INSIGHTS INTO STRONGLY COUPLED PLASMAS VIA ADS/CFT

KRISHNA RAJAGOPAL (MIT)

QUARK MATTER 2008, JAIPUR
Expts @ RHIC suggest that quark-gluon plasma is so strongly coupled at $T \sim 1.5 \text{ TeV}$ accessible at RHIC that it is better thought of as a liquid than a gas.

$\gamma \to \frac{\eta}{s} \Rightarrow \text{well-described with ideal hydrodynamics (zero m.f.p.)}$

$\gamma / s \sim 1$ according to perturbative QCD calculations

$\gamma / s \sim 10$ in water
LATTICE CALCULATION OF $\eta/s + 5/s$ IN $N_f = 0$ QCD

$N_T = 8 \quad N_f = 0 \quad$ Harvey Meyer '07

- Conformality $\Rightarrow 5/s = 0$
  $\eta/s = T$-independent
- And, no sign of $T$-dependence for $\eta/s$ over $T_{RHIC} \Rightarrow T_{LHC}$
- Suggests QGP as liquid-like at LHC as at RHIC.
Expansion of a rotating strongly interacting Fermi gas

\[
\frac{\Omega_0}{\omega_z} = 0
\]

\[
\frac{\Omega_0}{\omega_z} = 0.40
\]

\[
\frac{\Omega_0}{\omega_z} = 1.12
\]
Viscosity/entropy density (units of $\hbar/k_B$)

$\eta/s$

$E/E_F$

$^3$He, $^4$He near $\lambda$-point

Rotating cloud

String-theory $1/4\pi$

John Thomas, talk at BEC 07, Sant Feliu
How to calculate properties of strongly coupled QGP liquid?

1. Lattice QCD
   - Perfect for thermodynamics (i.e., static properties)
   - Calculation of \( g \) and other transport coefficients, beginning
   - Jet quenching and other dynamic properties not in sight

2. Perturbative QCD
   - Right theory but wrong approximation

3. Calculate QGP properties in other theories that are analyzable at strong coupling.
   - Are some dynamical properties universal? I.e., same for strongly coupled plasmas in a large class of theories. What properties? What class of theories?
\[ N=4 \text{ SUPERSYMMETRIC YANG MILLS} \]

- A gauge theory specified by two parameters: \( N_c \) and \( g^2 N_c = \lambda \).
- Conformal. (\( \lambda \) does not run.)
- If we choose \( \lambda \) large, at \( T \to 0 \) we have a strongly coupled plasma.
- This 3+1 dimensional gauge theory is equivalent to a particular string theory in a particular spacetime: \( \text{AdS}_5 \times S^5 \) → 5 “curled up” dim. 4+1 “big” dimensions.
- In the \( N_c \to \infty, \lambda \to \infty \) limit, the string theory reduces to classical gravity. ⇒ calculations easy at strong coupling.
PURPOSE OF THIS TALK

- Describe some of the insights into properties of strongly coupled plasmas via calculations done in $\mathcal{N}=4$ SYM and the infinite classes of other gauge theories dual to gravity in higher dimensional spacetime.

- Because of time constraints, I will not describe how the calculations that have led to those insights are done. I am happy to do so privately for any of you. All of them are easy enough to be done by undergraduates, once you learn the rules.
THERMODYNAMICS

In the $N_c \to \infty$, $\lambda \to \infty$ limit,

\[
\frac{E_{\lambda=\infty}}{E_{\lambda=0}} = \frac{P_{\lambda=\infty}}{P_{\lambda=0}} = \frac{S_{\lambda=\infty}}{S_{\lambda=0}} = \frac{3}{4}
\]

Gubser Klebanov Pohlt Tseytlin...

- Teaches us that thermodynamics of very weakly coupled plasmas and very strongly coupled plasmas can be rather similar.
- Reminds us that (approximate) conformal nature above $T_c$ need not mean weak coupling. $\rightarrow$ Fig.
- $\frac{1}{\lambda^2}$ corrections known. [$\frac{3}{4}$ becomes 0.77 for $\frac{g^2}{4\pi} = \frac{1}{2}$, $N_c = 3 \rightarrow \lambda = 6\pi$]
- $\frac{1}{N_c^2}$ corrections not known
$T(\text{MeV})$, assuming $T_c = 170$ MeV. (estimate is $140 < T < 190$)

\[ \frac{\varepsilon}{T^4} \]

$pion \ gas$

ideal QGP

DECONFINEMENT

(IONIZING THE HADRONS)

Karsch, Laermann, Peikert (Heine)

$T/T_c$

$\langle \bar{\psi}\psi \rangle$

Blum, Detar, MILC Collab.

$N_f = 2$

$M_q \neq 0$

=: smooth crossover

CHIRAL SYMMETRY RESTORATION (MELTING THE VACUUM) ON THE LATTICE

$T$ (funky units)
**Shear Viscosity**

\[ \frac{\eta}{s} = \frac{1}{4\pi} \]

Policastro, Starinets, Son

- For any theory with a gravity dual, in the \( N_c^2 \rightarrow \infty, \lambda \rightarrow \infty \) limit.

- Examples known for theories that are:
  - conformal or not
  - confining at \( T=0 \) or not
  - have fundamentals or not
  - supersymmetric or not
  - varying numbers of degrees of freedom

- \( \frac{1}{\lambda^{3/12}} \) corrections known.
  \[ \frac{1}{\lambda^{3/12}} \] becomes \( \frac{1.25}{4\pi} \) for \( \lambda = 6\pi \)

- \( \frac{1}{N_c^2} \) corrections not known

- \( \frac{\eta}{s} \geq \frac{1}{4\pi} \) conjectured as a lower bound for all materials.

Kovtun, Son, Starinets
AdS/CFT

We now know of infinite classes of different gauge theories whose quark-gluon plasmas:
- are all equivalent to string theories in higher dimensional spacetimes that contain a black hole
- all have

$$\frac{\varepsilon}{\lambda^4} = \frac{3}{4} \left( \frac{\varepsilon}{\lambda^4} \right)_0$$

Gubser Klebanov Tseytlin Polchinski

$$\frac{\eta}{\xi} = \frac{1}{4\pi}$$

Son Polchasto Starinets Kovtun Buchel Liu

in the limit of strong coupling and large number of colors.

Not known whether QCD in this class...
UNIVERSALITY?

Is there a new notion of universality for (nearly) scale invariant liquids?

To what systems does it apply?
  - quark-gluon plasma dual to string theory + black hole?
  - QCD quark-gluon plasma?
  - unitary fermionic atom gas?

To what quantities does it apply?
  - $\tau/s$?
  - other suggestions on the QCD side relate to “jet quenching”...
    Could you study “atom quenching”??
Further evidence that QGP@RHIC is strongly coupled.

**Radiative energy loss**

\[ xE, k_T \rightarrow (1-x)E \]

dominates in high \(E\) limit, \((E \gg k_T \gg T)\). If so (RHIC? LHC?), energy loss sensitive to medium through one parameter \(\hat{q}\), \(k_T\) picked up by radiated gluon per distance \(L\) travelled.

Spectrum of radiated gluons: \[ \frac{dI}{dw} \sim \alpha \sqrt{\frac{\hat{q}}{w}} L \]

Energy loss \(\Delta E \sim \alpha \hat{q} L^2\) for \(w < \alpha \hat{q} L^2\).
Jet Quenching Parameter $\hat{q}$

- Assume $E \gg k_T \gg T$
- Assume weak $\alpha_s(k_T)$.
  \[ \alpha_s \sim \text{(radiative energy loss)} \]
- If $\alpha_s(T)$ were weak,
  \[ \hat{q} \sim \frac{\alpha_s^2}{\lambda} \sim \text{mean free path} \]
  \[ \sim N_{\text{gluons}} \cdot \alpha_s^2 \]
  \[ \approx 3.1 \cdot \alpha_s^2 \cdot N_c^2 \cdot T^3 \]  \text{Baijer-Schiff}
  \[ \approx 0.9 \text{ GeV}^2/\text{fm} \]  \text{($N_c = 3$, $\alpha_s = 1/3$, $T = 300 \text{ MeV}$)}
- But: Smallness of $\hat{q}/s$ indicates QCD at scales $\sim T$ not weakly coupled
- AND: $\hat{q}$ extracted via comparison with RHIC data is
  \[ \approx 4-14 \text{ GeV}^2/\text{fm} \]  \text{Dainese Loizides Paic}
  \[ \approx 3 \text{ GeV}^2/\text{fm} \]  \text{Zhaug Owens Wang Wang}
  \[ \approx 8-19 \text{ GeV}^2/\text{fm}, \text{ at 20}, \text{ neglecting theoretical uncertainty PHENIX} \]
- WANTED: Strong coupling calculation of $\hat{q}$
$\hat{q}$ in $N=4$ SYM

In $Nc^2 \to \infty$, $\lambda \to \infty$ limit,

$$\hat{q} = \frac{2 \pi^2 \Gamma(5/4)}{\Gamma(3/4)} \sqrt{\alpha_N} Nc^2 T^3 = 27\sqrt{\alpha} Nc^2 T^3$$

+ Liu, KR, Wiedemann

- $1/Nc^{1/2}$ corrections partially known, Arnosto Edelstein, Mas

- $1/Nc^2$ corrections not known.

- $\hat{q}$ is not proportional to $T$, or to $N_{\text{scatterers}}$. These are $\sim Nc^2 T^5$.

- Multiple gluon correlations are as important as two gluon correlations.

  Liang, Wang, Zhou

- Reminds us that liquids do not have well-defined quasiparticles, so should not expect $\hat{q}$ to count number density of such.

- Try some numbers: $Nc = 3$, $\alpha = 1/2$

  $\hat{q} = 56 eV^2/fm$ for $T = 300$ MeV

- In ballpark of what RHIC data wants....
Towards QCD

- For any CFT with a gravity dual,
  \[ \hat{\eta} \frac{CFT}{\hat{\eta}_{N=4}} = \sqrt{\frac{S_{CFT}}{S_{N=4}}} \]
  Liu KR Wiedemann

  further highlighting the lesson.

- Is \( \frac{\hat{\eta} / \sqrt{\xi}}{\sqrt{S/N^2}} \) universal like \( \xi/\xi \)? Not known....

- Also suggests \( \frac{\hat{\eta}_{QCD}}{\hat{\eta}_{N=4}} \sim \sqrt{\frac{47.5}{120}} \sim 0.63 \)

- In one toy model, adding nonconformality (to a degree indicated by QCD thermo.) increases \( \hat{\eta} \) by 22\% at \( T = 200 \text{ MeV} \)
  9\% at \( T = 300 \text{ MeV} \).
  Liu KR Shi

- Still in ballpark of what RHIC data wants....
A PREDICTION FOR LHC

If we assume \( \sqrt{\alpha_{\text{LHC}}} \sim \sqrt{\alpha_{\text{RHIC}}} \) then \( \hat{\alpha}_v \sim T^3 \). This, plus Bjorken expansion, yields:

\[
\frac{\hat{\alpha}_{v, \text{LHC}}}{\hat{\alpha}_{v, \text{RHIC}}} = \frac{(dN/d\eta)_{\text{LHC}}}{(dN/d\eta)_{\text{RHIC}}}
\]

Liu KR Wiedemann

where

\[
\hat{\alpha}_v = \frac{z}{2\pi} \int_0^L dp \frac{d \hat{\alpha}_v(t)}{dt}
\]

is the time averaged \( \hat{\alpha}_v \) which determines parton energy loss and is extracted by comparison with data.
MOVING HEAVY QUARKS: DRAG AND DIFFUSION

For a quark with mass $M$ moving through the plasma with velocity $v$ such that $M > \frac{\sqrt{s}}{(1-v^2)^{1/4}}$ or $\frac{1}{\sqrt{s}} < \frac{M}{\sqrt{s}T}$, energy loss occurs via drag and diffusion:

$$\frac{dp}{dt} = -\gamma_{\text{Drag}} P + 5(4), \quad \langle 3v(4), 3v(4) \rangle = K \delta(t-\tau)$$

where $\gamma_{\text{Drag}} = \frac{\pi \sqrt{s} T^2}{M}$ and $D = \frac{2T^2}{K} = \frac{2}{\pi T \sqrt{s}}$

Her Zoe, Karch Kovtun Kovacs Yaffe; Gubser;
Casalderrey - Solana Teaney;...

This $D$, in the Langevin formalism of Moore + Teaney, yields $R_{AA}$ and $v_2$ for heavy quarks in broad agreement with RHIC data for non-photonic electrons.
WHERE DOES THE ENERGY GO?

For a heavy quark with \( u > \frac{1}{2} \) sound = \( \frac{1}{\sqrt[3]{3}} \)

but \( \frac{1}{(1-v^2)^{1/4}} < \frac{M}{\sqrt{T}} \)

through the strongly coupled plasma of \( N=4 \) SYM, we now know:

- Mach cone
- and wake

with relative strengths such that, according to hydrodynamic calculations with Cooper-Frye freezeout, the Mach cone should be considerably filled in the data.

- But: a point particle does make a Mach cone

- Remarkably similar cone + wake for point quark, with its color field, moving through QCD QGP, assuming small \( q \).

B. Müller, parallel talk on Friday
Energy density.

*NB: Specific heat $\propto Nc^2$ amplifies effect of heat over motion in $E$. So, this plot tells you where there is heating. I.e. compression. I.e. SOUND.*

Momentum flow.

Mach cone and wake.
SCREENING IN $N=4$

$V(L) = \text{potential between static } Q < Q$

$\sim \frac{1}{L}$  \hspace{1cm} $T=0$

$L_5 = 0.277/T$

$T \neq 0$

Similar to screening in QCD above QCD's $T_c$...
lattice QCD calculation

Unquenched, $N_f = 2$

Upon defining an $L_s$, the authors find $L_s \sim 0.5/T$
A PREDICTION FOR EXPERIMENT

H. Liu, KR, Wiedemann

Calculate force between $\bar{q} + q$ moving through the $N=4$ QGP. (Not known how to do this calculation in QCD.) Find:

$$L_s = \frac{f(v,0)}{\pi T} (1-v^2)^{1/4}$$

where $f$ is almost a constant. $(f(0,0) = 0.869)$

$$f(\Phi, \Xi) = 0.743$$

So, $L_s(v,T) \approx L_s(0,T) / \sqrt{8}$

Makes sense if $L_s$ controlled by $E$,
since $E \sim T^4$ and $E(v) = E(0) x^2$.

$J/\psi$ ($\Xi_c$) and $Y$ ($b\bar{b}$) mesons dissociate when $T$ reaches $T_{diss}$, at which $L_s \sim$ meson size.

Suggests: $T_{diss}(v) \sim T_{diss}(0) / \sqrt{8}$!
\[ T_{\text{dissociation}} \text{ vs. } p_T \]

- At \( p_T = 0 \), \( T_{\text{diss}} \approx 2.1 T_c \), from lattice QCD.
- I curve schematic. (Scaled rel. to \( J/\psi \) by meson size in vacuum.)

![Graph showing \( T_{\text{diss}}(p_T)/T_c \) vs. \( p_T \) in GeV]

- Our velocity scaling: \( T_{\text{diss}}(v) = T_{\text{diss}}(0)/\sqrt{8} \)
- Karsch KhareevSato model (i.e. \( 2.1 T_c < T_{\text{thick}} < 1.2 T_c \))

\( \Rightarrow J/\psi \) themselves dissociate for:
- \( p_T > 5 \text{ GeV} \) if \( T_{\text{thick}} \sim 1.5 T_c \)
- \( p_T > 9 \text{ GeV} \) if \( T_{\text{thick}} \sim 1.2 T_c \)
Hot-wind scenario in hydro+J/ψ model

Parallel talk at QM2008 by T. Gunji, Feb. 9th Session XVIII 15:20~15:40

Melting temperature in hot-wind

\[ T_{\text{melt}}(v) = T_{\text{melt}}(0)(1-v^2)^{1/4} \]

H. Liu et al.
PRL.98:182301, 2007

Melting temperatures: \((T_{J/\psi}, T_{\pi}) = (2.0T_c, 1.34T_c)\)
10% feed-down correction

1: Survival Probability of \(J/\psi\) vs. \(N_{\text{part}}\)

2: Survival Probability of \(J/\psi\) vs. pT

20-30% \(N_{\text{part}} \approx 170\)

3: \(v2\) of \(J/\psi\) vs. pT

- \(J/\psi\) suppression from Hot-wind scenario was calculated in hydro+J/ψ model.
- Overall suppression pattern is similar in both cases.
- Larger suppression and large \(v2\) (~3%) in the high pT region in a scenario with hot-wind.
Gunji et al show that the reduction in $T_{\text{diss}}$ for $J/\psi$'s that feel a hot wind is:
- a small effect in $S_{J/\psi}$
- significant in $S_{J/\psi}^P$
- $P_T$ at which effect sets in is:
  - sensitive to, and thus a measure of,
    - $T_{\text{diss}}(v=0) - T_{\text{reached}}$

Increasing $T_{\text{diss}}(v=0)$ from 2.0 to 2.2 GeV pushes this $P_T$ up to 4-5 GeV

- and, i.e., higher in Cu-Cu than Au-Au

- $R_{AA}(P_T)$ will be interesting to watch as error bars come down

- $\Upsilon'/\Upsilon$ vs. $P_T$ (e.g., at LHC) [or $\Upsilon'/J/\psi$]

Even better since any $P_T$-dependence of $b+\bar{b}$ ($c+\bar{c}$) production [e.g., Cronin; eg regeneration; eg gluon en. loss, ...] cancels
Nuclear modification factor $R_{AA}$

- Double the $p_T$ range to 10GeV/c
- Consistent with no suppression at high $p_T$:
  \[ R_{AA}(p_T>5 \text{ GeV/c}) = 0.89 \pm 0.20 \]
- Indicates $R_{AA}$ increase from low $p_T$ to high $p_T$
- Different from expectation of most models:
  AdS/CFT:
  H. Liu, K. Rajagopal and U.A. Wiedemann, 
PRL 98, 182301(2007) and hep-ph/0607062
  Two Component Approach:
  X. Zhao and R. Rapp, hep-ph/07122407

Zebo Tang, USTC/BNL  
Quark Matter 2008, Jaipur, India, Feb. 4-10, 2008
Suppose $T \sim 3$TeV. Then, $Y'$ unaffected for $p_T \leq 10$ GeV, dissociated for $p_T \geq 10$ GeV. Expect $Y'$ dissociated at any $p_T$. 
Upon introducing dynamical, but heavy, quarks into $N=4$ SYM, mesons ("quarkonia") exist as bound states in the plasma as long as

$$T < T_{\text{diss}}(v) = f(v) \frac{T_{\text{diss}}(0)}{\sqrt{8}}$$

with $1.01 < f(v) < 0.92$ for $0 < v < 1$.

Justifies inference drawn from static potential

**BARYON SCREENING**

No quarks in a circle of radius $L$

feel a potential only if

$$L < L_{s}(v) = L_{s}(0) / \sqrt{8}.$$

Further confirmation of robustness of the velocity-dependence of screening...
FROM $N=4$ SYM TO QCD

- The two theories differ on various "axes." (But, much more similar at $T=0$ than $T=0$)
- To make insights semiquantitative for QCD, need to take steps toward QCD on each such axis, and see how results change.
- Degrees of freedom at weak coupling differ: Define $\eta$ by $\eta = \pi T^4 / 30$
  Then: $\eta_{\text{QCD}} = 2(N_c^2 - 1) + 3N_c / 2 = 47.5$ (for $N_c = 3$)
  $\eta_{N=4} = 15(N_c^2 - 1) = 120$ (for $N_c = 3$)

  Need observables that are insensitive to this. (E.g., $\pi/5$, $\eta$, perhaps, $q / \sqrt{\eta/N_c}$)

NB: liquids have no quasiparticles anyway

- $N=4$ calculations easy when $\chi = g^2 N_c = 4\pi \alpha N_c \gg 1$. Leading corrections ($\sim 1/\lambda^{3/2}$) computed for $\pi/5$, and small; partially computed for $q$. 
• $N = 4$ is conformal. QCD is not.
  - But, for $2T_c < T < \infty$, QGP thermodynamics is quite conformal
    $[E \sim T^4; P \sim T^4; S \sim T^3; \mathcal{V}_5 \sim \frac{1}{5} ]$
  and early indications from lattice are that $\phi/5 \sim \text{const}$ and $S/\phi \sim \text{small}$.
  - So, perhaps strongly coupled QGP of QCD well-modelled as conformal
  - In studies to date, adding a level of nonconformality as in QCD thermodynamics to $N=4$ SYM has
    no effect on $\phi/5$, little effect on $S$.
• $N=4$ calculations tractable when $1/N_c \ll 1$. Leading $1/N_c^2$ corrections to any of quantities in this talk
  not currently known.
WHAT IS $g^2 N_c$ IN QGP@RHIC?

Need a quantity that is calculable at strong coupling in QCD, at finite

$\lambda = g^2 N_c$ in $N=4$ SYM, and that does not depend on # of degrees of freedom.

Two examples, one classic, one new:

$\frac{\pi}{\text{Enoninteracting}} = 0.78 - 0.82$ (Lattice QCD)

$\frac{\pi}{\text{Enoninteracting}} = \frac{3}{4} + 1.69 x^{-3/2}$ ($N=4$ SYM)

$\rightarrow 9 \leq x \leq 15$

$\frac{\pi}{5} = 0.134 (33)$ at $T=1.65 T_c$

$\frac{\pi}{5} = 0.102 (56)$ at $T=1.24 T_c$

$\frac{\pi}{5} = \frac{1}{4\pi} (1 + 20.3 x^{-3/2})$ ($N=4$ SYM)

$\rightarrow \frac{\pi}{5} = 0.134 \rightarrow x = 10$

$\frac{\pi}{5} = 0.102 \rightarrow x = 17$
INSIGHTS I DID NOT HAVE TIME TO DESCRIBE

include those arising from investigations of:

- Time-dependent backgrounds with gravity duals. (As opposed to dynamical probes of eqbm plasma.)
  - Nastase; Shuryak; Sin; Zahed; Janik; Peschanski; Nakamura; Lin; Shuryak; Friess; Gubser; Michalogiorgakis; Pufu; Kajantie; Tahkokallio; Louko;
  - Parallel talk by Shu Lin this afternoon

- DIS off $N=4$ QGP Hatua Iancu Mueller
  - Parallel talk by Edmond Iancu this afternoon

- "photon" and "dilepton" emission from $N=4$ QGP Caron-Huot; Moore; Koutun; Starinets; Yaffe

- Magnetic description of $N=4$ SYM plasma
  - Liao; Shuryak
  - Parallel talk by Jinfeng Liao this afternoon

- Meson spectral functions for Starinets;
  - Koutun; Starinets; Teaney; Myers; Starinets; Thompsons; Myers; Sinha;
  - Parallel talk by Aninda Sinha this afternoon
1. Thermodynamics within 15-25% of that at zero coupling arises at strong coupling.

\[ \eta/s = \frac{1}{4\pi} \] in $N_c^{-1}, \lambda^{-1}$ limit, for plasma of any gauge theory with a gravity dual.

\[ \eta/s \] in QCD plasma (lattice; RHIC) and for unitary cold atom gas seems comparable.

\[ q \propto \sqrt{\frac{S}{N_c^2 T^3}} \sqrt{\lambda} T^3 \] for an infinite class of strongly coupled plasmas. Jet quenching does not count gluons; all multiple gluon correlations equally important.

\[ q \approx 3-5 \text{ GeV}^2/\text{fm} \text{ at } T = 300 \text{ MeV}. \]

\[ \frac{dN/d\Omega}{dN/d\Omega_{\text{RHIC}}} \sim \frac{(dN/d\Omega)_{\text{LHC}}}{(dN/d\Omega)_{\text{RHIC}}} \]

2. In a strongly coupled plasma, heavy POINT-LIKE quarks drag, diffuse, and excite a Mach cone.

3. Heavy quarkonia mesons, bound above $T_c$, dissociate at lower temperatures when moving.

\[ T_{\text{diss}}(v) \approx T_{\text{diss}}(0) (1-v^2)^{1/4} \]

Also for heavy quark baryons.