Baryon number Strangeness and electric Charge fluctuations at zero and non zero Density

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Outline

• Introduction

• Taylor expansion in B, S, Q chemical potentials
  
  \[ \mu_B > 0 \quad (\mu_S = \mu_Q = 0) \]

• Correlations between charges

• Conclusions
Fluctuations of $B$, $S$, $Q$ can be measured experimentally and indicate criticality.

LGT at $\mu = 0$ → RHIC, LHC

LGT at $\mu > 0$ → RHIC at low energies, FAIR@GSI

Introduction: The phase diagram of QCD
**Taylor expansion in:** $\mu_{B,S,Q}$

QCD is naturally formulated with quark chemical potentials $\mu_{u,d,s}$

- We start from Taylor expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \mu_u, \mu_d, \mu_s) = \sum_{i,j,k} c_{u,d,s}^{i,j,k} \left( \frac{\mu_u}{T} \right)^i \left( \frac{\mu_d}{T} \right)^j \left( \frac{\mu_s}{T} \right)^k$$

- Use unbiased, noisy estimators to calculate $c_{u,d,s}^{i,j,k}$. → See C. Miao, CS, PoS (Lattice 2007) 175.

- Line of constant physics: $m_q = m_s / 10$
  (physical strange quark mass)

- Measure currently up to $\mathcal{O}(\mu^8) \leftrightarrow (N_t = 4)$
  $\mathcal{O}(\mu^4) \leftrightarrow (N_t = 6)$
QCD is naturally formulated with quark chemical potentials $\mu_{u,d,s}$

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- expansion coefficients $c_{u,d,s}^{i,j,k}$ are related to B,S,Q-fluctuations

$$n_B = \frac{\partial (p/T^4)}{\partial (\mu_B/T)} = \frac{1}{3} (n_u + n_d + n_s)$$
$$n_S = \frac{\partial (p/T^4)}{\partial (\mu_S/T)} = -n_s$$
$$n_Q = \frac{\partial (p/T^4)}{\partial (\mu_Q/T)} = \frac{1}{3} (2n_u - n_d - n_s)$$

$$\mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q$$
$$\mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q$$
$$\mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S$$

- choice of $\mu_u \equiv \mu_d$ is equivalent to $\mu_Q \equiv 0$
Hadronic fluctuations \((\mu_B = 0)\)

In general we have:

\[
\frac{\partial^p (p/T^4)}{\partial (\mu_X/T)^p} \quad \text{related to} \quad \langle n^p_X \rangle_{\mu=0}
\]

\[
\frac{\partial^{p+q} (p/T^4)}{\partial (\mu_X/T)^p \partial (\mu_Y/T)^q} \quad \text{related to} \quad \langle n^p_X n^q_Y \rangle_{\mu=0}
\]

to be more precise:

\[
2c_2^X = \frac{\partial^2 (p/T^4)}{\partial (\mu_X/T)^2} = \frac{1}{VT^3} \langle (\delta N_X)^2 \rangle = \frac{1}{VT^3} \langle N_X^2 \rangle
\]

\[
24c_4^X = \frac{\partial^4 (p/T^4)}{\partial (\mu_X/T)^4} = \frac{1}{VT^3} \left( \langle (\delta N_X)^4 \rangle - 3 \langle (\delta N_X^2) \rangle^2 \right)_{\mu=0}
\]

\[
c_{11}^{XY} = \frac{\partial^2 (p/T^4)}{\partial (\mu_X/T) \partial (\mu_Y/T)} = \frac{1}{VT^3} \left( \langle N_X N_Y \rangle - \langle N_X \rangle \langle N_Y \rangle \right)_{\mu=0}
\]

\[
\text{with: } \delta N_X = N_X - \langle N_X \rangle
\]
The Resonance gas ($T < T_c$)

- the pressure is given by the free quantum gas pressure, summed over all particles

\[
\ln Z(T, V, \mu_B, \mu_S, \mu_Q) = \sum_{i \in \text{hadrons}} \ln Z_{m_i}(T, V, \mu_B, \mu_S, \mu_Q) \\
\sum_{i \in \text{mesons}} \ln Z_{m_i}^B(T, V, \mu_S, \mu_Q) + \sum_{i \in \text{baryons}} \ln Z_{m_i}^F(T, V, \mu_B, \mu_S, \mu_Q)
\]

- the contribution of particle $i$ with mass $m_i$ and quantum numbers $B_i, S_i, Q_i$

\[
\begin{align*}
\frac{p_i}{T^4} &= \frac{d_i}{\pi^2} \left( \frac{m_i}{T} \right)^2 \sum_{l=1}^{\infty} \frac{1}{l^2} \sum_{l=1}^{\infty} (+1)^{l+1} K_2(lm_i/T) \cosh(lS_i \mu_S / T + lQ_i \mu_Q / T) \\
\frac{p_i}{T^4} &= \frac{d_i}{\pi^2} \left( \frac{m_i}{T} \right)^2 \sum_{l=1}^{\infty} (-1)^{l+1} K_2(lm_i/T) \cosh(lB_i \mu_B / T + lS_i \mu_S / T + lQ_i \mu_Q / T)
\end{align*}
\]

→ for a dilute baryonic gas the Boltzmann approximation is valid:
(only $l = 1$ contributes for $(m_N - \mu_B) >> T$)

\[
\frac{p_i}{T^4} = \frac{d_i}{\pi^2} \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(B \mu_B / T + S \mu_S / T + Q \mu_Q / T)
\]
The Resonance gas \((T < T_c)\)

- Baryon number fluctuations in Boltzmann approximation:

\[
2c_2^B = \frac{\partial^2 (p/T^4)}{\partial (\mu X/T)^2} = F(m, T) B^2 \cosh(B\mu_B/T)|_{\mu_B=0}
\]

\[
24c_4^B = \frac{\partial^4 (p/T^4)}{\partial (\mu X/T)^4} = F(m, T) B^4 \cosh(B\mu_B/T)|_{\mu_B=0}
\]

\[
\text{with } F(m, T) = \sum_{i \in \text{baryons}} \frac{d_i}{\pi^2} \left(\frac{m_i}{T}\right)^2 K_2(m_i/T)
\]

- \(\mu_B\)-dependence factorize (all baryons have \(B_i = 1\))

- ratio of fourth and second order cumulant gives „unit charge“

\[
12 \frac{c_4^B}{c_2^B} = B^2
\]

mass and temperature independent ratio

- Strangeness and electric charge fluctuations are more difficult: multiple charged particles, light mesons (Boltzmann approximation not valid)
Critical scaling \( (T = T_c) \)

At \( \mu = 0 \) and \( m_u, d \) small, we expect O(4)-critical behavior

- **scaling field:**
  \[
  t = \left| \frac{T - T_c}{T_c} \right| + A \left( \left( \frac{\mu_u}{T_c} \right)^2 + \left( \frac{\mu_d}{T_c} \right)^2 \right) + B \frac{\mu_u \mu_d}{T_c T_c}
  \]

- **singular part:**
  \[
  f_s(T, \mu_u, \mu_d) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{(2-\alpha)}
  \]

- **O(4)/O(2): \( \alpha < 0 \), small**
  \[
  \langle (\delta n_{u,d})^2 \rangle \rightarrow \text{dominated by } T \text{ dependence of regular part}
  \]
  \[
  \langle (\delta n_{u,d})^4 \rangle \rightarrow \text{develops a cusp!}
  \]
Results for expansion coefficients $c_{i,j,k}^{u,d,s}$

**Preliminary**

- $C_2^u$
- $C_4^s$
- $C_{1,1}^{u,s}$

**small cut-off effects**
(similar to cut-off effects in other bulk thermodynamic quantities)

RBC-Bielefeld, 2+1 flavor
$m_\pi \approx 220\text{MeV}$

preliminary
Baryon number fluctuations ($\mu_B = 0$)

\[ \langle B^2 \rangle \]

\[ \langle B^4 \rangle - 3 \langle B^2 \rangle^2 \]

- Fluctuations increase with decreasing mass.
- Fluctuations increase over the resonance gas value.

Red: RBC-Bielefeld, preliminary
Strangeness fluctuations \( (\mu_B = 0) \)

\[
\langle S^2 \rangle \\
\langle S^4 \rangle - 3 \langle S^2 \rangle^2
\]

fluctuations increase over the resonance gas value

red: RBC-Bielefeld, preliminary
Electric charge fluctuations \((\mu_B = 0)\)

- Fluctuations increase with decreasing mass.
- Fluctuations increase over the resonance gas value.

**Graphs:**
- Left graph: \(<Q^2>\) vs. \(T\) [MeV] for different \(n_f\) values.
- Right graph: \(<Q^4> - 3 <Q^2>^2\) vs. \(T\) [MeV] for different \(m_\pi\) values.

**Data Points:**
- Red: RBC-Bielefeld, preliminary
Consequences for the phase diagram: the radius of convergence

- The radius of convergence can be estimated from the Taylor coefficients of the pressure:

  \[ \rho = \lim_{n \to \infty} \rho_n \]

  with

  \[ \rho_n = \sqrt{\frac{c_B}{c_{n+2}}} \]

- For \( T > T_c \), \( \rho_n \to \infty \)

- For \( T < T_c \), \( \rho_n \) is bounded by the transition line

- Non monotonic behavior of the convergence radius

  \[ \rightarrow \text{ first hint for a critical region?} \]

- Higher order approximations are needed to locate the critical point

\[ N_t = 4, \ m_\pi \approx 220M\text{eV} \]
Consequences for the phase diagram: the radius of convergence

- The radius of convergence can be estimated by the Taylor coefficients of the pressure:
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  with
  \[ \rho_n = \sqrt{c_B^n / c_{Bn+2}} \]

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\( \rightarrow \) first hint for a critical region?
Hadronic fluctuations $\mu_B > 0$ ($\mu_S = \mu_Q = 0$)

\[
\frac{\chi_B}{T^2} = 2c_2^B + 12c_4^B \left( \frac{\mu_B}{T} \right)^2
\]

\[
\frac{\chi_S}{T^2} = 2c_2^S + 2c_{22}^{BS} \left( \frac{\mu_B}{T} \right)^2
\]

\[
\frac{\chi_Q}{T^2} = 2c_2^Q + 2c_{22}^{BQ} \left( \frac{\mu_B}{T} \right)^2
\]

→ large baryon number fluctuations

→ enhanced strangeness and electric charge fluctuations ($\sim$ factor 3 at $T_c$)
Correlations of $S,B$ and $S,Q$

- Small quenching effect
  - $\to$ Normalization effect?

\[
\frac{\langle N_S N_X \rangle}{\langle N_S^2 \rangle} = \frac{f_X c_s^{qS} + 2c_s^s}{2c_2^s}
\]

\[
\frac{1}{T} \left( \langle N_B N_S \rangle - \langle N_B \rangle \langle N_S \rangle \right) = \tilde{c}_{11}^{BS} + 3\tilde{c}_{31}^{BS} \left( \frac{\mu_B}{T} \right)^2
\]

\[
\langle N_S \rangle \equiv 0
\]

$\rightarrow$ Correlations increase for $\mu_B > 0$

- full: RBC-Bielefeld, nf=2+1, preliminary
- open: Gavai, Gupta, nf=2, partially quenched

Conclusions / Summary

• Taylor expansion method at 2+1 flavor provides lots of input for HIC phenomenology.
• All results on fluctuations and correlations develop a peak with increasing chemical potential and are consistent with a gas of quasi-free quarks, already at 1.2-1.5 Tc.
• Ratios of cumulants are robust quantities and should be well suited for comparison with the experiment.
• The fluctuations increase with decreasing mass and rise over the resonance gas value close to Tc.  
  *(This is a necessary condition for the determination of the critical point with means of the radius of convergence.)*
• Strangeness and electric charge chemical potentials have to be tuned to meet the conditions of HIC.