Parton picture for a strongly–coupled plasma

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Collaboration with Yoshitaka Hatta and Al Mueller
(arXiv:0710.2148 and 0710.5297 [hep-th])
Hard probes for sQGP

- QGP just above the phase transition: $T \approx 2 \div 5 T_c$
  - deconfined
  - nearly conformal: $(E - 3P)/E \lesssim 20\%$
  - strongly coupled: $\lambda \equiv g^2 N_c \approx 3 \div 6$
    (RHIC data: $v_2$, low viscosity, early thermalization)

- a cousin of the strongly-coupled $\mathcal{N} = 4$ SYM plasma
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- AdS/CFT–based studies of medium properties ...
  - thermodynamics, hydrodynamics, transport coefficients

- ... and also of ‘hard probes’: \( \omega, q \gg T \)
  - energy loss, momentum broadening, meson dissociation

- How does a strongly–coupled plasma look, when probed on short distances and at high energies?
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- DIS: the best suited (Gedanken) experiment to measure this
The proton structure functions

\[ F_{1,2}(x, Q^2) \sim \text{Im} \int d^4x \, e^{-i q \cdot x} \langle P | [J_\mu(x), J_\nu(0)] | P \rangle \]

- Virtual photon absorbed by a quark excitation with
  - transverse size \( \Delta x_\perp \sim 1/Q \), where \( Q^2 \equiv -q^\mu q_\mu \geq 0 \)
  - longitudinal momentum \( k_z = xP \), where \( x \equiv \frac{Q^2}{2P \cdot q} \approx \frac{Q^2}{s} \)

\[ F_2(x, Q^2) : \text{the quark distribution in the proton} \]
DIS off a $\mathcal{N} = 4$ SYM plasma

- An Abelian $\mathcal{R}$–current: $J_{\mu}(q) \propto e^{-i\omega t + i q z}$ with $Q^2 = q^2 - \omega^2$

\[
\Pi_{\mu\nu}(q) \equiv \int d^4x e^{-i q \cdot x} i\theta(x_0) \langle [J_{\mu}(x), J_{\nu}(0)] \rangle_T
\]

- $Q^2 > 0 \implies$ DIS: $F_{1,2}(x, Q^2) \sim \text{Im} \Pi_{\mu\nu}(q)$
DIS off a $\mathcal{N} = 4$ SYM plasma

- An Abelian $\mathcal{R}$–current: $J_\mu(q) \propto e^{-i\omega t+iqz}$ with $Q^2 = q^2 - \omega^2$

$$\Pi_{\mu\nu}(q) \equiv \int d^4x e^{-iq\cdot x} i\theta(x_0) \langle [J_\mu(x), J_\nu(0)] \rangle_T$$

- $Q^2 < 0 \implies$ Jet physics: Energy loss, jet quenching ...

$q=(\omega,0,0,q)$

$Q^2 < 0$
The string dual: $AdS_5$ Black Hole

- Strong coupling limit $\lambda \to \infty$ ($N_c \to \infty$): supergravity
- Metric perturbation in $AdS_5$:
  \[ A_\mu(t, x, u) = e^{-i\omega t + i q z} A_\mu(u) \]

- Inelastic scattering ($\text{Im} \Pi_{\mu\nu} \neq 0$) \iff Absorption by the BH
The gravitational wave equation

- Einstein equations linearized around $AdS_5$
- Effective Schroedinger equation: $\psi'' - V\psi = 0$

- Repulsive barrier due to the virtuality $Q^2$
- Attractive potential due to the black hole

- Gravitational interactions are proportional to the energy
- Change of regime when increasing $\omega$ and/or $T$ at fixed $Q^2$
Low energy (or low temperature): $\omega T^2 \ll Q^3$

- Potential barrier: energy–momentum conservation
- Vacuum–like dynamics: a space–like current cannot decay
- Very small imaginary part due to double tunnel effect
Saturation momentum: $\omega T^2 \sim Q^3$

The barrier disappears when $Q \sim Q_s \equiv (\omega T^2)^{1/3}$

The same as the meson screening length

$$L \sim \frac{1}{Q} \quad \& \quad \gamma \sim \frac{\omega}{Q} \quad \Rightarrow \quad L_s \sim \frac{1}{\sqrt{\gamma T}} = \frac{(1 - v^2)^{1/4}}{T}$$

[Liu, Rajagopal, Wiedemann; Chernicoff, Garcia, Guijosa; 2006]
From current to meson

- At very high energy ($\omega \sim q \gg Q$) the current is like a bunch of light–like partons with transverse size $L \sim 1/Q$

- The current dynamics can be studied also after the ‘meson’ breaking $\Rightarrow$ a bunch of partons falling in the black hole
High energy (or high temperature): $\omega T^2 \gg Q^3$

- The wave falls into the BH $\implies$ large imaginary part
- No reflected wave $\implies$ Total absorption (unitarity limit)
- The line of stationary phase: same as the ‘trailing string solution’ for a heavy quark (here, with $v \simeq 1$) [Herzog, Yaffe, et al; Gubser et al, 2006]
Partonic interpretation

- **‘Partonic’ variables:** $Q^2$ and $x \equiv \frac{Q^2}{2\omega T}$

\[
Q_s \simeq (\omega T^2)^{1/3} \quad \iff \quad x_s \simeq \frac{T}{Q}
\]

- **Low energy** ($x \gg x_s$) : $F_2 \simeq 0 \implies$ No large–$x$ partons

- **High energy** ($x \lesssim x_s$) : $F_2(x, Q^2) \sim x N_c^2 Q^2$

Interpretation :

\[
\frac{1}{x} F_2(x, Q^2) \sim \int_{Q_s}^{Q^2} d^2 k_\perp \frac{dn}{d^2 x_\perp d^2 k_\perp}
\]

\[
\implies \frac{1}{N_c^2} \frac{dn}{d^2 x_\perp d^2 k_\perp} \sim \mathcal{O}(1)
\]

\[\implies\] one parton of each color per unit cell in phase–space

\[\implies\] parton saturation (occupation numbers of $\mathcal{O}(1)$)
Parton branching: **weak vs. strong coupling**

- **Weak coupling:** very little energy loss per branching
  - energy is carried by the few remaining partons at large $x$

- **Strong coupling:** energy is democratically divided
  - all partons fall down at very small $x \lesssim x_s \equiv T/Q$
  - the energy is carried by the partons with $x \simeq x_s$
Saturation line: weak vs. strong coupling

Saturation exponent: \( Q_s^2(x) \propto \frac{1}{x^\lambda} \equiv e^{\lambda Y} \)

- weak coupling (pQCD): \( \lambda \approx 1.23 g^2 N_c \) (BFKL Pomeron)
- strong coupling (plasma): \( \lambda = 2 \) (graviton)
The string dual of the $\mathcal{N} = 4$ SYM plasma

- The $AdS_5 \times S^5$ black hole:

\[
ds^2 = \frac{r^2}{R^2} \left( -f(r) dt^2 + d\mathbf{x}^2 \right) + \frac{R^2}{r^2 f(r)} dr^2 + R^2 d\Omega_5^2
\]

where $f(r) = 1 - \frac{r_0^4}{r^4}$ and the horizon $r_0 = \pi R^2 T$

- Strong coupling limit $\lambda \to \infty$ ($N_c \to \infty$): supergravity

- The black hole entropy: $S_{\text{BH}} = A/4G$, with $A =$ horizon area

\[
\implies S \equiv \frac{S_{\text{BH}}}{V_{3D}} = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} S_0
\]

- For generic values of $\lambda = g^2 N_c$:

$S = f(\lambda) N_c^2 T^3$ where $f$ interpolates between 1 and $3/4$

\[
\implies \text{Same number of d.o.f. at either weak or strong coupling!}
\]
The wave equations

- **Classical EOM** (Maxwell eqs. in the $AdS_5$ BH metric):

  \[ \partial_m \left( \sqrt{-g} g^{mn} F_{np} \right) = 0, \quad F_{mn} = \partial_m A_n - \partial_n A_m \]

- **Boundary conditions at $r \to \infty$**

  \[ A_\mu(t, x, r \to \infty) = e^{-i\omega t + iqz} A_\mu, \quad A_r(t, x, r \to \infty) = 0 \]

- **Boundary conditions at the BH horizon $r = r_0$**

  No wave returning from the horizon $\implies$ purely outgoing wave

  N.B. The origin of the imaginary part!

- **The classical action $\implies$ the current–current correlator**:

  \[ R_{\mu\nu}(q) = \frac{\partial^2 S_{cl}}{\partial A_\mu \partial A_\nu} \]

- **The classical action involves just** \( (A_\mu, \partial_r A_\mu) \big|_{r \to \infty} \)