Equation of State and more from lattice regularized QCD

Frithjof Karsch, BNL& Bielefeld University

- Bulk thermodynamics

  cut-off effects in QCD thermodynamics
  the equation of state and velocity of sound at $\mu_q = 0$

- Characterizing the QCD transition

  deconfinement and chiral symmetry restoration

- Conclusions

RBC-Bielefeld and hotQCD collaborations

- RIKEN-BNL-Columbia (RBC) - Bielefeld:
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- hotQCD:
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Bulk thermodynamics

Goal: QCD thermodynamics with realistic quark masses in (2+1)-f QCD and controlled extrapolation to the continuum limit;

$\Rightarrow$ T_c, EoS,.. for $\mu_q \geq 0$

- $N_\tau = 4,\ 6$: bulk thermodynamics on a line of constant physics (LCP):

  (i) use $m_l = 0.1 m_s$, corresponding to $m_\pi \simeq 220$ MeV;

  (ii) tune m_s to physical strange quark mass using $m_K,\ m_{\bar{s}s}$ at all values of the cut-off

RBC-Bielefeld collaboration

- analyze EoS in a wide T-range: $140\ \mathrm{MeV} \lesssim T \lesssim 800\ \mathrm{MeV}$

- extend analysis to $N_\tau = 8$;
  compare p4 and asqtad results:
  joint project of RBC, Bielefeld, MILC, LANL and LLNL
  $\Rightarrow$ hotQCD collaboration

see poster by R.Gupta/R.Soltz

EoS with $\mathcal{O}(a^2)$ improved SF

Goal: QCD thermodynamics with realistic quark masses in (2+1)-f QCD and controlled extrapolation to the continuum limit;

$\Rightarrow$ T_c, EoS,.. for $\mu_q \geq 0$

$\mu_q > 0$: Christian Schmidt
session X @ 14:40 today

- use an improved staggered fermion action that removes $\mathcal{O}(a^2)$ errors in bulk thermodynamic quantities and reduces flavor symmetry breaking inherent to the staggered formulation

  RBC-Bielefeld choice: p4-action + 3-link smearing (p4fat3)

  M. Cheng et al., PRD77, 014511 (2008)

  MILC: Naik-action + (3,5,7)-link smearing (asqtad)

  C. Bernard et al., PRD75, 094505 (2007)

- use RHMC algorithm to remove 'step-size errors'
- perform detailed $T = 0$ study of vacuum subtractions and scale setting for ALL $T > 0$ parameter sets

Cut-off effects and staggered fermions

the situation is more complex than in SU(3)

I) we have to deal with $\mathcal{O}(a^2)$ discretization errors;
just like in SU(3) but more severe!

$\mathcal{O}(a^2)$ improved actions for thermodynamics: Naik, p4

II) in addition we have to deal with $\mathcal{O}(a^2)$ violations of chiral symmetry

fat links in various variants, 3-link staple, 7-link staple (asqtad), stout,..

Cut-off effects with SF

the ideal gas (infinite temperature) limit (I):

- standard staggered fermions lead to $\mathcal{O}(a^2)$ errors in bulk thermodynamics
- p4-action and Naik action remove $\mathcal{O}(a^2)$ errors in bulk thermodynamics

  $\Rightarrow$ $\mathcal{O}(a^2)$ improved pressure;

  $\Rightarrow$ small higher order corrections

see poster by Prasad Hegde

arXiv:0801.4883

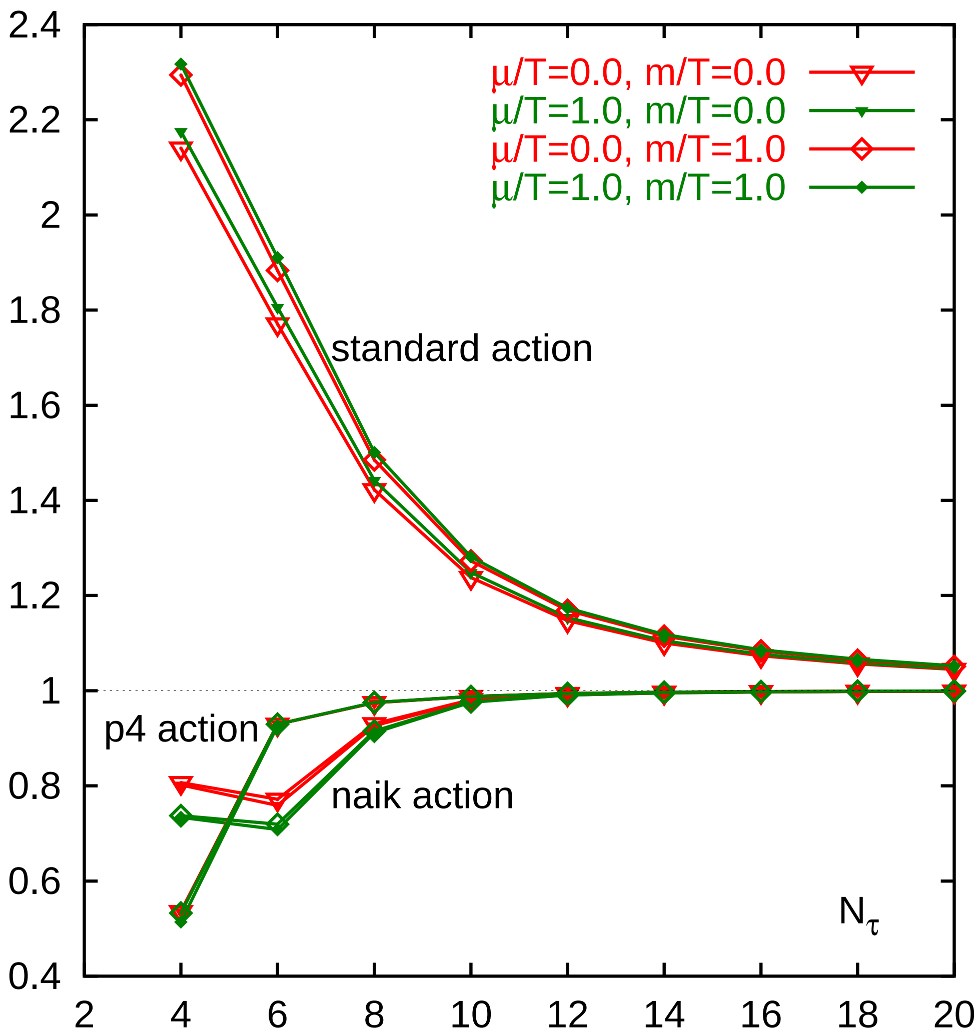
$$\frac{p}{p_{SB}} = 1 + \frac{248}{147}\left(\frac{\pi}{N_\tau}\right)^2 + \frac{635}{147}\left(\frac{\pi}{N_\tau}\right)^4 + \ldots \quad (standard)$$

$$\frac{p}{p_{SB}} = 1 + 0 - \frac{1143}{980}\left(\frac{\pi}{N_\tau}\right)^4 + \frac{73}{2079}\left(\frac{\pi}{N_\tau}\right)^6 + \ldots \quad (p4)$$

$$\frac{p}{p_{SB}} = 1 + 0 - \frac{1143}{980}\left(\frac{\pi}{N_\tau}\right)^4 - \frac{365}{77}\left(\frac{\pi}{N_\tau}\right)^6 + \ldots \quad (Naik)$$

Cut-off effects with SF

the ideal gas (infinite temperature) limit (II):



cut-off effects similar for

$\mu = 0$ and $\mu > 0$

$\Rightarrow$

quark number susceptibilities

$\chi_{q,s}/T^2 \sim \partial^2 \ln Z/\partial(\mu_{q,s}/T)^2$

show similar cut-off effects as

pressure $p/T^4 \sim \ln Z$

Calculating the EoS on lines of constant physics (LCP)

- The interaction measure for $N_f = 2+1$ $\Leftrightarrow$ Trace Anomaly

$$\frac{\epsilon - 3p}{T^4} = T\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{p}{T^4}\right) = \left(a\frac{\mathrm{d}\beta}{\mathrm{d}a}\right)_{LCP} \frac{\partial p/T^4}{\partial \beta}$$

$$= \left(\frac{\epsilon - 3p}{T^4}\right)_{gluon} + \left(\frac{\epsilon - 3p}{T^4}\right)_{fermion} + \left(\frac{\epsilon - 3p}{T^4}\right)_{\hat{m}_s/\hat{m}_l}$$

- The pressure

$$\left.\frac{p}{T^4}\right|_{\beta_0}^{\beta} = \int_0^T \mathrm{d}T\, \frac{1}{T}\left(\frac{\epsilon - 3p}{T^4}\right)$$

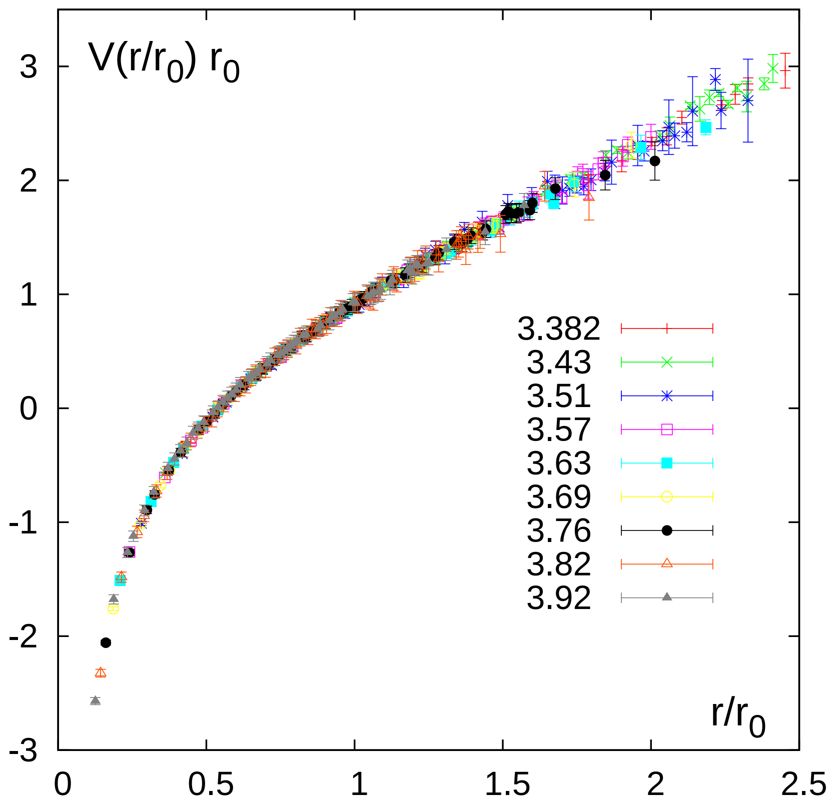
- need T-scale, $aT = 1/N_\tau$ and its relation to the gauge coupling $a \equiv a(\beta)$

N.B.: $a(\beta)$ is only defined through physical observables
$\Rightarrow$ choose a simple one

$T = 0$ scale setting using the heavy quark potential

use r_0 or string tension to set the scale for $T = 1/N_\tau a(\beta)$

$$V(r) = -\frac{\alpha}{r} + \sigma r \quad , \quad r^2 \frac{\mathrm{d}V(r)}{\mathrm{d}r}\bigg|_{r=r_0} = 1.65$$



no significant cut-off dependence when cut-off varies by a factor 5

i.e. from the transition region on $N_\tau = 4$ lattices ($a \simeq 0.25$ fm) to that on $N_\tau = 20$ lattices ($a \simeq 0.05$ fm) !!

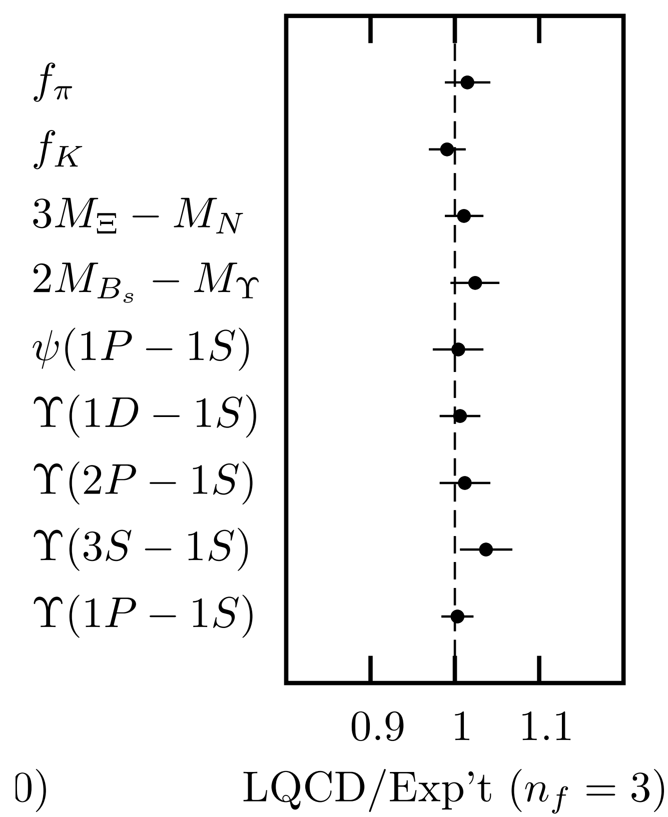
scales extracted from 'gold plated observables'

- high precision studies of several experimentally well known observables in lattice calculations with staggered (asqtad) fermions led to convincing agreement $\Rightarrow$ gold plated observables
- simultaneous determination of r_0/a in these calculations determines the scale r_0 in MeV

knowing any of these experimentally accessible quantities accurately from a lattice calculation is equivalent to knowing r_0, which is a fundamental parameter of QCD

C.T.H. Davies et al., PRL 92 (2004) 022001

A. Gray et al., PRD72 (2005) 094507



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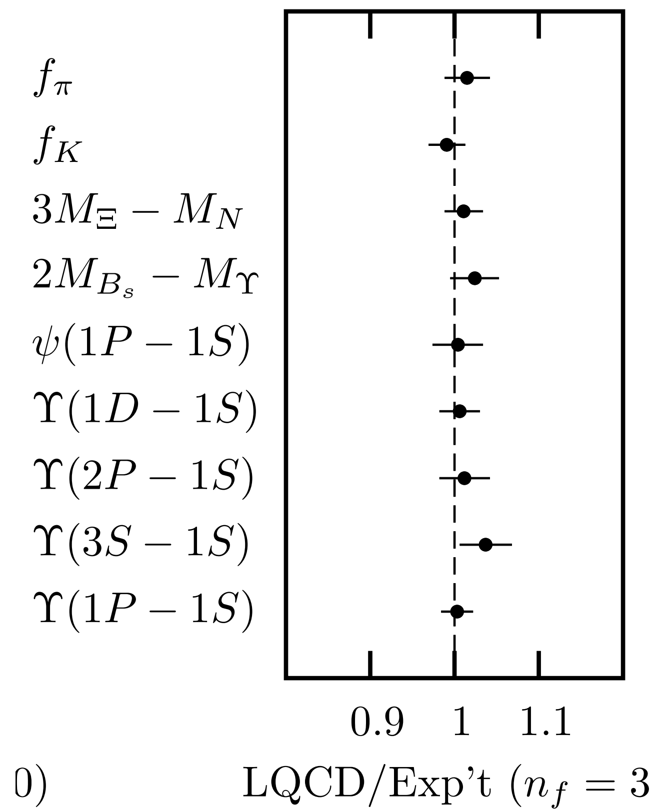
we use $r_0 = 0.469(7)$ fm determined from quarkonium spectroscopy
A. Gray et al, Phys. Rev. D72 (2005) 094507



C.T.H. Davies et al., PRL 92 (2004) 022001
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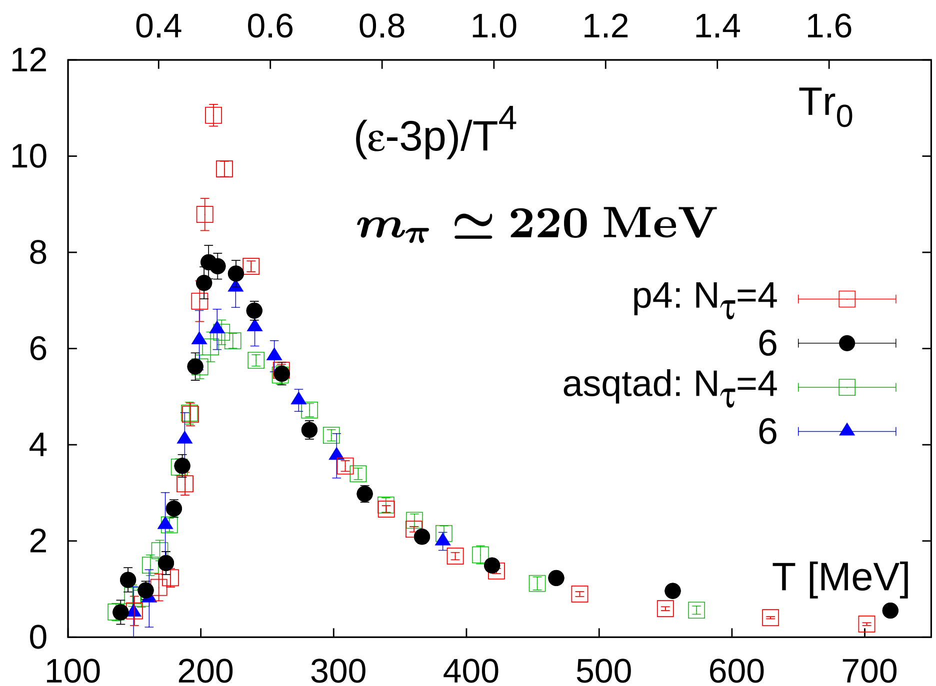
$(\epsilon - 3p)/T^4$ on LCP

- requires good control over $T > 0$ observables (action differences, chiral condensates); difficult: CPU requirement $\sim a^{-(10-12)}$
- requires accurate determination of $T = 0$ scales $\Leftrightarrow$ interplay with studies of hadron spectroscopy



p4 vs. asqtad: overall good agreement

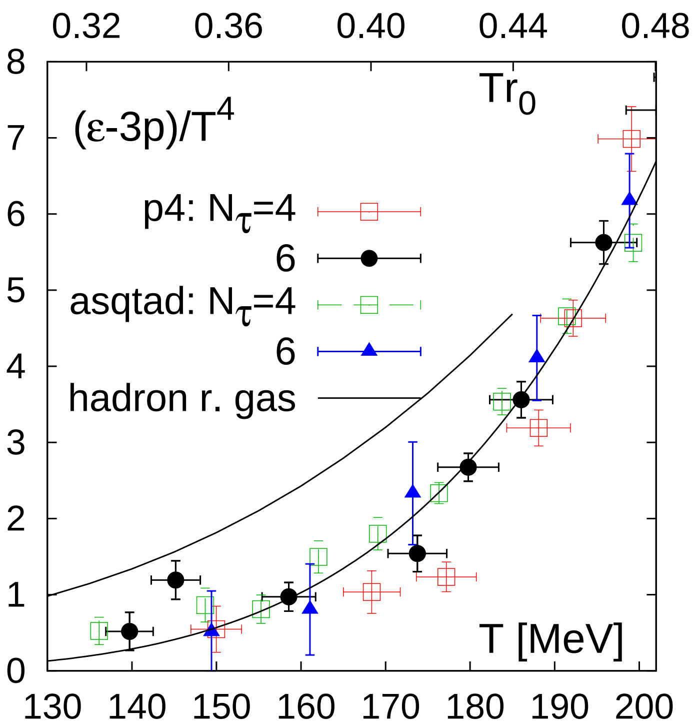
However: We still need to...

$T < T_c$	make contact to hadron gas phenomenology
$T < 2T_c$	analyze large deviation from conformal limit ($\epsilon = 3p$)
$T > 2T_c$	make contact to (resummed) perturbation theory

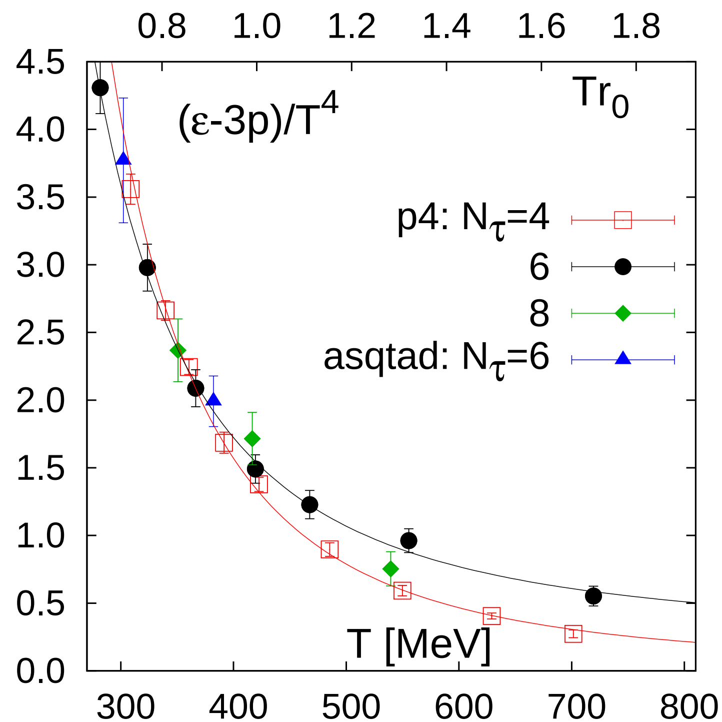
p4-data: RBC-Bielefeld
M. Cheng et al, PRD77, 014511 (2008)

asqtad data: MILC
C. Bernard et al., PRD 75, 094505 (2007)

EoS: low and high T regime



LGT vs resonance gas

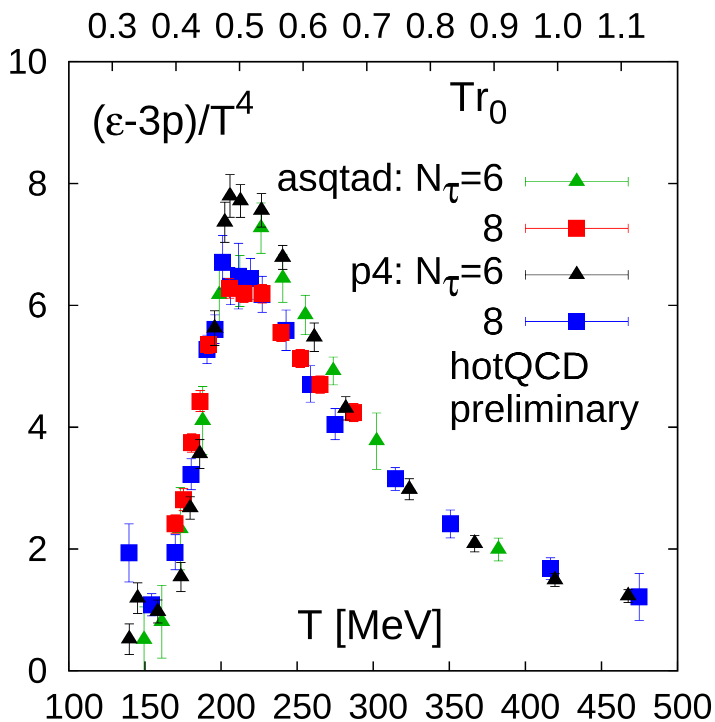


LGT vs. pert. theory

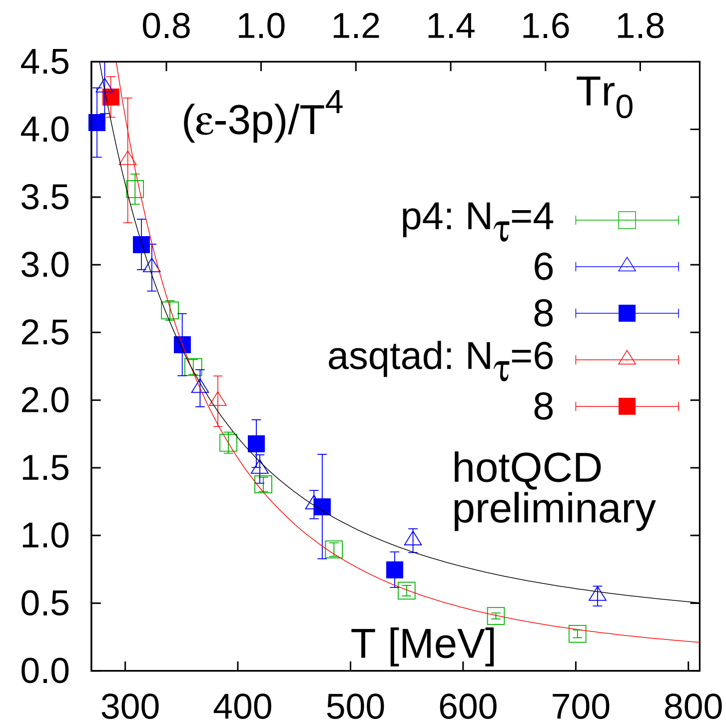
- approach to physical quark masses
  $\rightarrow\ \mathcal{O}(5MeV)$ shift of T-scale
- approach to continuum limit
  $\rightarrow\ N_\tau = 8, ..$
  hotQCD collaboration, ongoing
- strong deviations from conformal limit:
  find $(\epsilon - 3p)/T^4 \sim a/T^2 + b/T^4$ for
  $300\text{MeV} \lesssim T \lesssim 700\text{MeV}$
- need to control cut-off dependence for
  $T \gtrsim 400$ MeV to establish contact to
  (resummed) perturbative QCD

..towards the cont. limit: $N_\tau = 8$

hotQCD-collaboration PRELIMINARY



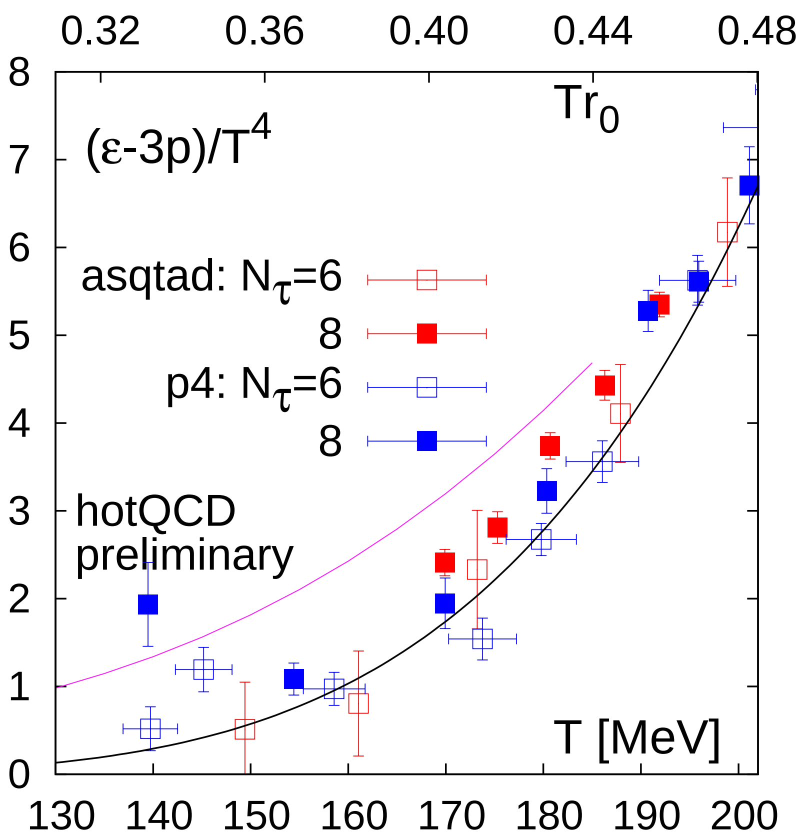
trace anomaly



high-T behavior

- $[(\epsilon - 3p)/T^4]_{max} \gtrsim 200$ MeV (∼ softest point of EoS)
- cut-off effects persist in the peak region
- high-T: good agreement between $N_\tau = 6$ and 8 results for $T \gtrsim 300$ MeV
- low-T: small shift of transition region; better agreement with HRG model

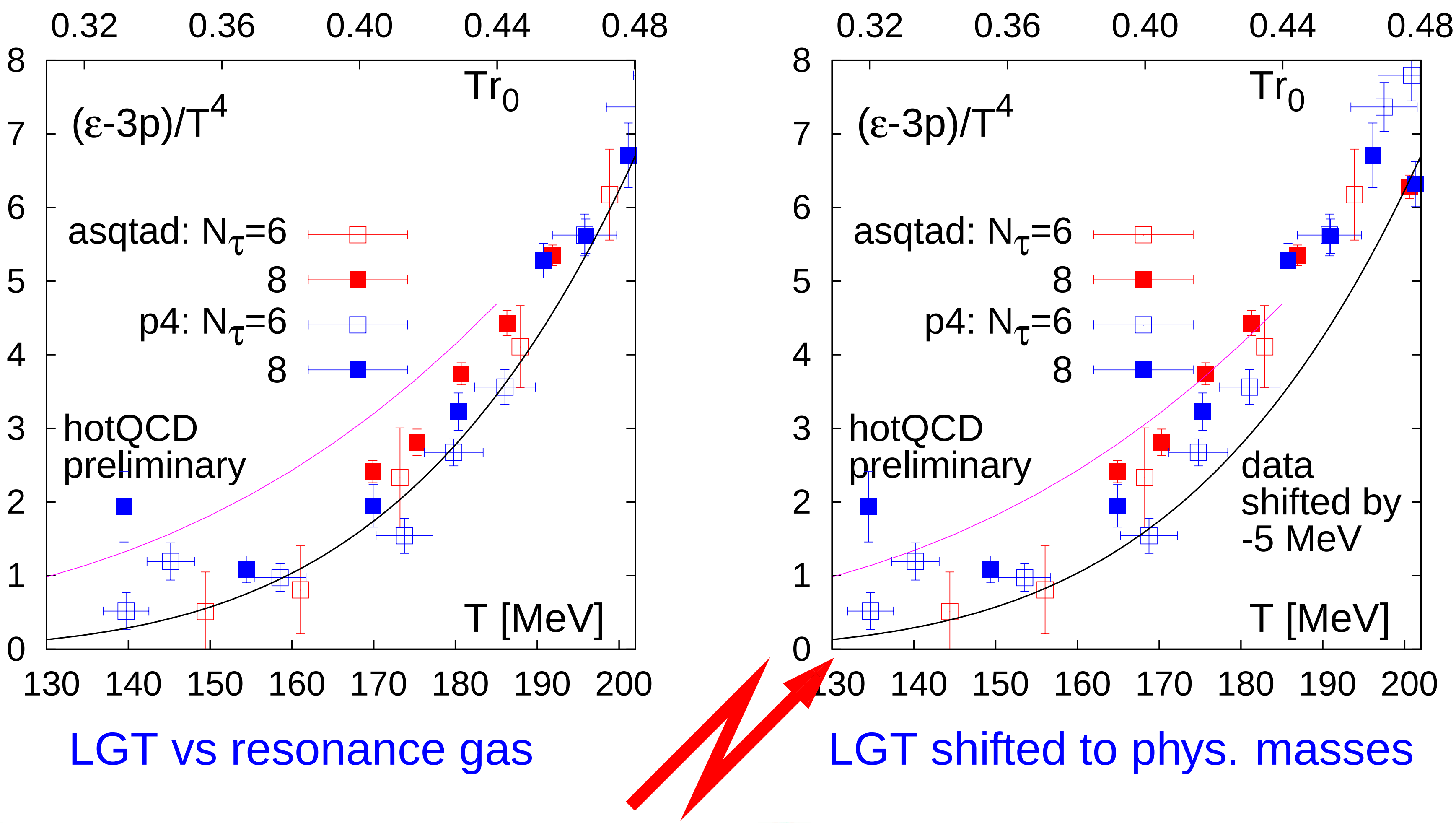
EoS: low T regime



LGT vs resonance gas

- approach to physical quark masses
  $\longrightarrow$ $\mathcal{O}(5MeV)$ shift of T-scale

EoS: low T regime



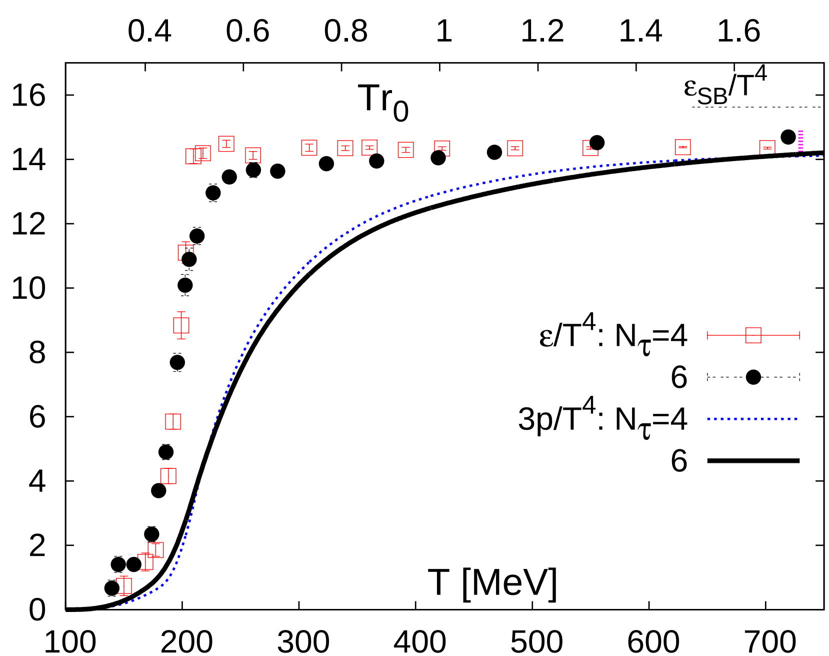
LGT vs resonance gas

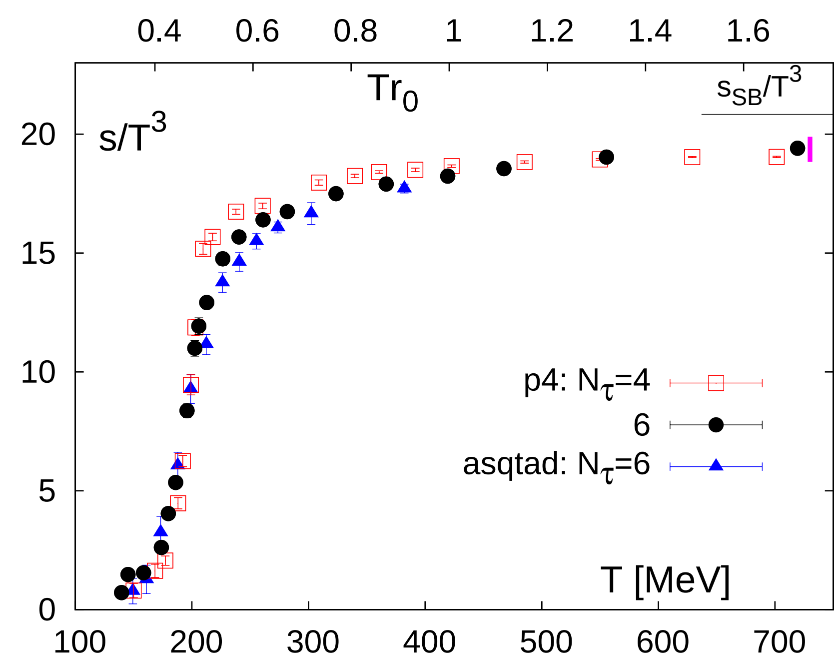
LGT shifted to phys. masses

- approach to physical quark masses
  $\rightarrow\ \mathcal{O}(5MeV)$ shift of T-scale
- good agreement with HRG model in the transition region

Pressure, Energy and Entropy

- p/T^4 from integration over $(\epsilon - 3p)/T^5$;
  systematic error arises from starting the integration at $T_0 = 100$ MeV with $p(T_0) = 0$;
  use hadron resonance gas to estimate systematic error: $[p(T_0)/T_0^4]_{HRG} \simeq 0.265$



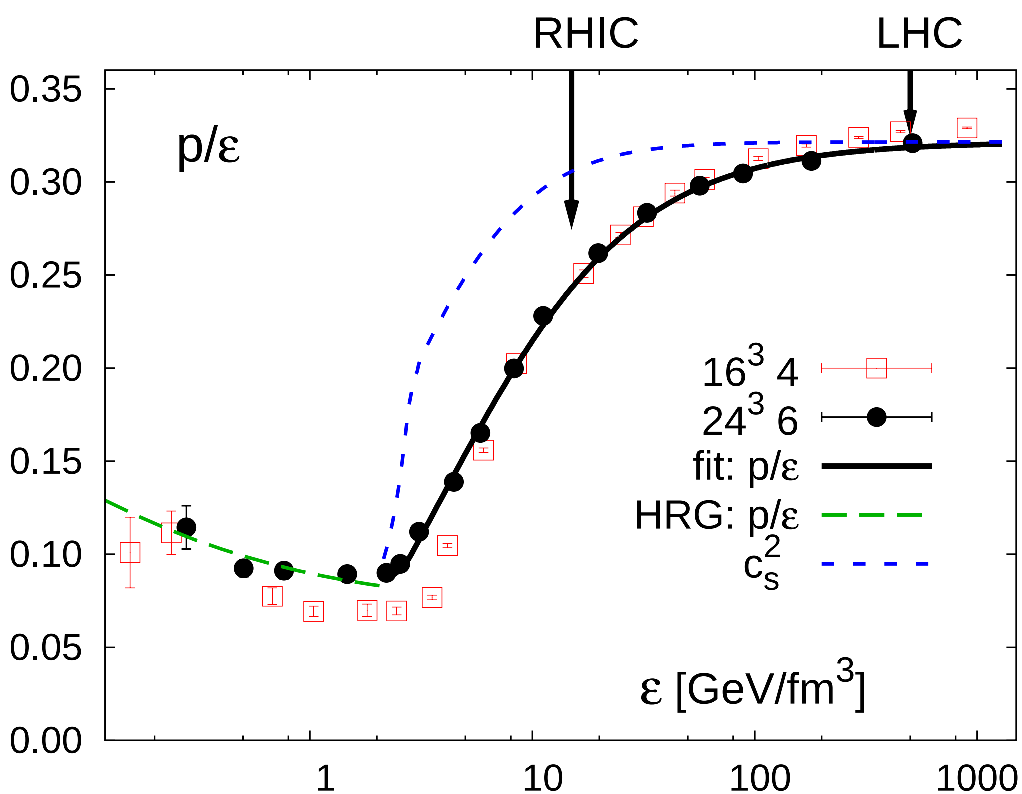


M. Cheng et al. (RBC-Bielefeld), PRD77, 014511 (2008)

EoS and velocity of sound

- $p/\epsilon \Rightarrow$ velocity of sound:

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\epsilon} = \epsilon \frac{\mathrm{d}(p/\epsilon)}{\mathrm{d}\epsilon} + \frac{p}{\epsilon} \equiv \frac{s}{c_V}$$



needs to be confirmed on larger lattices, closer to the continuum limit;

e.g. $N_\tau = 8$, ...

hotQCD-collaboration

M. Cheng et al. (RBC-Bielefeld), PRD77, 014511 (2008)

Deconfinement and χ-symmetry

- The chiral phase transition (i.e. at $m_q = 0$) is deconfining
  - true in QCD, i.e. SU(3) + fermions in the fundamental representation
  - SU(3) + fermions in the adjoint representation: $T_{deconf} < T_\chi$
- The transition in QCD with physical quark masses is a crossover

In which sense is the transition

deconfining and chiral symmetry restoring?

- deconfinement: heavy hadrons $\Rightarrow$ light quarks and gluons; liberation of many new light degrees of freedom
  $\Rightarrow$ rapid change in ϵ/T^4, s/T^3, ...
- chiral symmetry restoration: vanishing mass splittings, no new degrees of freedom
  $\Rightarrow$ minor effect on bulk thermodynamics, but
  rapid change of chiral condensate

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Deconfinement: Polyakov loop and quark number susceptibility

- rapid rise in the Polyakov loop reflects change in the free energy of static quark sources; screening of the heavy quark potential
- rapid change in quark/baryon/strangeness number susceptibility reflects change in mass of the carrier of these quantum numbers

  $\Leftrightarrow$ DECONFINEMENT
- quark number susceptibilities $\Leftrightarrow$ coefficients of the leading order correction to the pressure

$$\left(\frac{p}{T^4}\right)_\mu - \left(\frac{p}{T^4}\right)_0 = \frac{1}{2}\frac{\chi_l}{T^2}\left(\frac{\mu_l}{T}\right)^2 + \frac{1}{2}\frac{\chi_s}{T^2}\left(\frac{\mu_s}{T}\right)^2 + \frac{\chi_{ls}}{T^2}\frac{\mu_l}{T}\frac{\mu_s}{T} + \mathcal{O}(\mu^4)$$

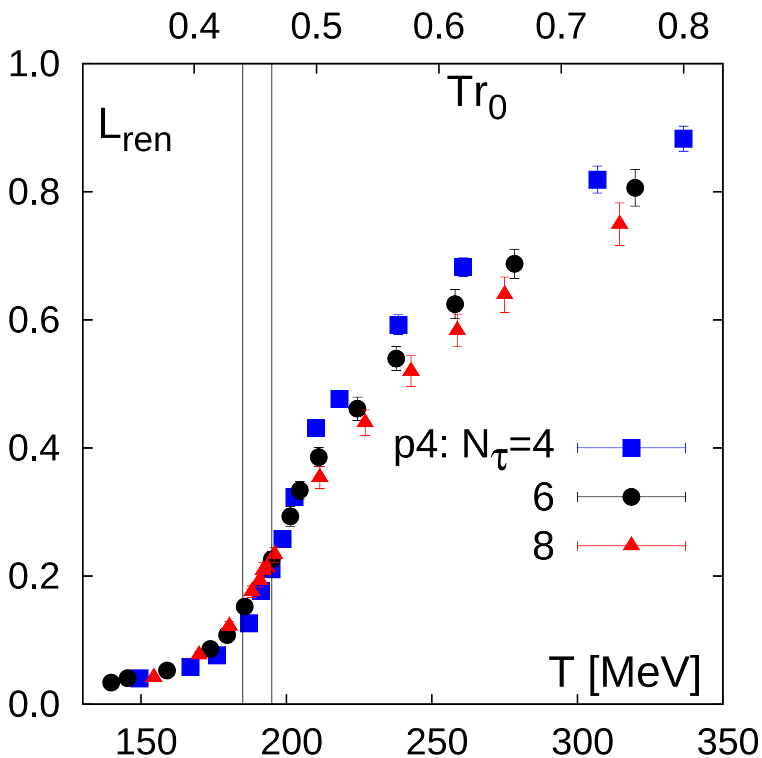
$\Rightarrow$ Expect close relation between DECONFINEMENT and T-dependence of bulk thermodynamics

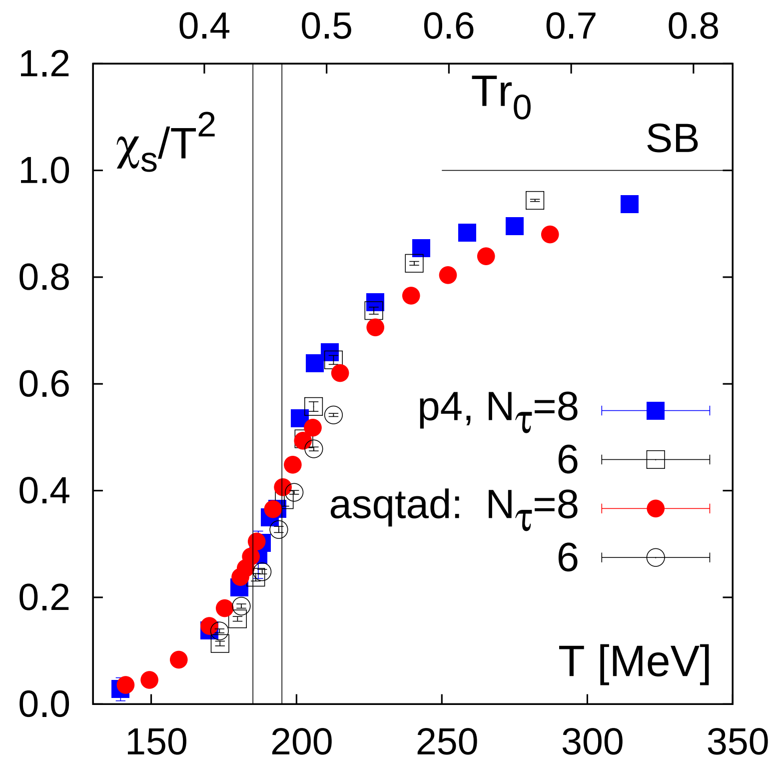
Deconfinement

- renormalized Polyakov loop and strange quark number susceptibility

$$L_{ren} \sim \mathrm{e}^{-F_Q(T)/T}, \qquad \chi_s/T^2 \sim \langle N_s^2 \rangle$$

band: $185\mathrm{MeV} \le T \le 195\mathrm{MeV}$





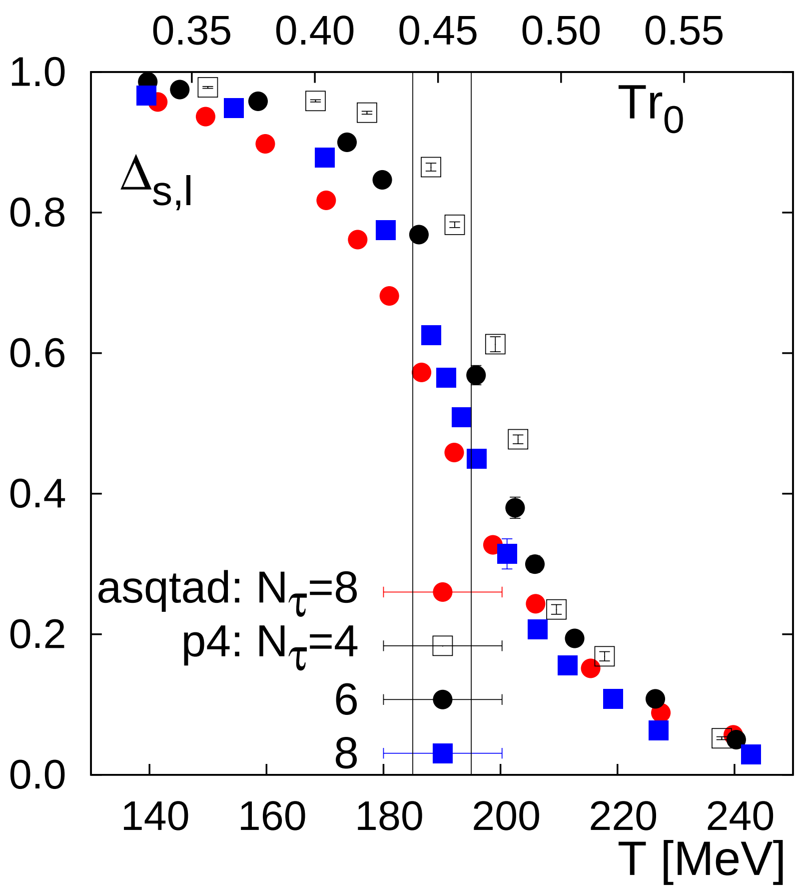
$N_\tau = 4,\ 6$ (p4): RBC-Bielefeld, PRD77, 014511 (2008)

$N_\tau = 8$, and $N_\tau = 6$ (asqtad): hotQCD, preliminary

Chiral condensates

- sudden change in ratios of finite and zero temperature condensates reflects chiral symmetry restoration

$$\Delta_{l,s}(T) = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}$$



subtracted a fraction of the strange quark condensate to eliminate additative renormalization terms

- 'normal' cut-off dependence of the subtracted and normalized chiral condensate; no 'unusually' large effects for $N_\tau = 8$
- consistent with confinement observables
- good agreement between p4 and asqtad results for $N_\tau = 8$

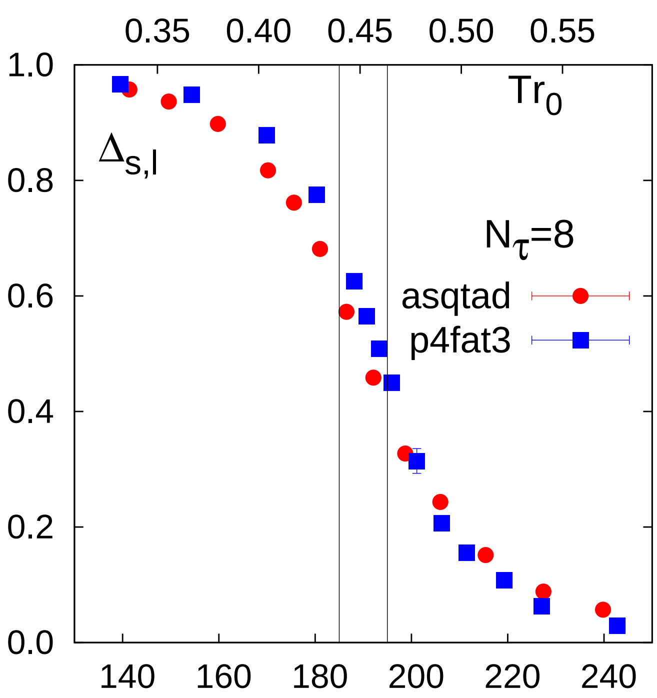
$N_\tau = 4,\ 6$ (p4): RBC-Bielefeld, PRD77, 014511 (2008)
$N_\tau = 8$, and $N_\tau = 6$ (asqtad): hotQCD, preliminary

χ-condensate and susceptibility

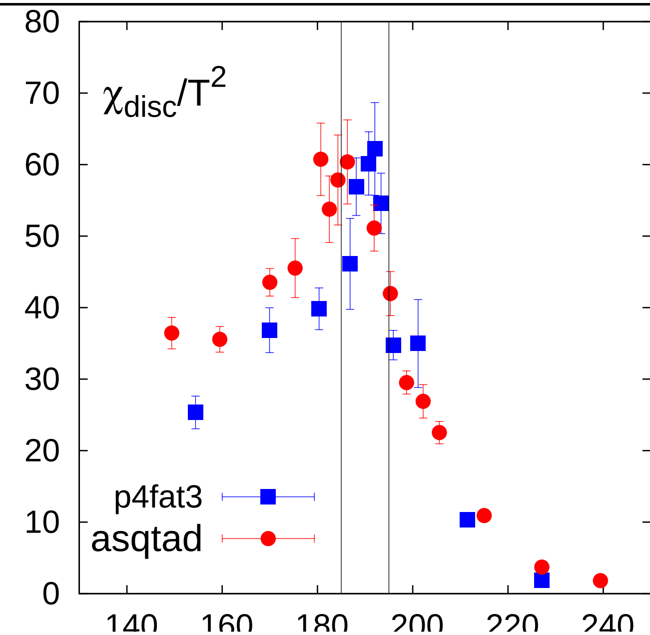
- sudden change in chiral condensate is, of course, related to peaks in the (singlet) chiral susceptibility

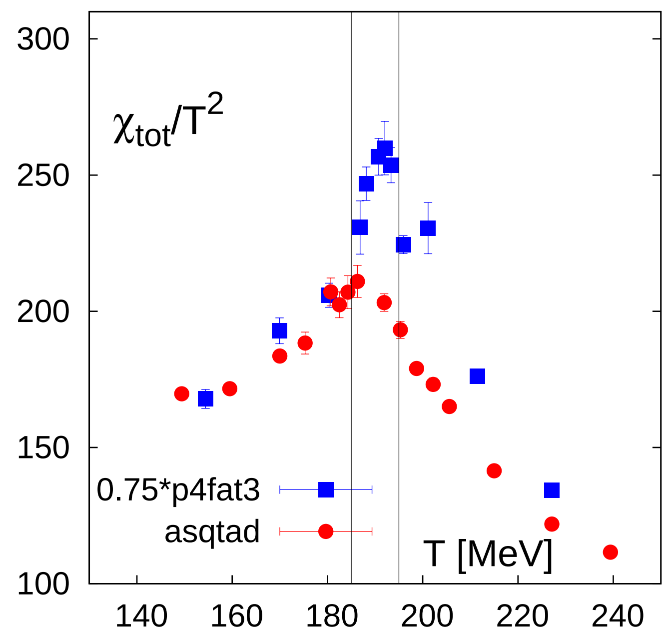
$$\chi_{tot}/T^2 = 2\chi_{dis}/T^2 + \chi_{con}/T^2$$

band: 185 MeV $< T <$ 195 MeV



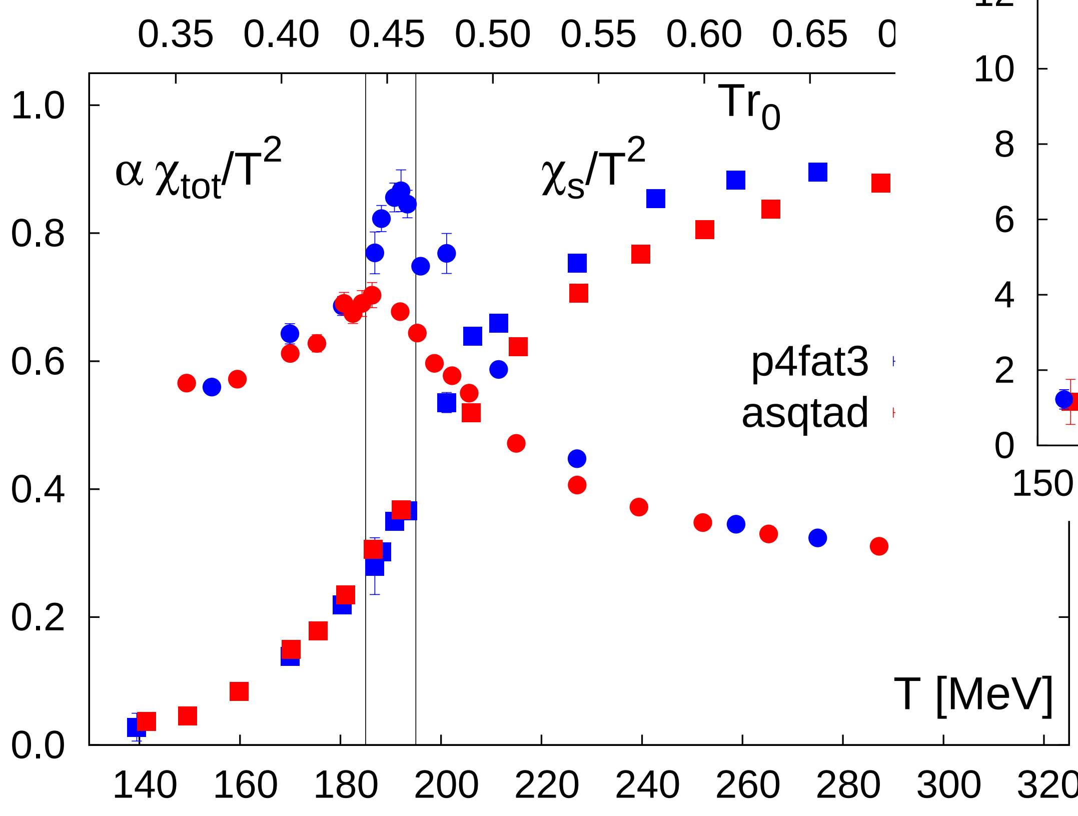
p4 and asqtad: hotQCD, preliminary

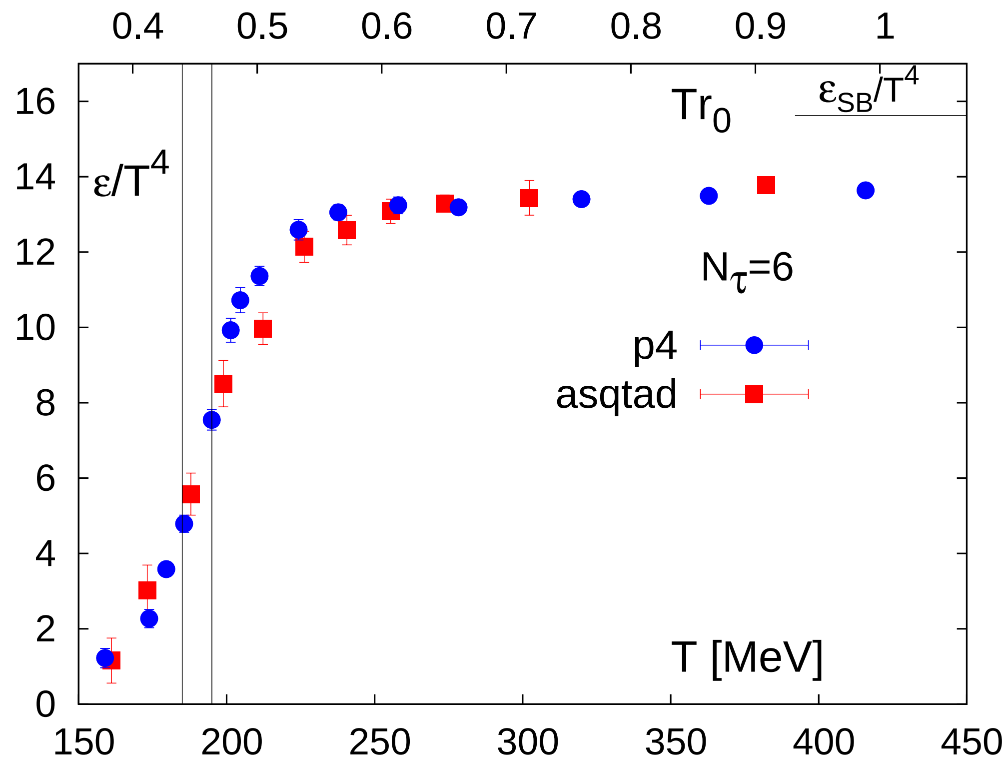




Deconfinement and χ-symmetry and bulk thermodynamics

- most prominent features of bulk thermodynamics are related to deconfinement:
  e.g. rapid rise in energy density and quark number susceptibility





band: $185\text{MeV} \leq T \leq 195\text{MeV}$

- no indication for 'low' chiral transition temperature

Conclusions

- $\mathcal{O}(a^2)$ improved actions drastically reduce cut-off effects
  - p4 and asqtad actions lead to consistent thermodynamics on lattices of temporal extent $N_\tau = 6$ and 8, although the handling of flavor symmetry breaking (fat-links) and $\mathcal{O}(a^2 g^2)$ corrections as well as cut-off effects in the free limit are quite different
- deconfinement and chiral symmetry restoration happen at roughly the same temperature that also characterizes the crossover region seen in bulk thermodynamics
- T_c analysis on $N_\tau = 8$ lattices, including updated $N_\tau = 6$ results is in progress