluctuations
CERES
40, 80, 158 AGeV Pb+Au collisions
Mean $P_T$ Fluctuation

\[
\sigma_{P_T,dyb}^2 \equiv \langle \Delta M_{P_T}^2 \rangle - \frac{\Delta P_T^2}{\langle N \rangle}
\]

\[
\langle M_{P_T}^2 \rangle = \frac{\sum_{j=1}^{N} (M_j^2 - \langle M_j \rangle)^2}{\sum_{j=1}^{N} N_j}
\]

\[
\Sigma_{P_T} \equiv \text{sgn}(\sigma_{P_T,dyb}^2) \sqrt{\sigma_{dyb}^2}
\]

No unusually large fluctuation or non-monotonic behavior

\[
\Sigma_{P_T} (\%) = 2.5 \times 10^{-5} s_{NN}^{1/2} (\text{GeV})
\]

CEP: attractor of isentropic trajectories
Similar correlation length and fluctuation is observed near CEP.

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Fluctuations (II)

NA49

- Suggestion of QCD critical end point?

- Fluctuation → Conservation
Kinetic Freeze-out Temperature

Low \( T_f \) comes from large flow.

- Kinetic Freeze-out mean (elastic) collision rate expansion rate
- \( \pi^- - N \) cross section
  \( \frac{(p + \bar{p})}{(\pi^+ + \pi^-)} \)
  - SPS \( \sim 0.09 \)
  - RHIC \( \sim 0.09 \)
- Expansion rate
  - RHIC > SPS
- Collision rate
  - SPS \( \sim \) RHIC

Xu and Kaneta, nucl-ex/0104021(QM2001)
BRAHMS nucl-ex/0404011
Au+Au Collisions

\[
\frac{\eta}{s} \sim T \lambda_f c_s
\]

\[
\begin{align*}
T &= 165 \pm 3 \text{ MeV} \\
\lambda_f &= 0.3 \pm 0.03 \text{ fm} \\
c_s &= 0.35 \pm 0.05
\end{align*}
\]

\[
\frac{\eta}{s} \sim 0.09 \pm 0.015
\]

Where is CEP on T-\(\mu_B\) plane?

- NJL/I, NJL/II
  Asakawa & Yazaki
- CO (composite operator)
  Barducci et al.
- NJL/inst (instanton NJL)
  Berges & Rajagopal
- RM (random matrix)
  Halaz et al.
- LSM (linear sigma model)
  Scavenius et al.
- CJT (effective potential)
  Hatta & Ikeda
- HB (hadronic bootstrap)
  Antoniou et al.

Stephanov:hep-ph/0402115

\[ T \]

\[ \mu_B \]
Where is the Critical Point?

Energy Scan:
- Necessity of lower energy experiments at RHIC
- SPS

Landmark study. Physicists have seen a smooth transition from bound quarks to quark-gluon plasma (dotted line). They now hope to find the point beyond which the transition becomes violent (white line).