3D Hydro + UrQMD Model with QCD Critical Point

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In collaboration with

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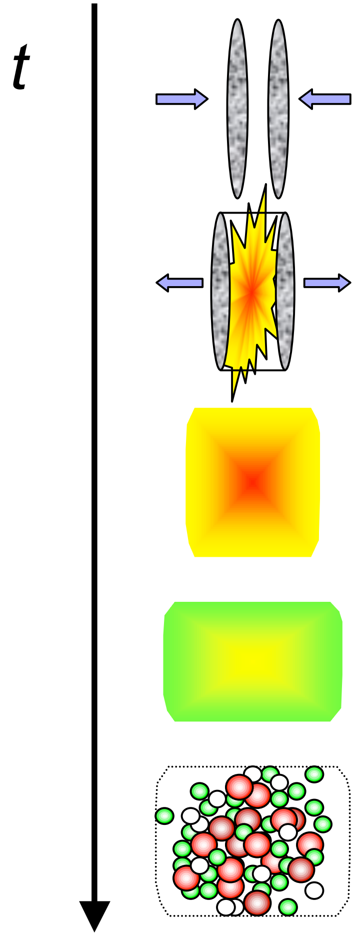
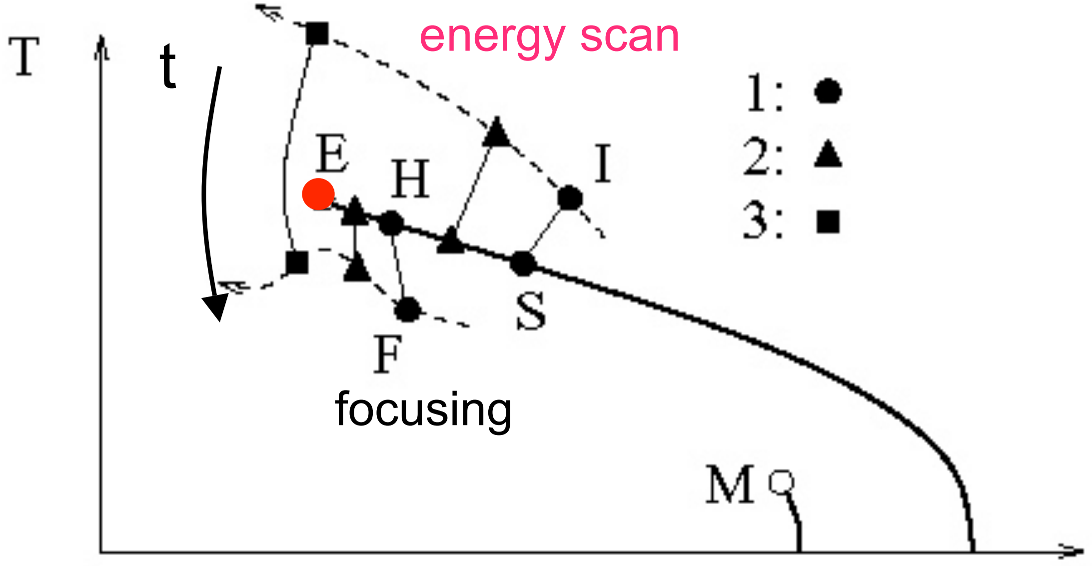
February 8, 2008 @QM2008, Jaipur, India

The QCD Critical Point in HIC

- **The QCD critical point search from phenomenology**





M. Stephanov, K. Rajagopal, and E.Shuryak,
PRL81 (1998) 4816

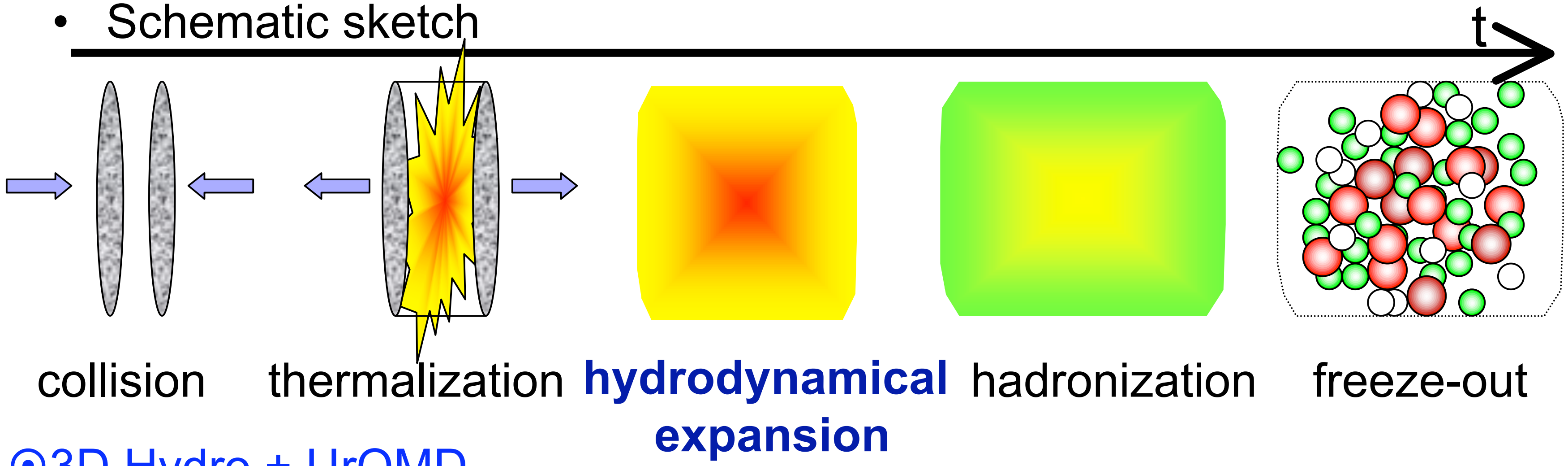
3D Hydro + UrQMD
Realistic dynamical model for Heavy Ion Collisions

3D Hydro+UrQMD Model

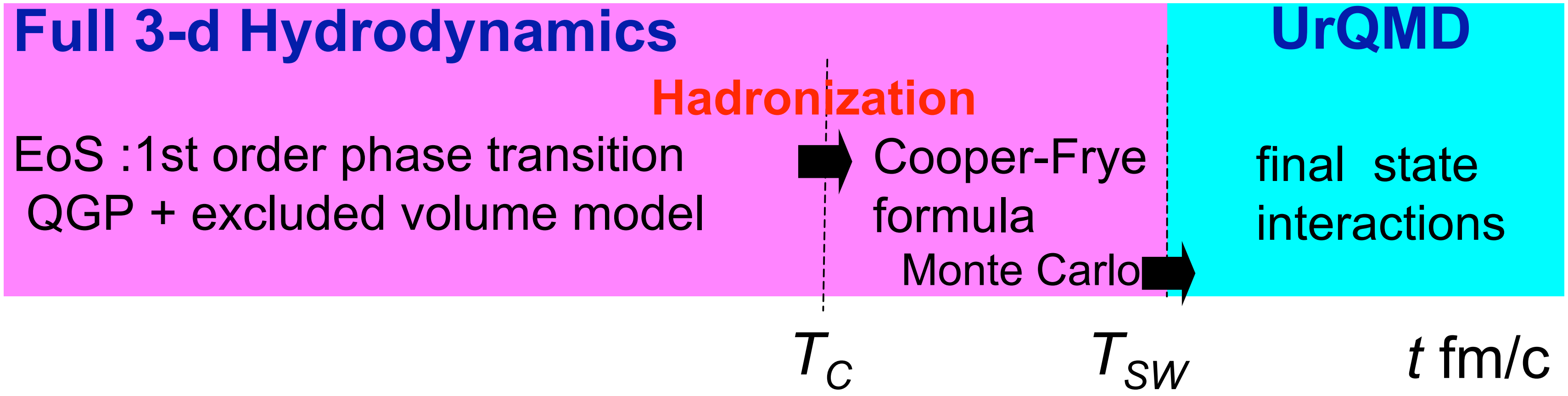
Nonaka and Bass PRC75:014902(2007)

Relativistic Heavy Ion Collision

- Schematic sketch



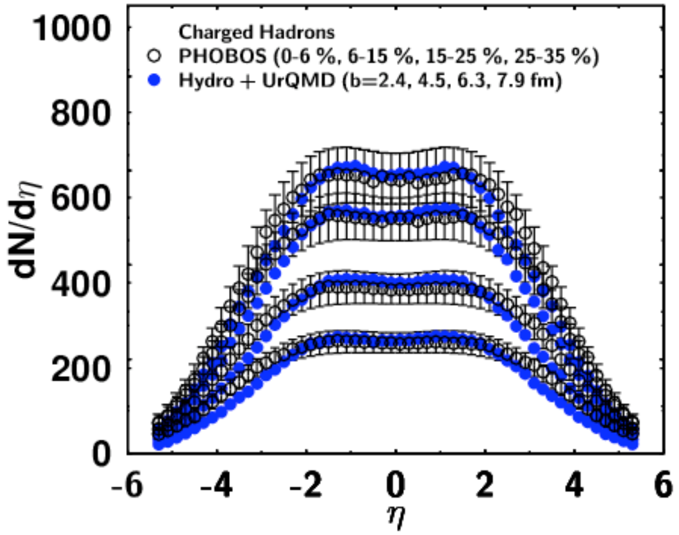
⊙3D Hydro + UrQMD

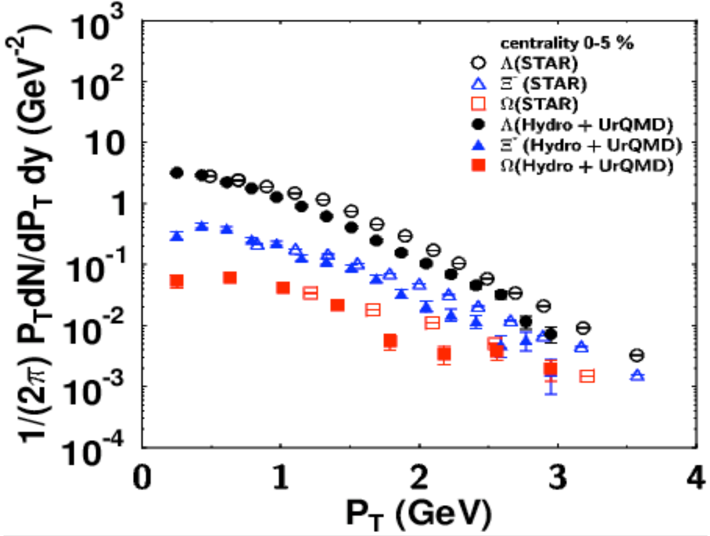


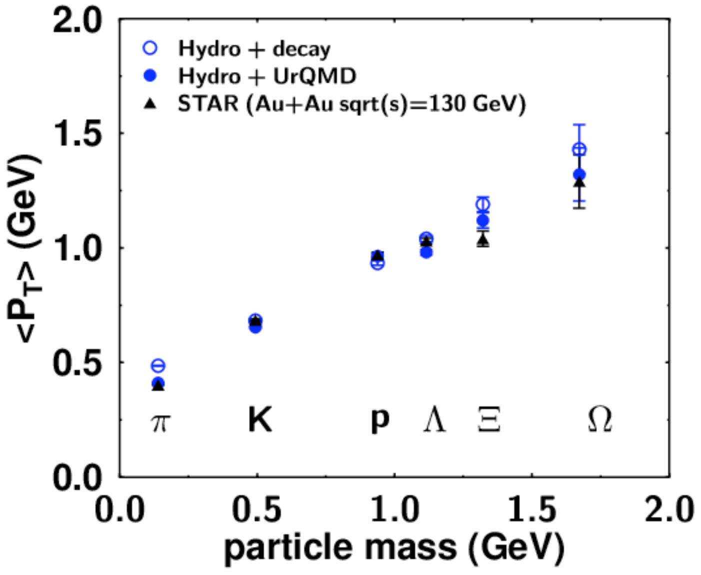
T_C:critical temperature > T_{SW}: Hydro ➔ UrQMD

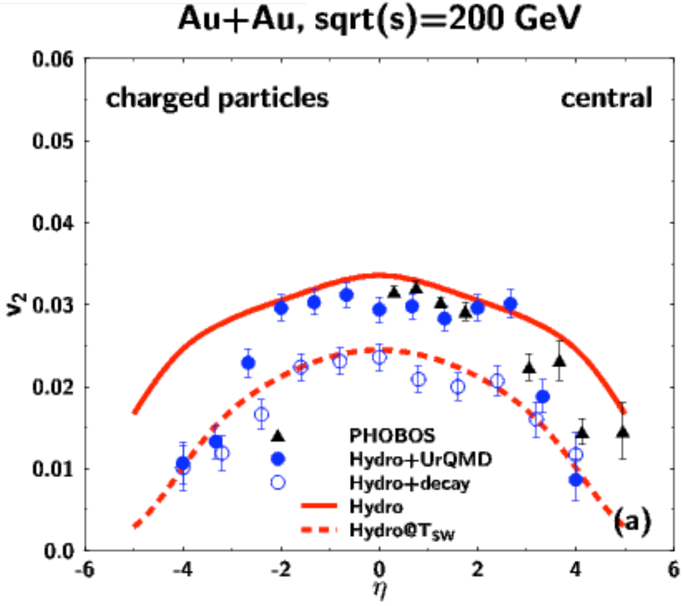
Highlight of 3D Hydro+UrQMD

Nonaka and Bass PRC75:014902(2007)









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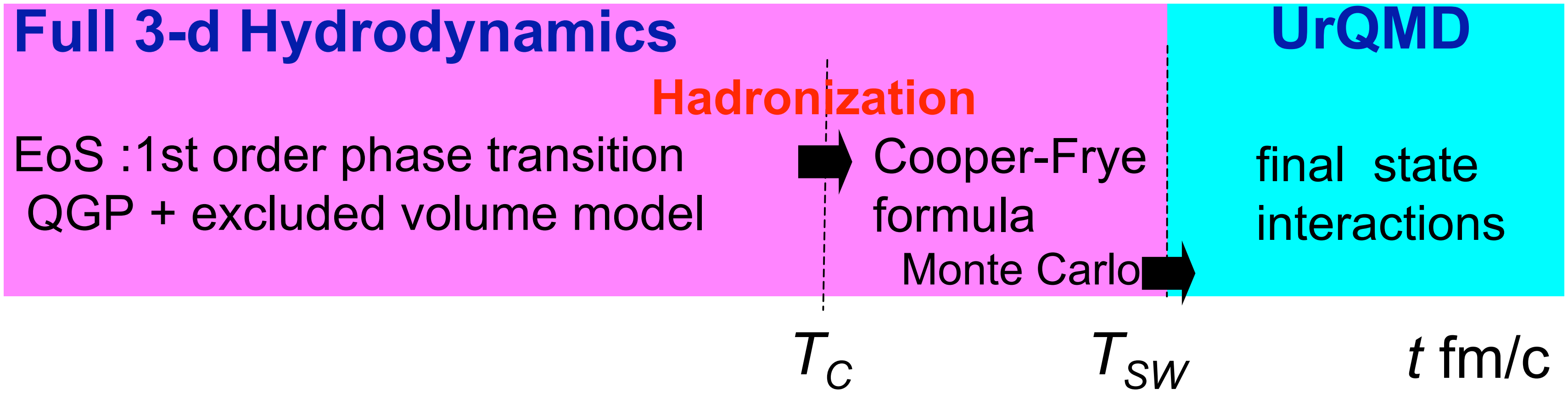
Realistic Equation of States

- **3D Hydro + UrQMD**



Full 3-d Hydrodynamics

UrQMD

Hadronization

EoS :1st order phase transition
QGP + excluded volume model

Cooper-Frye formula

Monte Carlo

final state interactions

T_C:critical temperature > T_{SW}: Hydro → UrQMD

initial conditions
- parametrization

equation of states
- bag model

freezeout process
- Viscosity effect of hadron phase
- Final state interactions

↓

Realistic EOS with QCD critical point

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EOS with QCD Critical Point

Nonaka and Asakawa, PRC71,044904(2005)

- **Singular part near QCD critical point + Non-singular part**
  - Non-singular part

    QGP phase and hadron phase
  - Singular part

3d Ising Model ⟷ **QCD**

Same Universality Class

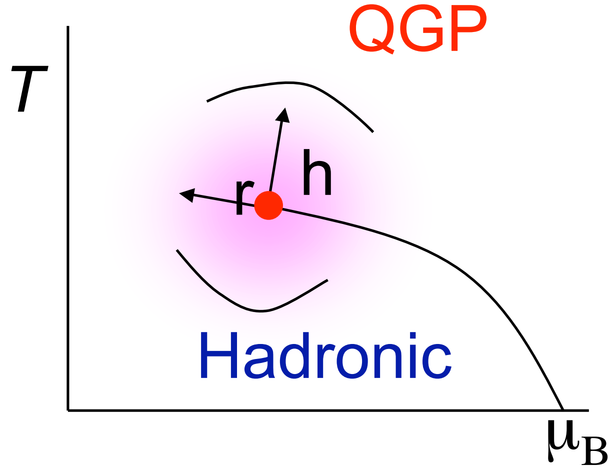
$r = \frac{T - T_C}{T_C}$

h: extermal magnetic field

$T\,,\ \mu_B$

$(r, h) \longleftrightarrow (T, \mu_B)$

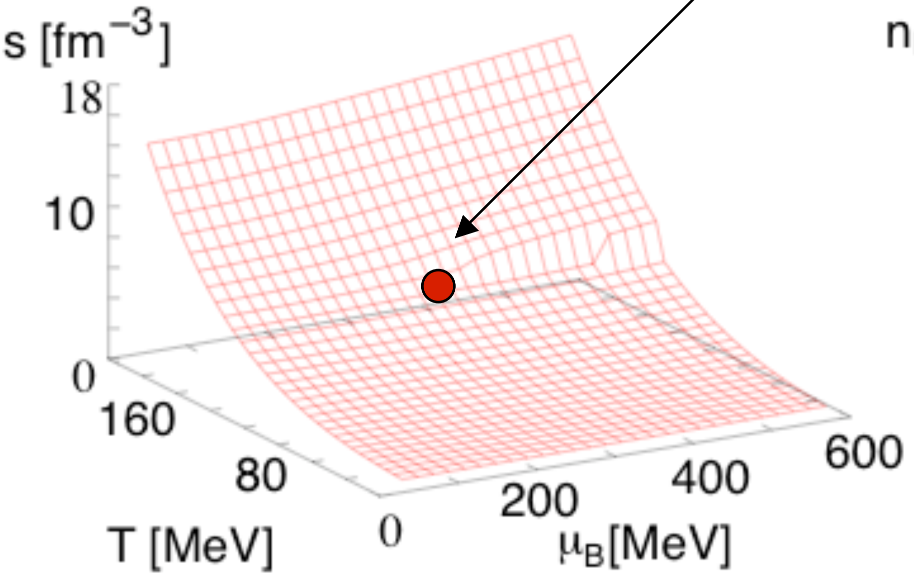
- Mapping (r, h) ➔ (T, μ_B)
- Matching with known QGP and hadronic entropy density
- Thermodynamical quantities



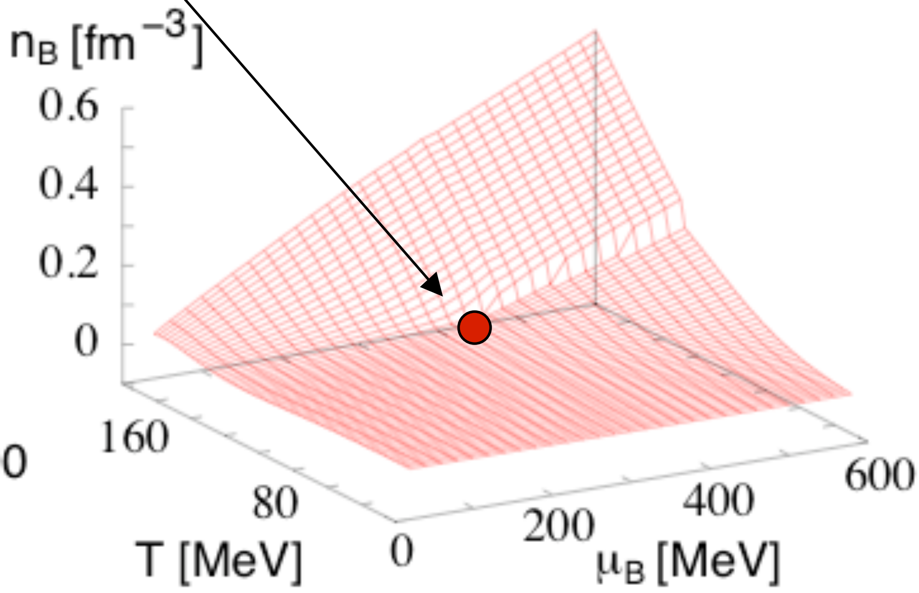
Equation of State

$T_E = 154.7$ [MeV], $\mu_E = 367.7$ [MeV]

QCD critical point



entropy density

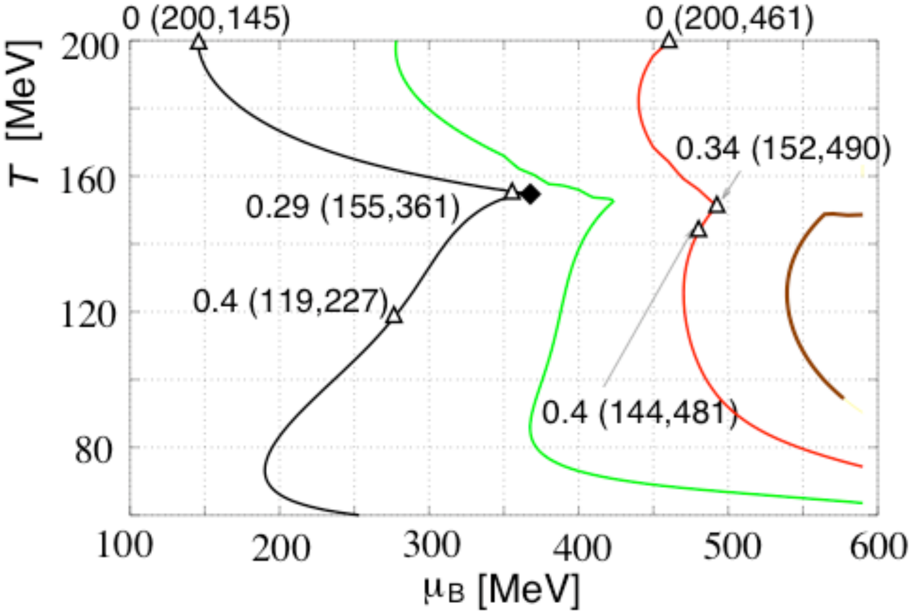


baryon number density

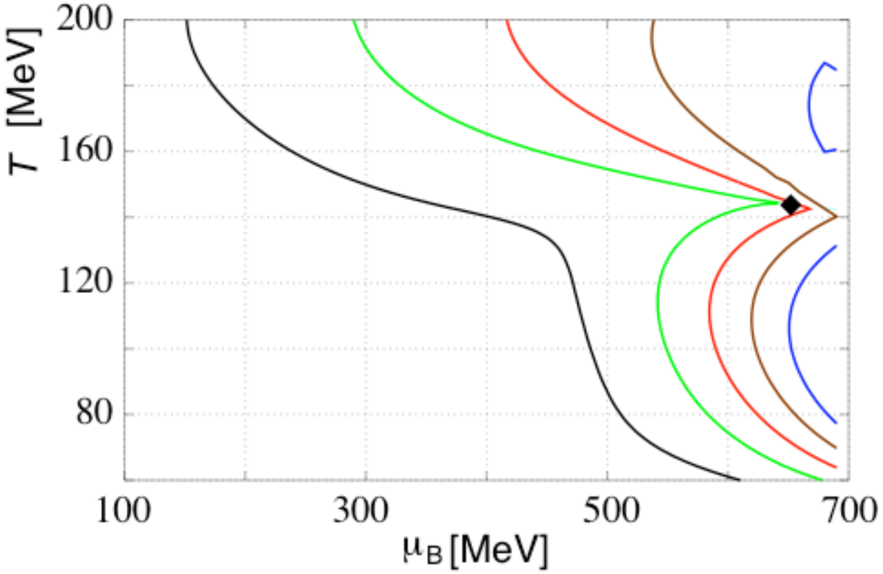
Isentropic Trajectories

- **Isentropic trajectories: n_B/s=const. line**

$T_E = 154.7$ MeV, $\mu_E = 367.7$ MeV



$T_E = 143.7$ MeV, $\mu_E = 652.0$ MeV



•Behavior of focusing depends on the location of QCP on T-μ_B plane.
•The focusing effect appears stronger in the case (T,μ_B)=(143,652).
•The location of QCP is a parameter. → Experiments

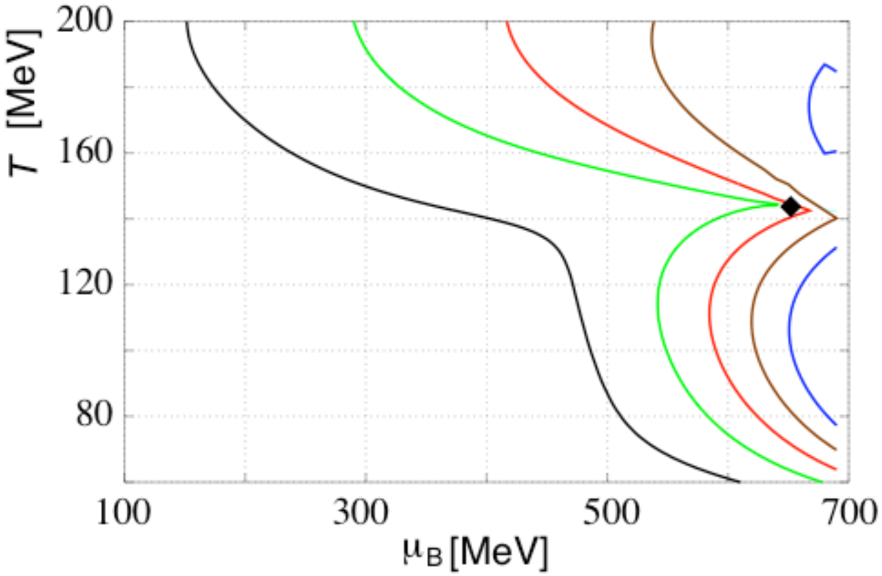
Comparison with Bag Model

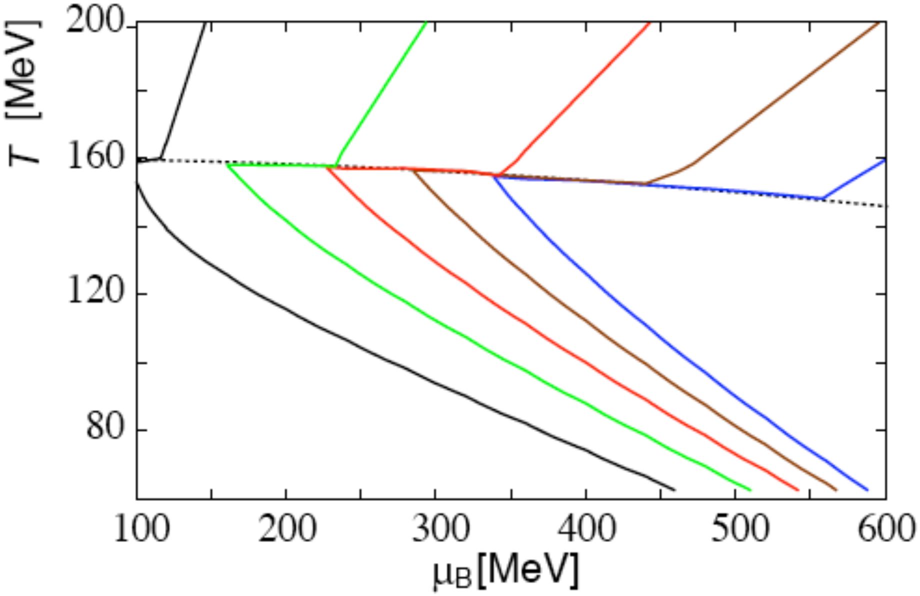
- **Isentropic trajectories on *T*- μ_B plane**

With QCD critical point

Bag Model +
Excluded Volume Approximation
(No Critical Point)

= Usual Hydro Calculation





Focused

Not Focused

3D Hydro+UrQMD with QCP

QCP: $T_E = 143.7$ MeV, $\mu_E = 652.0$ MeV

Initial Conditions

- Energy density

  $$\varepsilon(x,y,\eta) = \varepsilon_{\max} W(x,y;b) H(\eta)$$

- Baryon number density

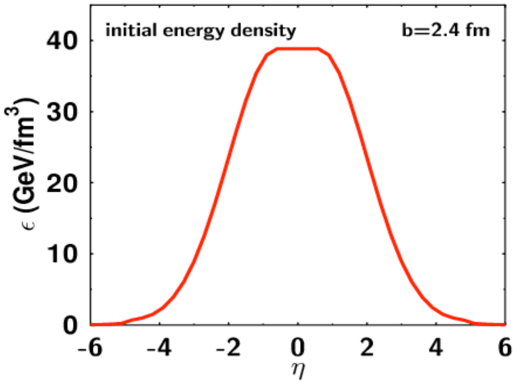
  $$n_B(x,y,\eta) = n_{B\max} W(x,y;b) H(\eta)$$

- Parameters $\begin{cases} \tau_0 = 0.6 \text{ fm/c} \\ \eta_0 = 0.5 \;\; \sigma_\eta = 1.5 \end{cases}$

- Flow

  $v_L = \eta$ (Bjorken's solution); $v_T = 0$

•longitudinal direction: $H(\eta)$



	ε_{max} GeV/fm^3	n_{Bmax} fm^{-3}
1	**1.5**	**0.2**
2	**2.0**	**0.15**

EOS: QCP, Bag Model

Switching temperature

T_{SW}=150 [MeV]

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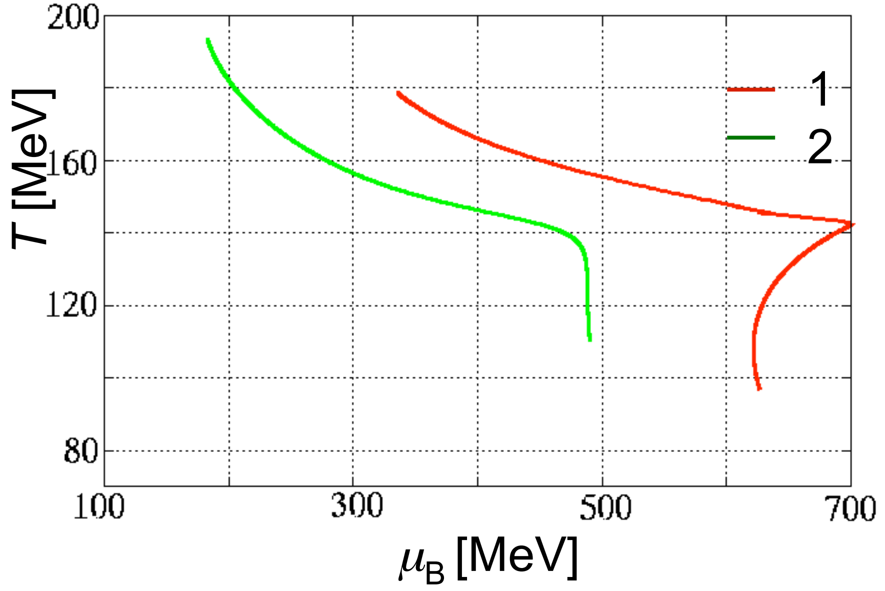
Isentropic Trajectories in Hydro

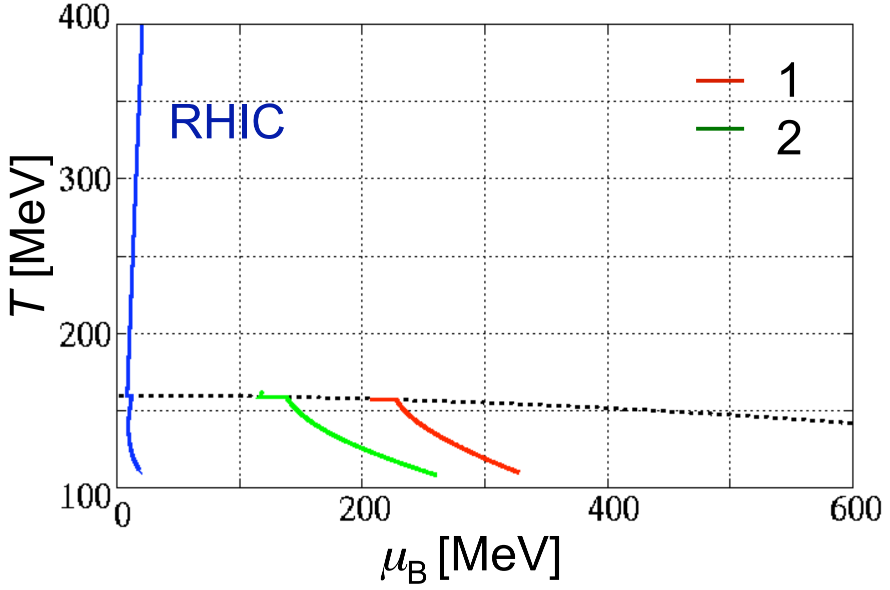
- T and μ_B in one volume element close to center

QCP: $T_E = 143.7$ MeV, $\mu_E = 652.0$ MeV

Hydro with QCD critical point

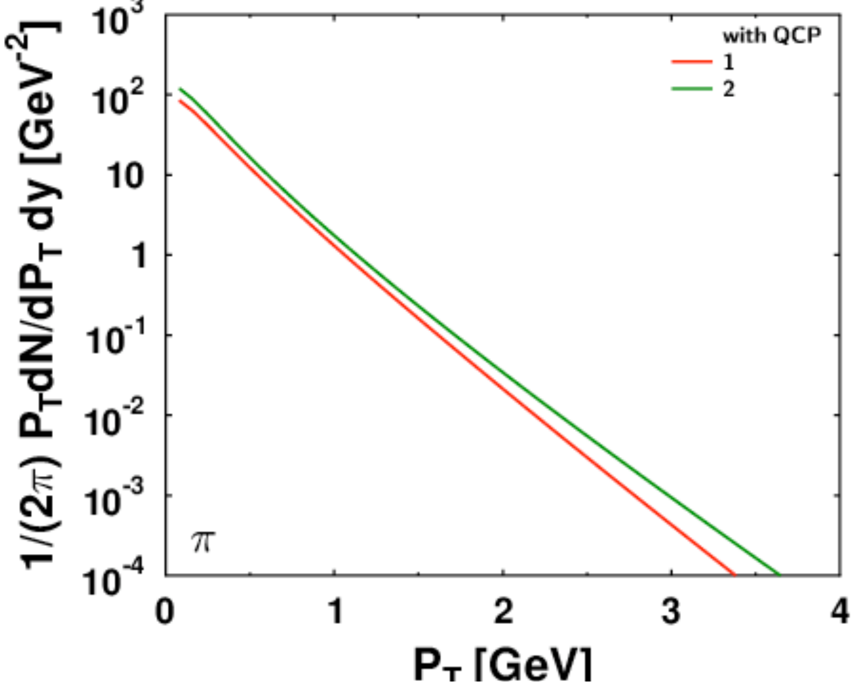


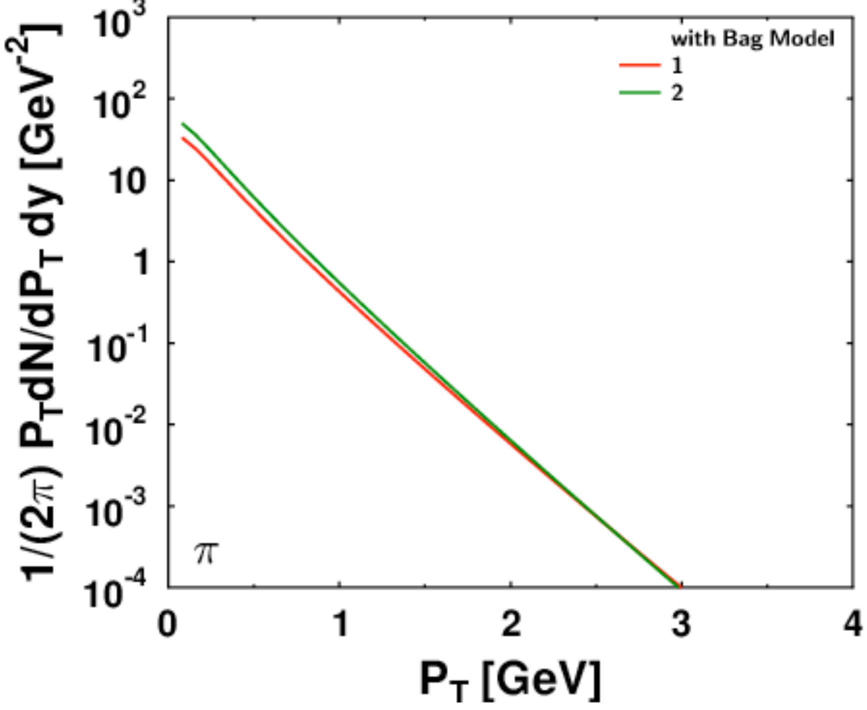
Hydro with Bag Model



- T_f=110 MeV
- Behavior of isentropic trajectories in hydro with QCP is different from one in hydro with bag model.
- Focusing effect appears in hydro with QCD critical point.

P_T Spectra in 3D Hydro

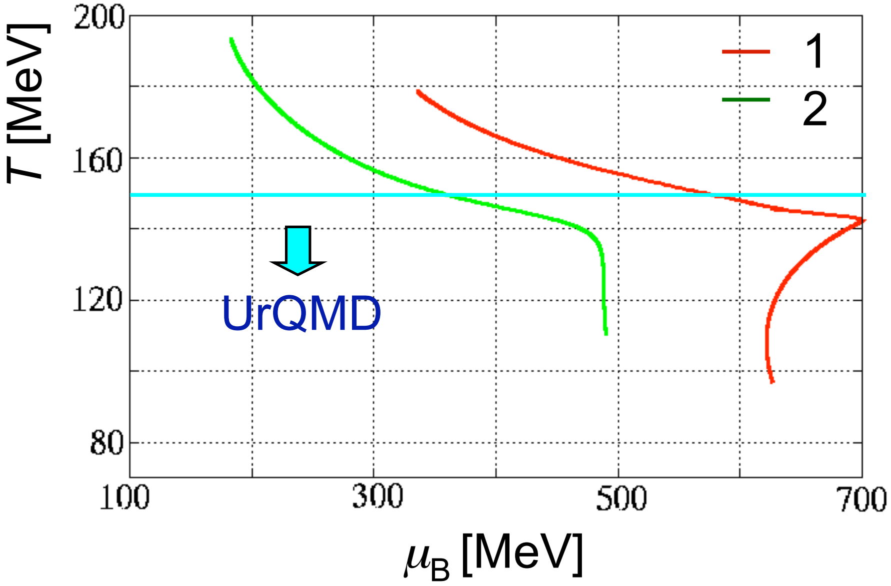

Hydro with QCD critical point


Hydro with Bag Model

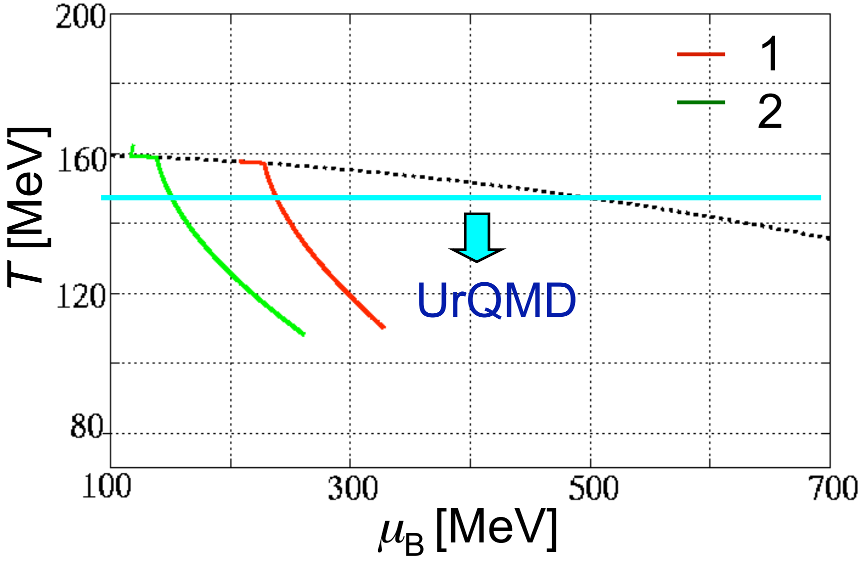
- T_f=110 MeV
- P_T slope is almost the same.

3D Hydro + UrQMD

Hydro with QCD critical point



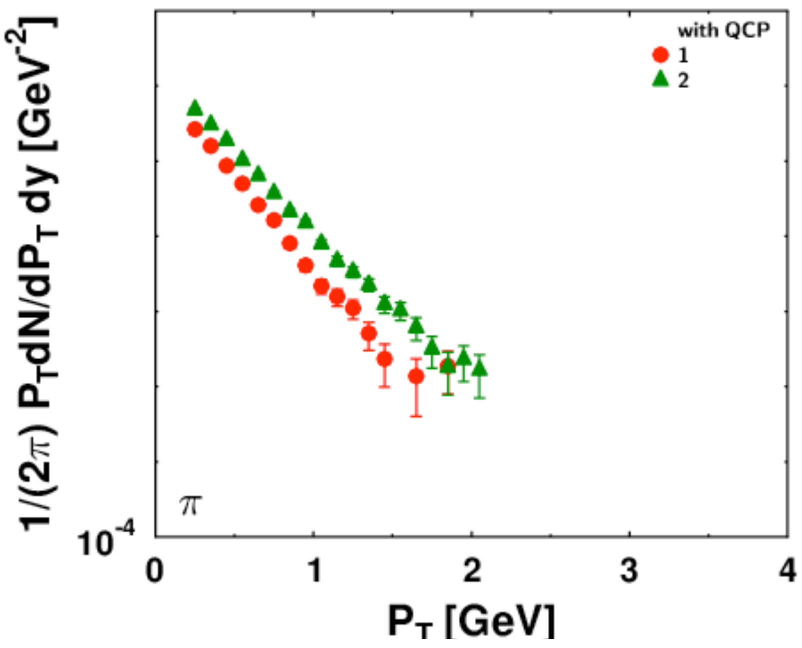
Hydro with Bag Model



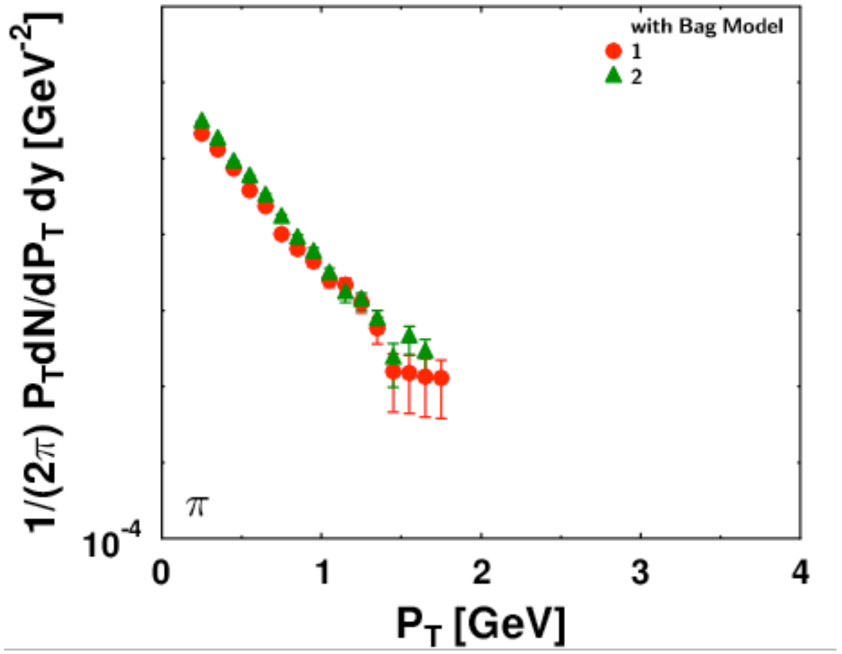
Switching temperature: 150 MeV

P_T Spectra

QCD critical point



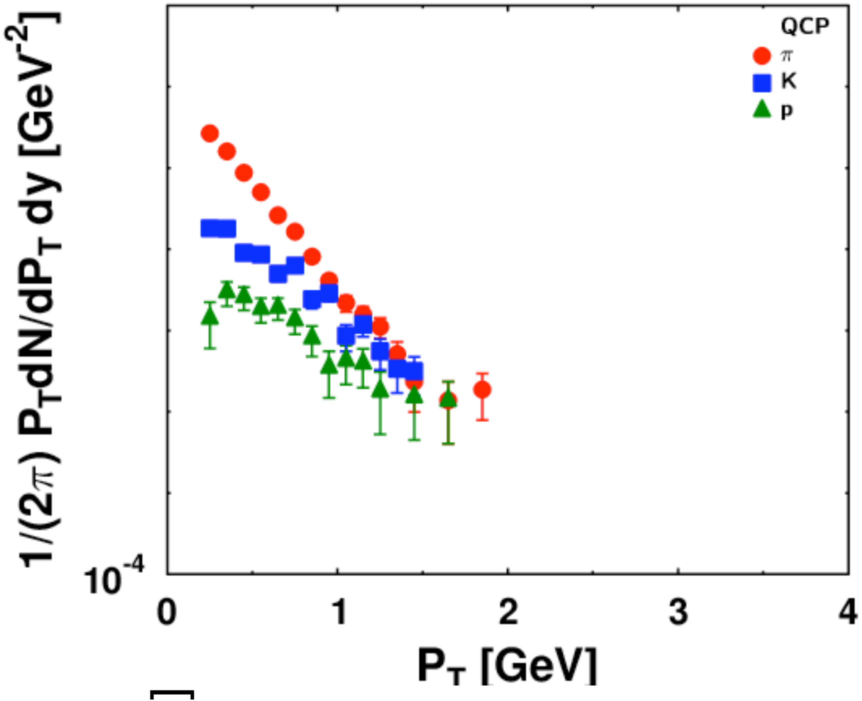
Bag Model



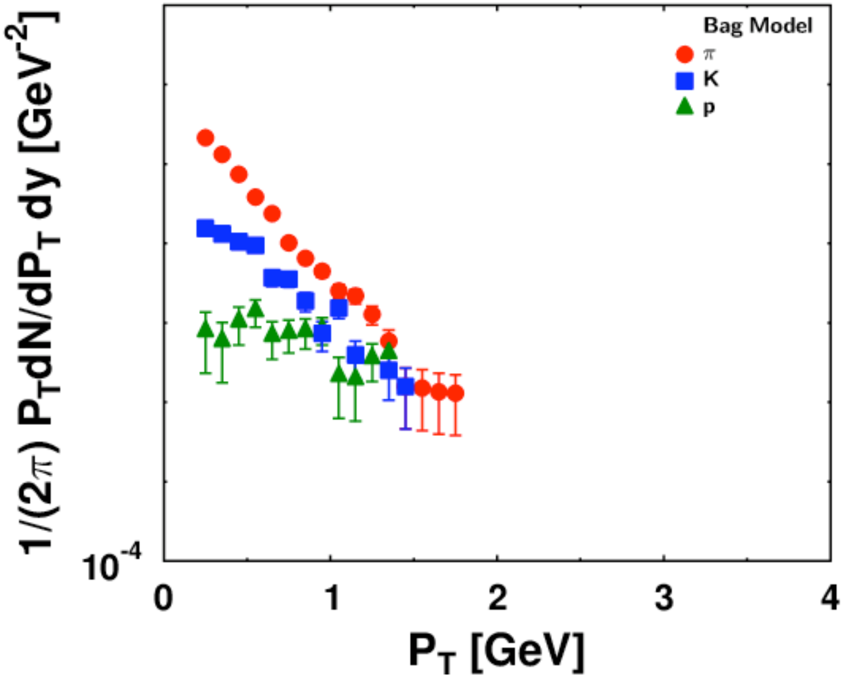
- P_T slope is almost the same.
- Larger difference of different initial conditions appears in the case of QCP.

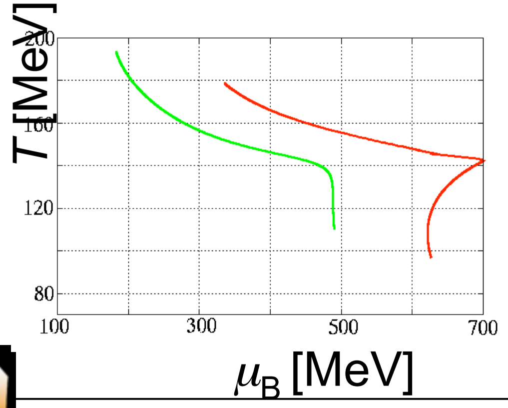
Hadron Ratios

QCD critical point



Bag Model



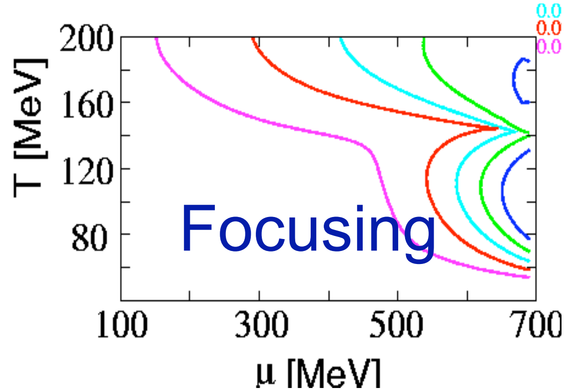


Because of focusing effect

At T_{SW} $\langle \mu_B \rangle_{QCP} > \langle \mu_B \rangle_{BG} \Rightarrow \frac{p}{\pi}_{QCP} > \frac{p}{\pi}_{BG}$

Summary

- **3D Hydro + UrQMD Model with the QCD critical point**
  - Isentropic trajectories
  - P_T spectra, hadron ratio



- **The QCD critical point search**
  - Energy scan

    parameter sets of ε and n_B in initial conditions
  - Switching temperature dependence
  - Location of QCD critical point
- **Physical observables**
  - Fluctuations
  - Balance function

BACKUP

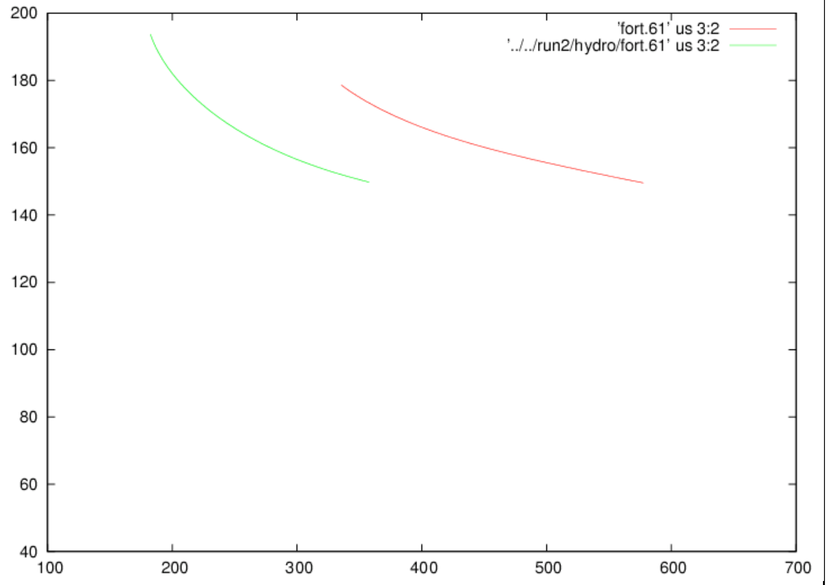
Mean P_T Fluctuation

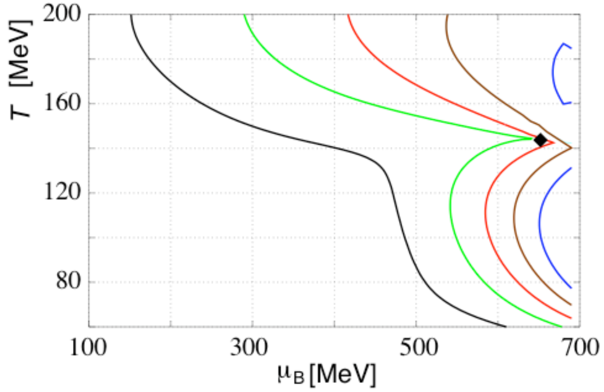
$P_T < 1$ GeV

	π	p	K
1	0.567E-02	0.266E-01	0.156E-01
2	0.416E-02	0.242E-01	0.115E-01

*In this calculation, definition of mean Pt fluctuation is different from CERES.

preliminary

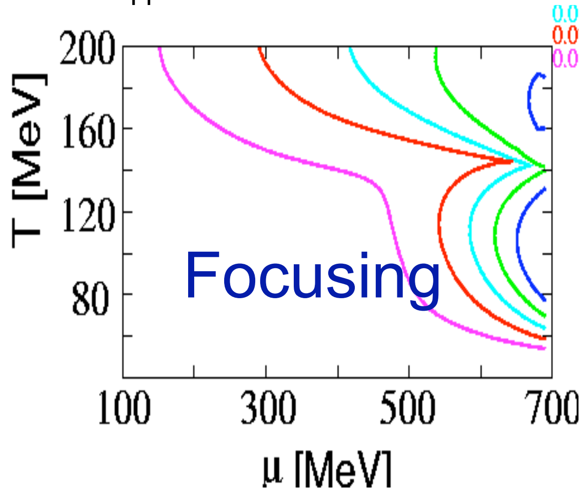




CEP and Its Consequences

Consequences

- Slowing out of equilibrium
  - Large fluctuation
- Freeze out temperature at RHIC
- Fluctuation



Existence and location of CEP in phase diagram:
Collision energy dependent experiments are indispensable.

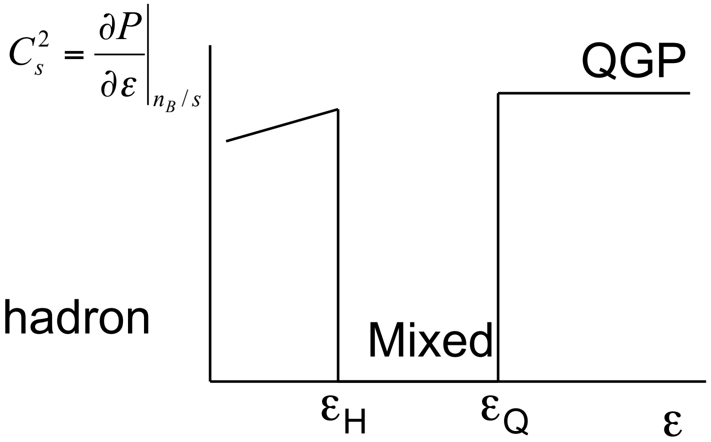
Work in Progress

- Realistic hydro calculation with critical point

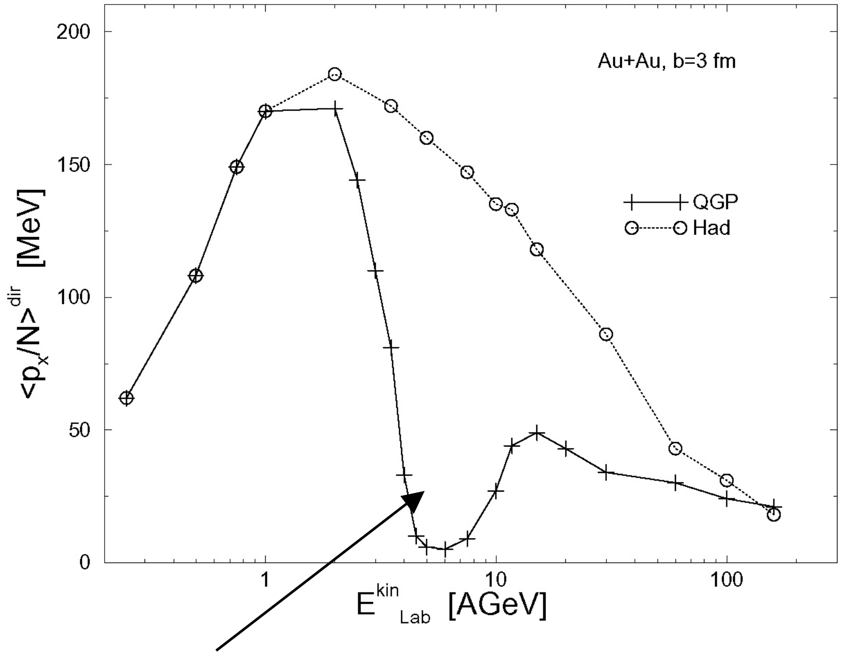
Sound Velocity

- **EoS** ⟷ **Hydrodynamic Expansion**

$$C_s^2 = \left.\frac{\partial P}{\partial \varepsilon}\right|_{n_B/s}$$



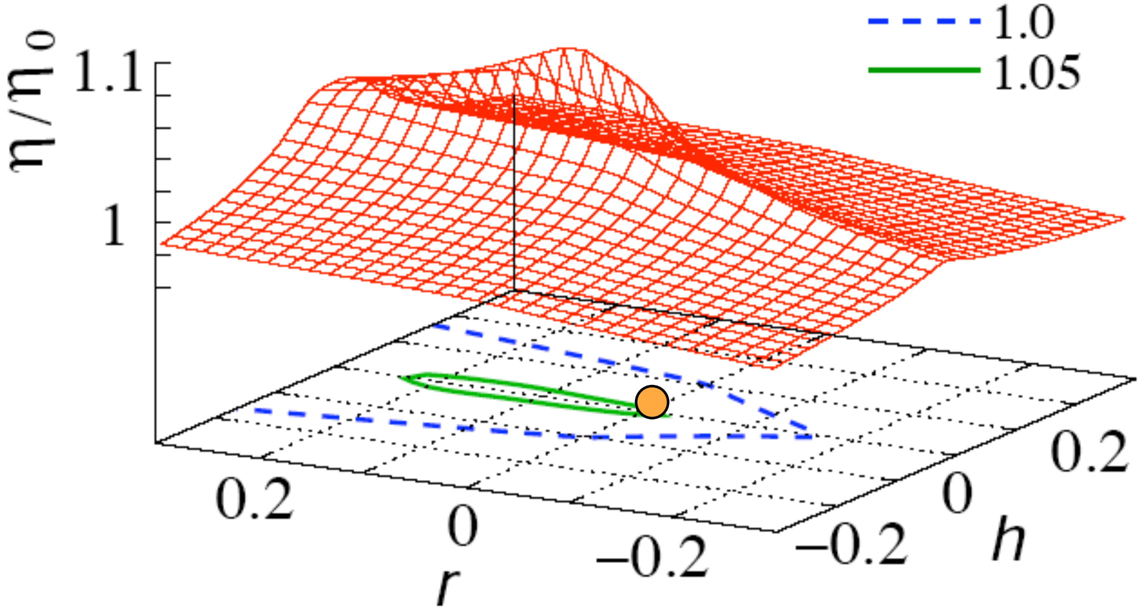
Ex. Rischke et al. nucl-th/9504021



Effect of mixed phase?

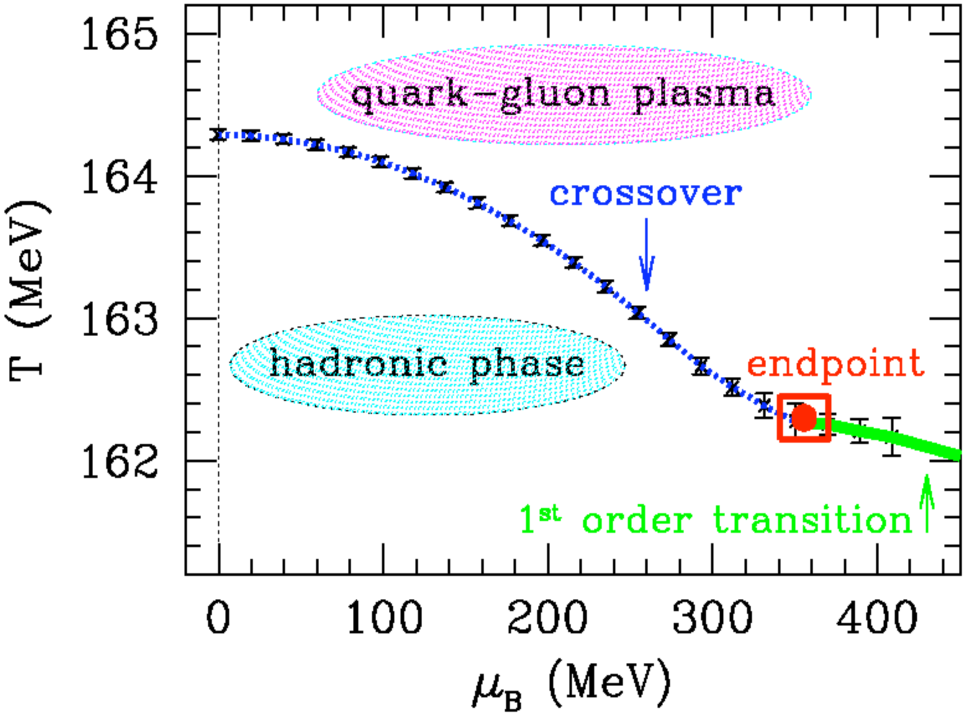
Shear Viscosity

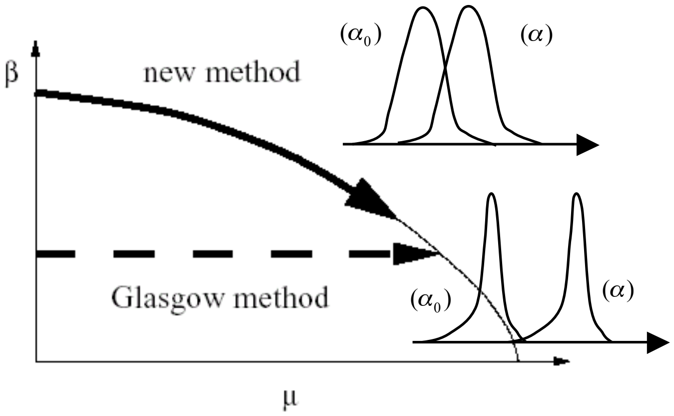
$$\frac{\eta}{\eta_0} = \left(\frac{\xi_{\mathrm{eq}}}{\xi_{\mathrm{eq},0}} \right)^{\left[\frac{1}{19}\varepsilon + O(\varepsilon^2) \right]}$$



Lattice QCD at finite T and μ_B

- Multi-parameter reweighting technique





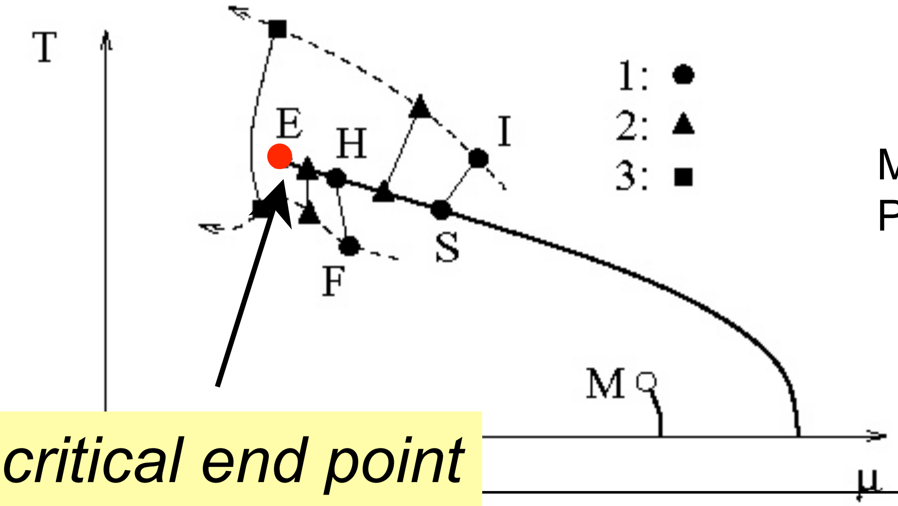
$$\alpha = (\beta, m, \mu)$$

- Fodor and Katz (JHEP 0203 (2002) 014)
  hep-lat/0402006
- Allton et al. (Bielefeld-Swansea)

Overlap problem:
The lattice size is small.

Maezawa san’s talk

Phenomenological Consequence ?



critical end point

M. Stephanov, K. Rajagopal, and E.Shuryak, PRL81 (1998) 4816

Divergence of Fluctuation

Correlation Length

If expansion is adiabatic.

Still we need to study

- Hadronic Observables : NOT directly reflect properties at CEP

  Fluctuation, Collective Flow

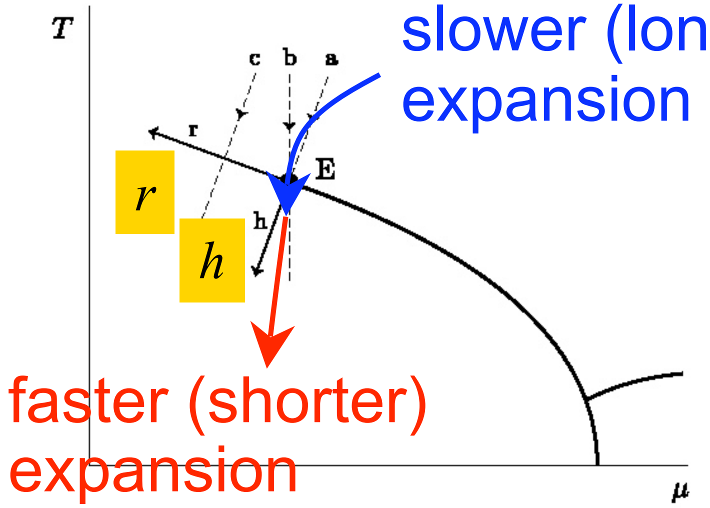
- EOS

  Focusing in n_B/s trajectories

- Dynamics (Time Evolution)

Time Evolution

- Berdnikov and Rajagopal's Schematic Argument

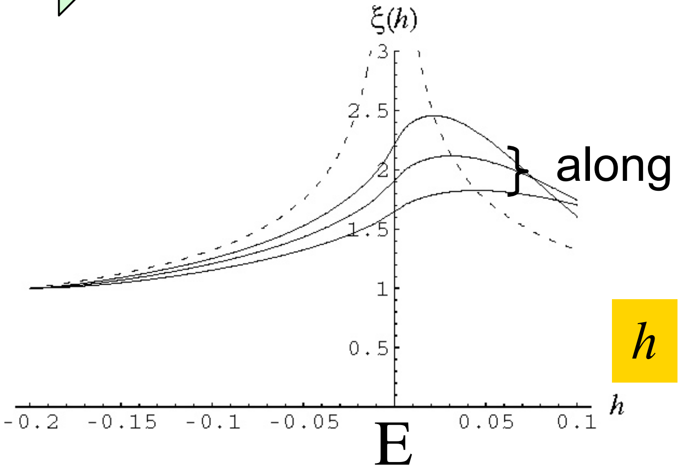


slower (longer) expansion

faster (shorter) expansion

B. Berdnikov and K. Rajagopal,
Phys. Rev. D61 (2000) 105017

Correlation Length longer than L_{eq}



} along $r = const.$ line

- What's missing:
  - Realistic Hydro Calculation with Realistic EOS

3-d Hydrodynamic Model

- **Hydrodynamic equation**

[illegible]

$$T^{\mu\nu} : T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - p(g^{\mu\nu} - u^{\mu} u^{\nu})$$

ε : energy density p : pressure u^{μ} : local velocity

  - Baryon number density conservation

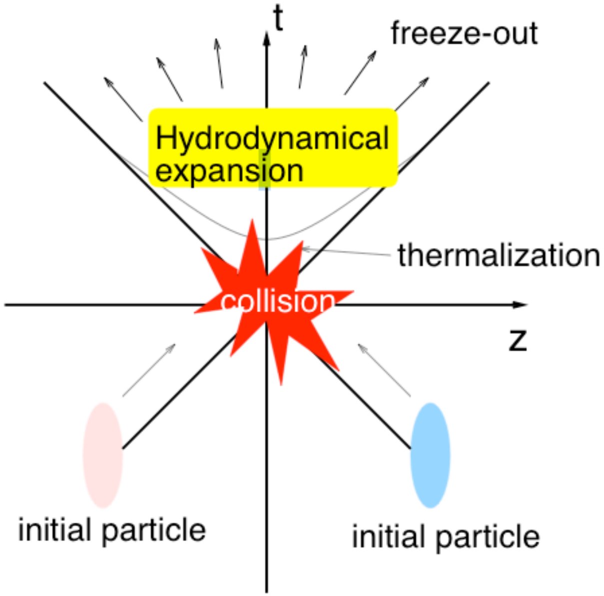
[illegible]

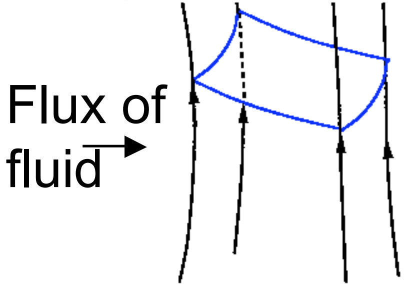
- **Coordinates**

[illegible]

- **Lagrangian hydrodynamics**
  - Tracing the adiabatic path of each volume element
  - Effects of phase transition of observables
- **Algorithm**
  - Focusing on conservation law

[illegible]

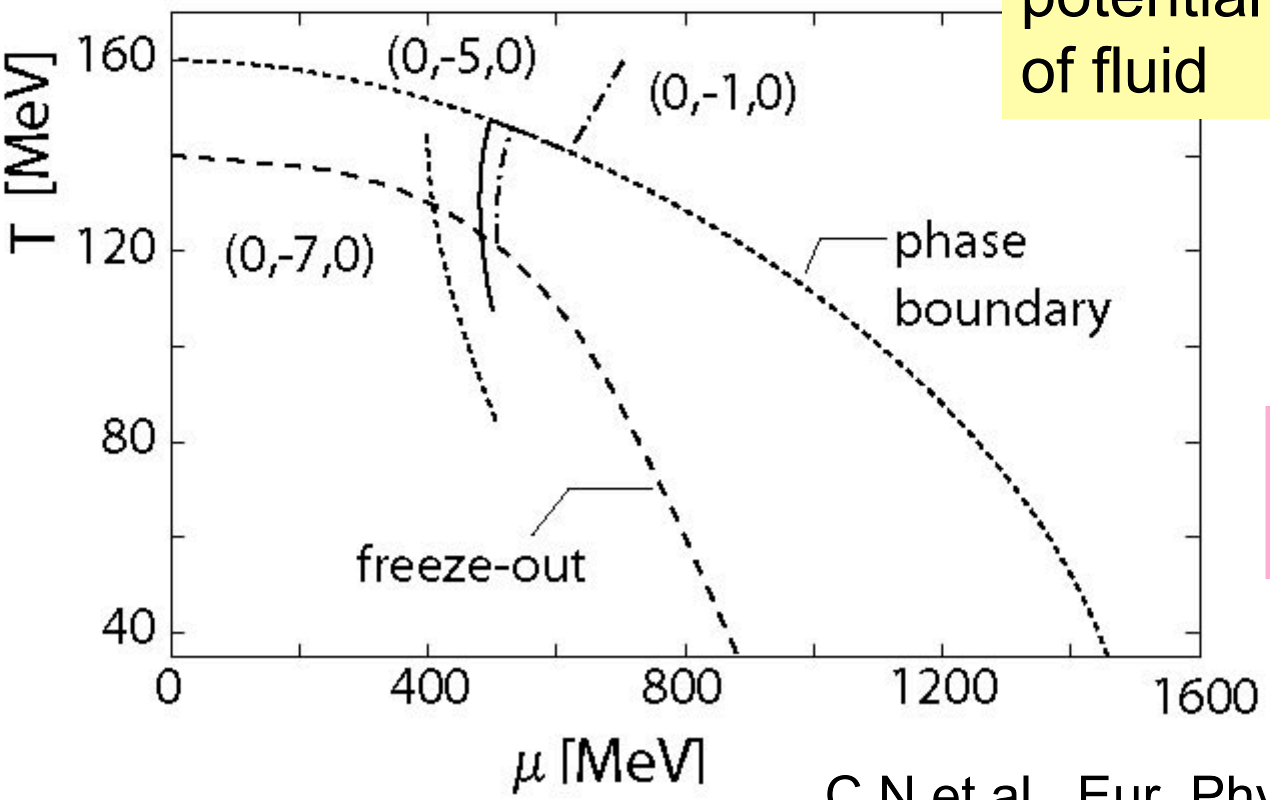




Trajectories on the phase diagram

- **Lagrangian hydrodynamics**

temperature and chemical potential of volume element of fluid



Effect of phase transition

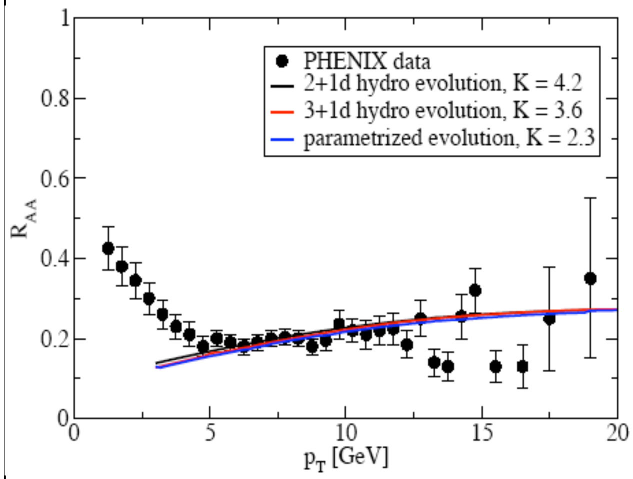
C.N et al., Eur. Phys.J C17,663(2000)

Jets in medium

Jet quenching mechanisms

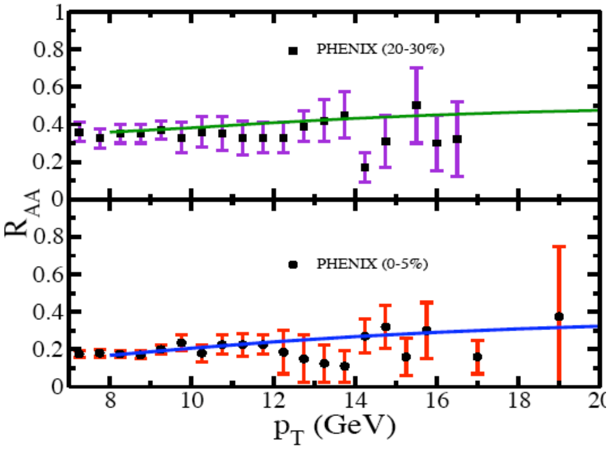
Ex. Nuclear modification factor in 3D hydro

- parton path



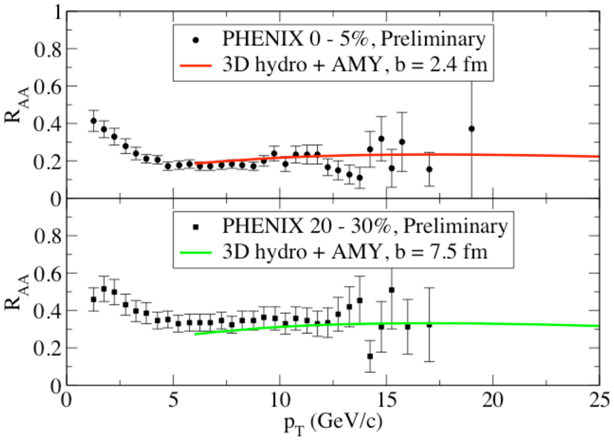
nucl-th/0611027
with Renk and Ruppert

- pQCD
- Mixed and hadron phase



nucl-th/0703019
with Majumder

- AMY
- gluon bremsstrahlung



arXiv:0705.2575(hep-ph)
with Qin, Ruppert, Turbide,
Gale

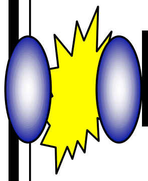
Soft + Hard

Soft
- Full 3-d Hydrodynamic Model
  - QGP formation, EoS

Improved Cooper-Frye formula (Reco)

UrQMD

t fm/c



Interaction between Soft and Hard

Hadronization

Final Interactions

Hard
- Hard scattering & jet production
- Propagation of jet in medium, energy loss

Fragmentation

- First schematic attempt
  Hirano & Nara
  PRC66:041901,2002,
  PRL91:082301,2003

- Dynamical effect on jets
- Jet correlations

Non-Singular Part

Hadron Phase

- Excluded volume model

$$P(T,\mu_B) = \sum_i P_i^{\text{ideal}}(T,\mu_{Bi} - V_0 P(T,\mu_{Bi}))$$

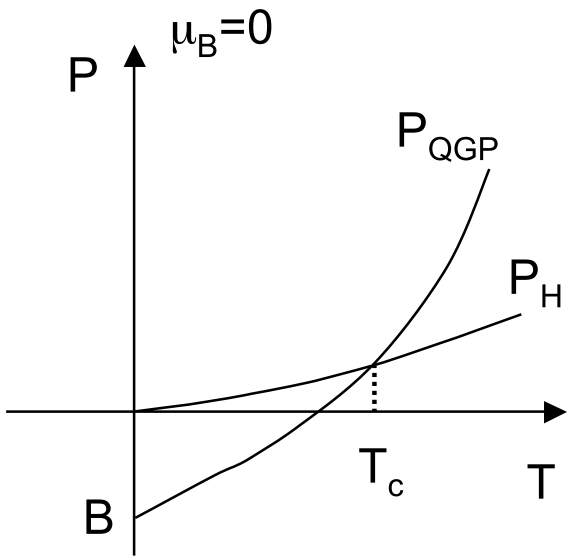
$$= \sum_i P_i^{\text{ideal}}(T,\tilde{\mu}_{Bi})$$

QGP Phase

$$P(T,\mu_B) = \frac{(32+21N_f)\pi^2}{180}T^4 + \frac{N_f}{2}\left(\frac{\mu_B}{3}\right)^2 T^2 + \frac{N_f}{4\pi^2}\left(\frac{\mu_B}{3}\right)^4 - B$$

$N_f=2$

B:Bag constant (220 MeV)^4



Isentropic Trajectories in Hydro

- **Conservation law in ideal fluid**

[illegible]

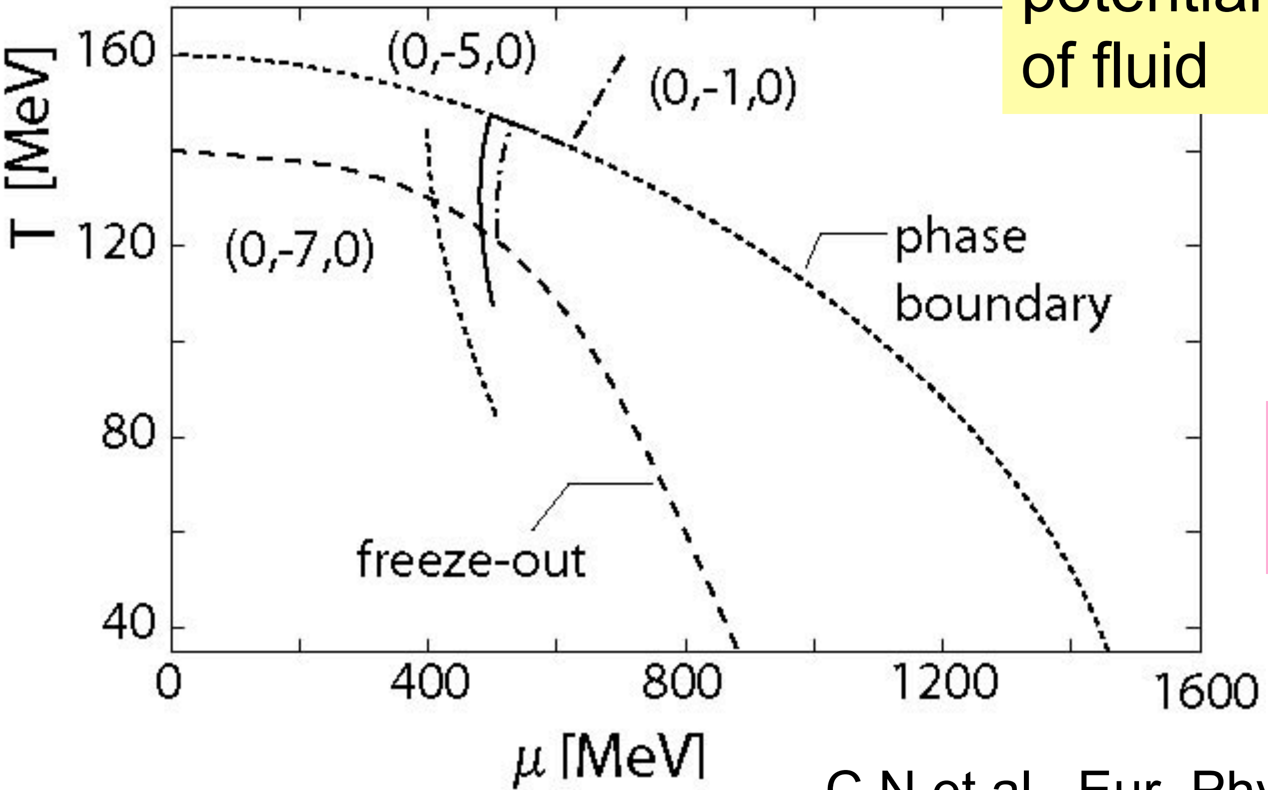
→ Hydrodynamic expansion: n_B/s =const.

→ n_B/s =const. lines on T-μ_B plane: behavior of expansion

Trajectories on the phase diagram

- **Lagrangian hydrodynamics**

temperature and chemical potential of volume element of fluid

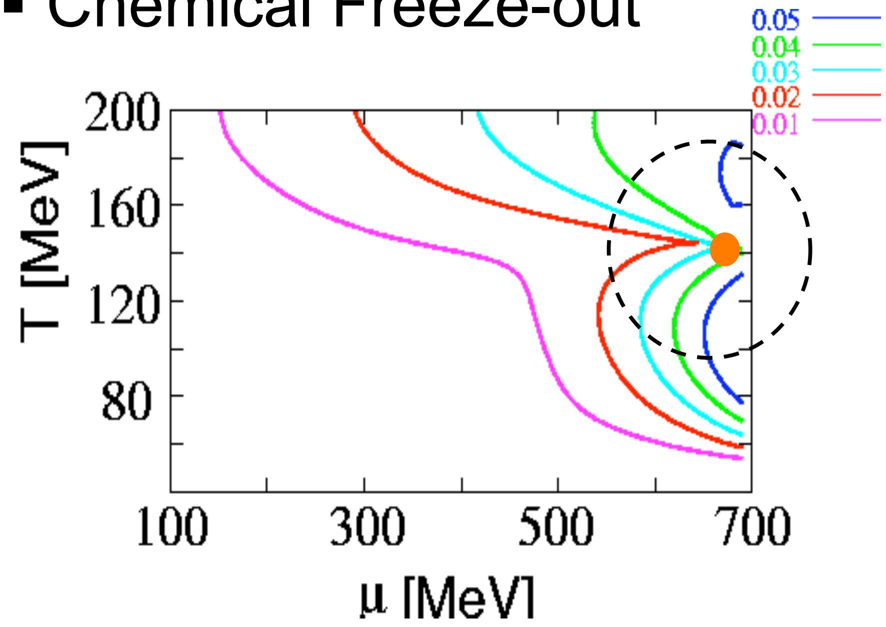


Effect of phase transition

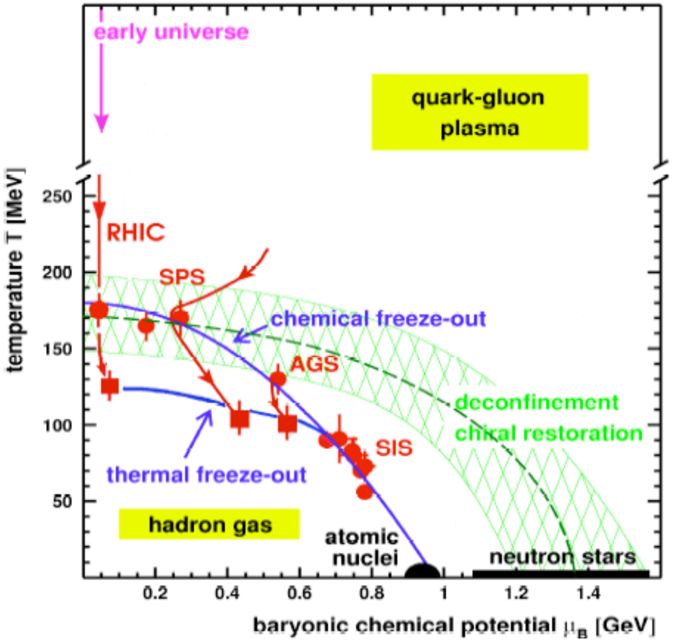
C.N et al., Eur. Phys.J C17,663(2000)

Hadron ratios near CEP

- Chemical Freeze-out



Heinz :hep-ph/0109006



•Hadron ratio (ex. p/π) is not sensitive to collision energy near CEP.

•Near chemical freeze-out temperature the contribution from recombination, resonances is small.

•Statistical Model
Free resonance gas model

⇕

•At chemical freeze-out
Quasi-particle state

EOS of 3-d Ising Model

- **Parametric Representation of EOS**

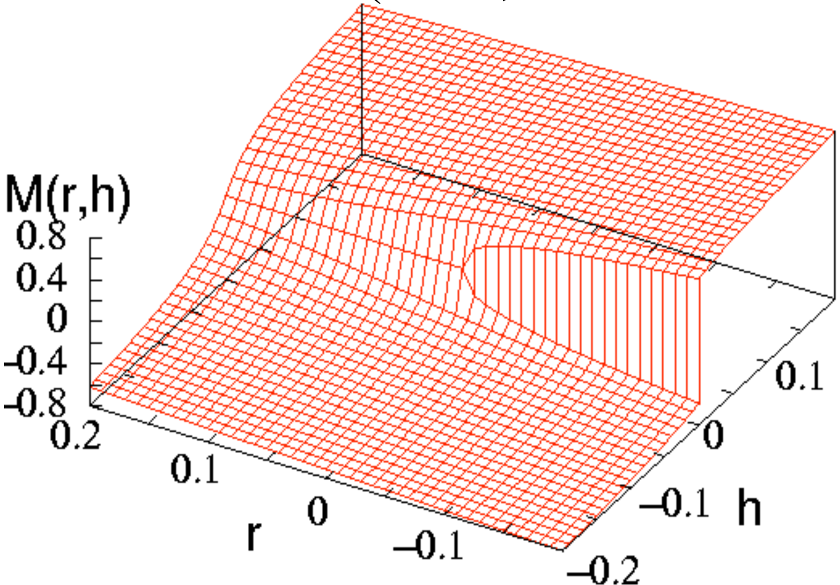
$$\left\{\begin{aligned} M &= M_0 R^\beta \theta \\ h &= h_0 R^\beta \tilde{h}(\theta) = h R_0^{\beta\delta}(\theta - 0.76201\theta^3 + 0.00804\theta^5) \\ r &= R(1-\theta^2) \qquad (R \geq 0,\ \ -1.154 \leq \theta \leq 1.154) \end{aligned}\right.$$

$$r = \frac{T - T_C}{T_C}$$

h : external magnetic field

$$\beta = 0.326$$
$$\delta = 4.8$$

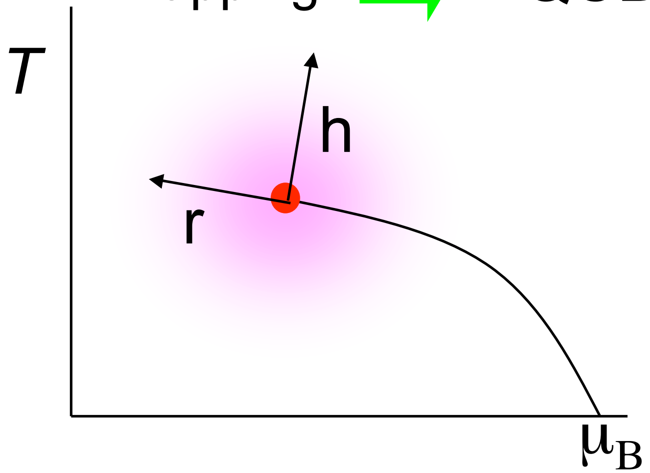
Guida and Zinn-Justin NPB486(97)626



Order parameter

$$M \leftrightarrow \langle \bar{q} q \rangle$$

Mapping → QCD



Singular Part of EOS

Gibbs Free Energy

Guida and Zinn-Justin NPB486(97)626

$$G(h,r) = F(M,r) - Mh$$

Free energy: $F(M,r) = h_0 M_0 R^{2-\alpha} \boxed{g(\theta)} \longleftarrow h = \left.\left(\frac{\partial F}{\partial M}\right)\right|_r$

$\alpha = 0.11$

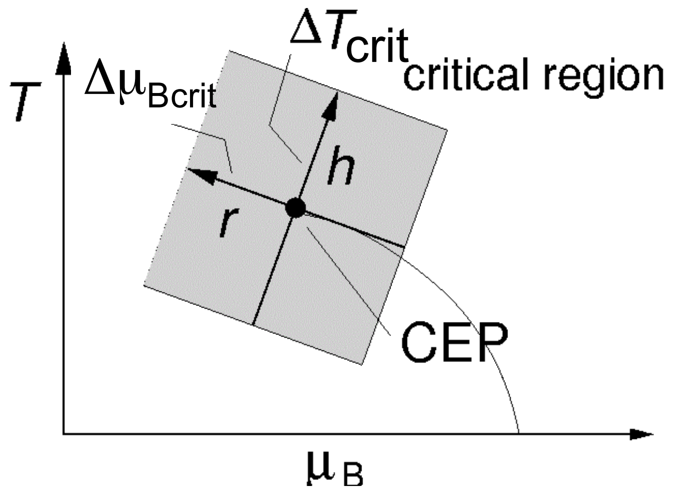
Entropy Density for Singular Part

$$s_c = -\left.\frac{\partial G}{\partial T}\right|_\mu = -\left.\frac{\partial G}{\partial h}\right|_r \boxed{\frac{\partial h}{\partial T}} - \left.\frac{\partial G}{\partial r}\right|_h \boxed{\frac{\partial r}{\partial T}}$$

$$\begin{cases} \left.\dfrac{\partial G}{\partial h}\right|_r = -M \\ \left.\dfrac{\partial G}{\partial r}\right|_h = \left.\dfrac{\partial F}{\partial r}\right|_h - \left.\dfrac{\partial M}{\partial r}\right|_h h \end{cases}$$

mapping

$$(r,h) \leftrightarrow (T,\mu_B)$$



Singular Part + Non-singular Part

Entropy Density

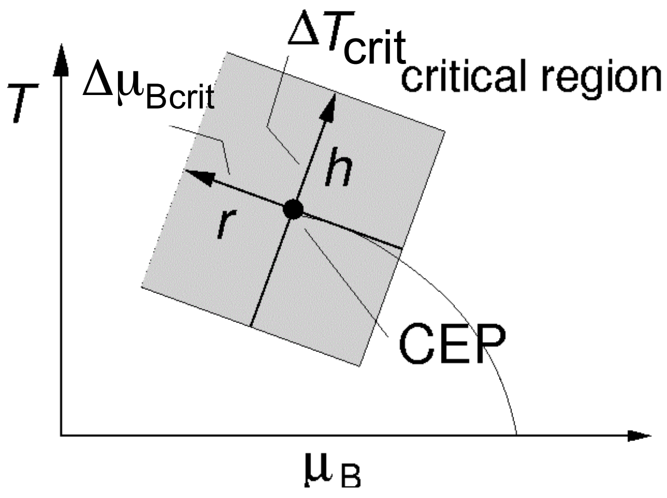
$$S_{\text{real}}(T,\mu_B)=\frac{1}{2}\left\{1-\tanh[S_c(T,\mu_B)]\right\}S_H(T,\mu_B)+\frac{1}{2}\left\{1+\tanh[S_c(T,\mu_B)]\right\}S_Q(T,\mu_B)$$

- $S_H(T,\mu_B)$ Hadron Phase (excluded volume model)
  $S_Q(T,\mu_B)$ QGP phase

- Dimensionless parameter: S_c

$$S_c(T,\mu_B)=s_c\sqrt{(\Delta T_{\text{crit}})^2+(\Delta\mu_{\text{crit}})^2}\times D$$

Critical domain



- Choice of parameters: $\Delta T_{\text{crit}}, \Delta\mu_{\text{crit}}, D$

Thermodynamical inequalities

$$\left(\frac{\partial S}{\partial T}\right)\Bigg|_{V,N}\geq 0,\quad \left(\frac{\partial P}{\partial V}\right)\Bigg|_{T,N}\geq 0,\quad \left(\frac{\partial \mu}{\partial N}\right)\Bigg|_{T,V}\geq 0$$

Critical exponent near CEP keeps correctly.

Chiho NONAKA

Thermodynamical Quantities

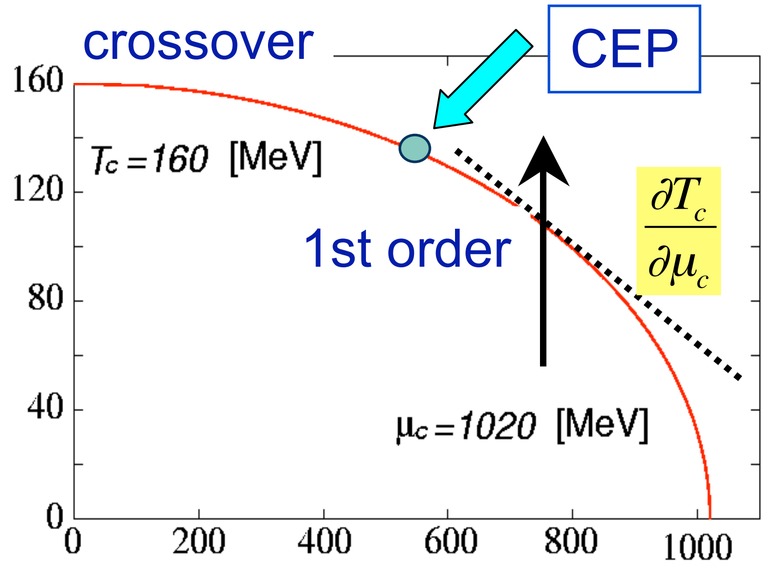
$$S_{\text{real}}(T,\mu_B) = \frac{1}{2}\{1-\tanh[S_c(T,\mu_B)]\}S_H(T,\mu_B) + \frac{1}{2}\{1+\tanh[S_c(T,\mu_B)]\}S_Q(T,\mu_B)$$

- **Baryon number density**

$$n_B(T,\mu_B) = \frac{\partial P}{\partial \mu_B} = \int_0^T \frac{\partial s}{\partial \mu_B}(T',\mu_B)\,dT' + n_B(0,\mu_B)$$

$$+\left|\frac{\partial T_C(\mu_B)}{\partial \mu_C}\right|\left(S_Q(T_C,\mu_B(T_C)) - S_H(T_C,\mu_B(T_C))\right)$$

1st order



- **Pressure**

$$P(T,\mu_B) = \int_0^T S_{\text{real}}(T',\mu_B)\,dT' + P(0,\mu_B)$$

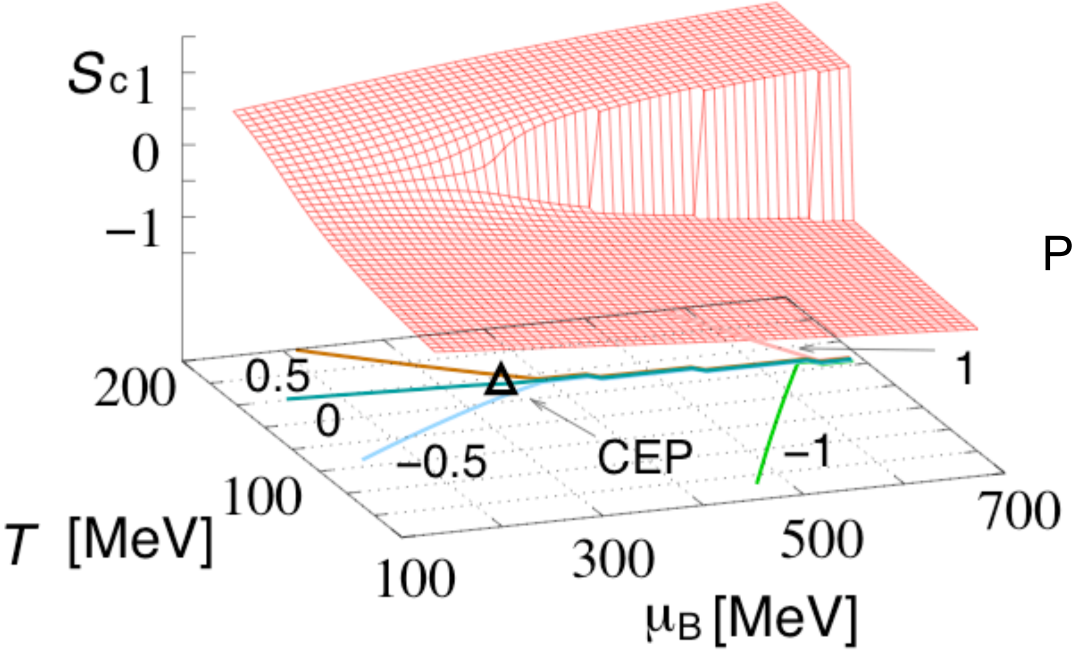
- **Energy Density**

$$\varepsilon = TS_{\text{real}} - P - \mu_B n_B$$

Dimensionless Parameter

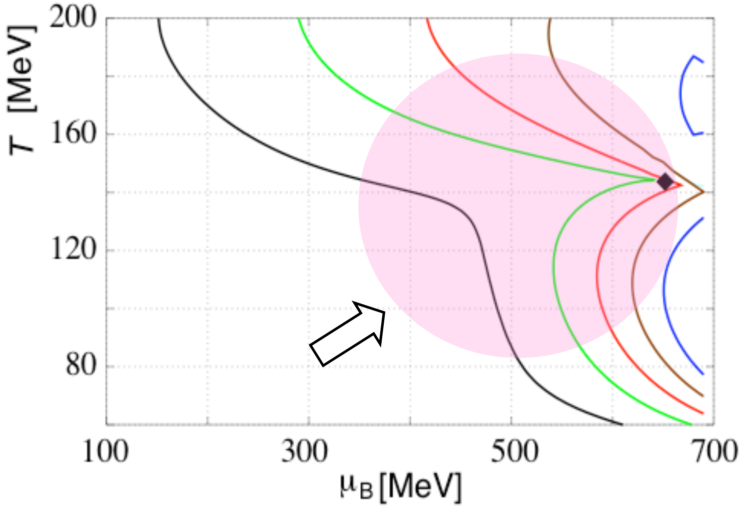
- $$s_c = -\left.\frac{\partial G}{\partial T}\right|_{\mu} = -\left.\frac{\partial G}{\partial h}\right|_{r}\frac{\partial h}{\partial T} - \left.\frac{\partial G}{\partial r}\right|_{h}\frac{\partial r}{\partial T}$$

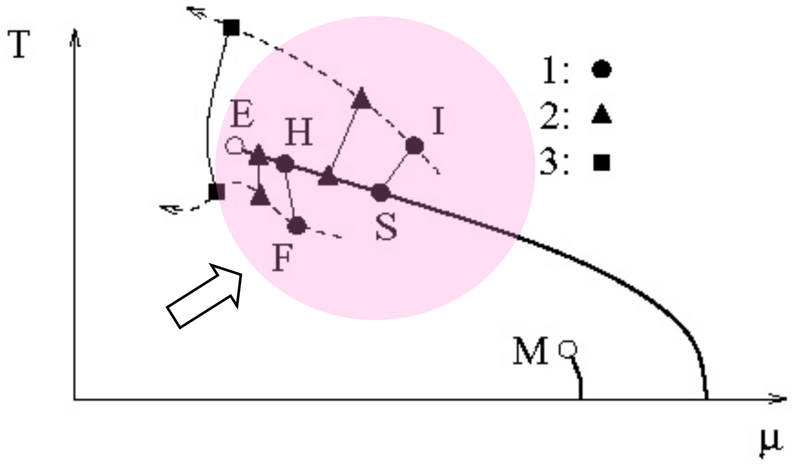
$$S_c(T,\mu_B) = s_c\sqrt{(\Delta T_{\text{crit}})^2 + (\Delta\mu_{\text{crit}})^2} \times D$$



Phase transition region

Focusing and QCD Critical Point





M. Stephanov, K. Rajagopal, and E.Shuryak,
PRL81 (1998) 4816

If the critical region is large enough

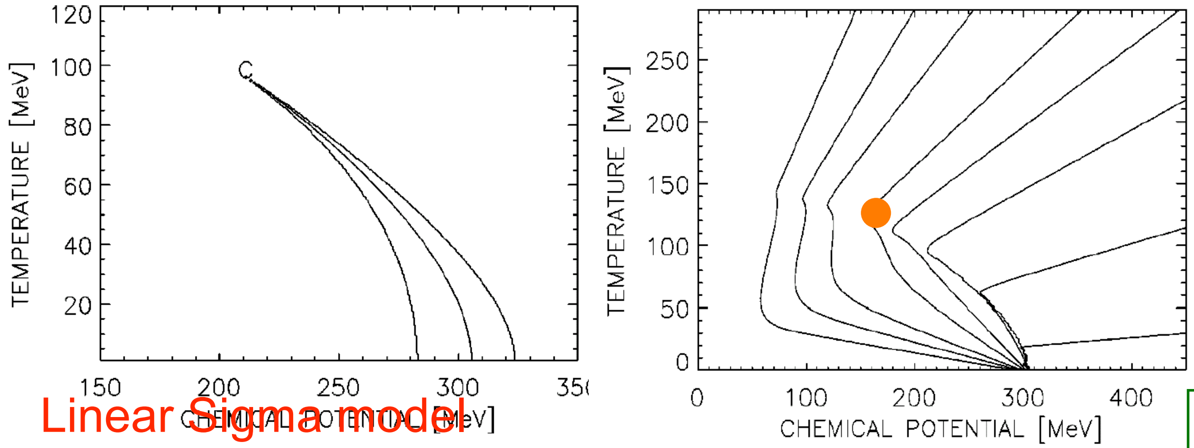
The fine-tuning of the collision energy is not necessary
not only on the high energy side but also on the low energy side.

Focusing

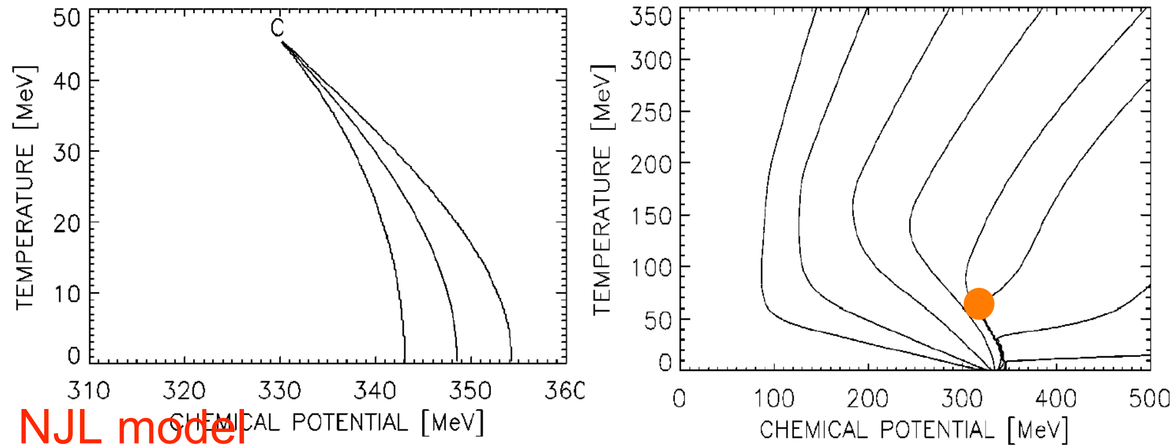
- n_B/s trajectories on T-μ plane ⟺ Hydrodynamic expansion

  Scavenius et al. PRC64(2001)045202

  entropy, baryon number conservatio



Linear Sigma model



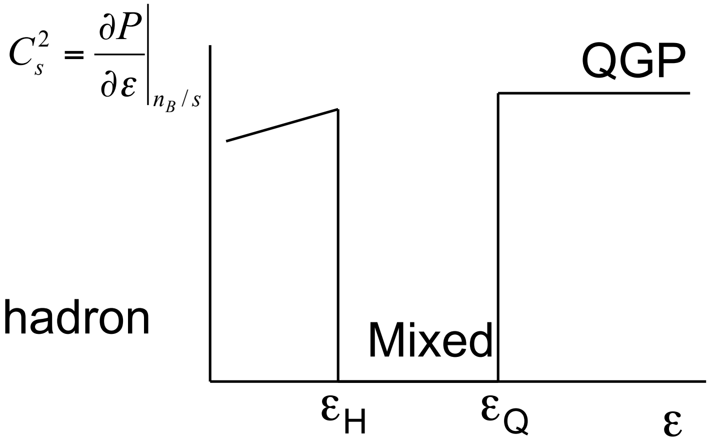
NJL model

- CEP is not an attractor for n_B/s trajectories.
- Critical phenomena near CEP ?

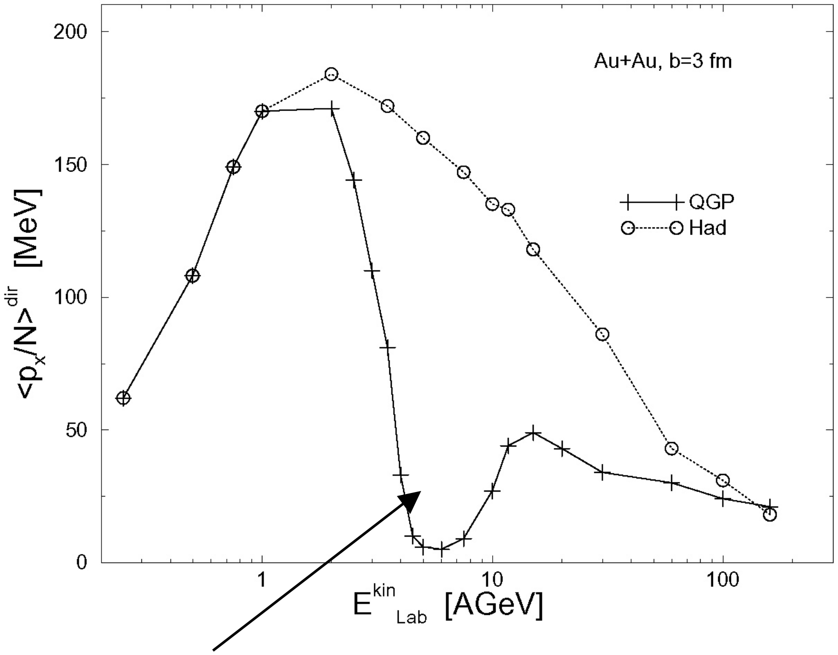
Sound Velocity

- **EoS** ⟺ **Hydrodynamic Expansion**

$$C_s^2 = \left.\frac{\partial P}{\partial \varepsilon}\right|_{n_B/s}$$



Ex. Rischke et al. nucl-th/9504021

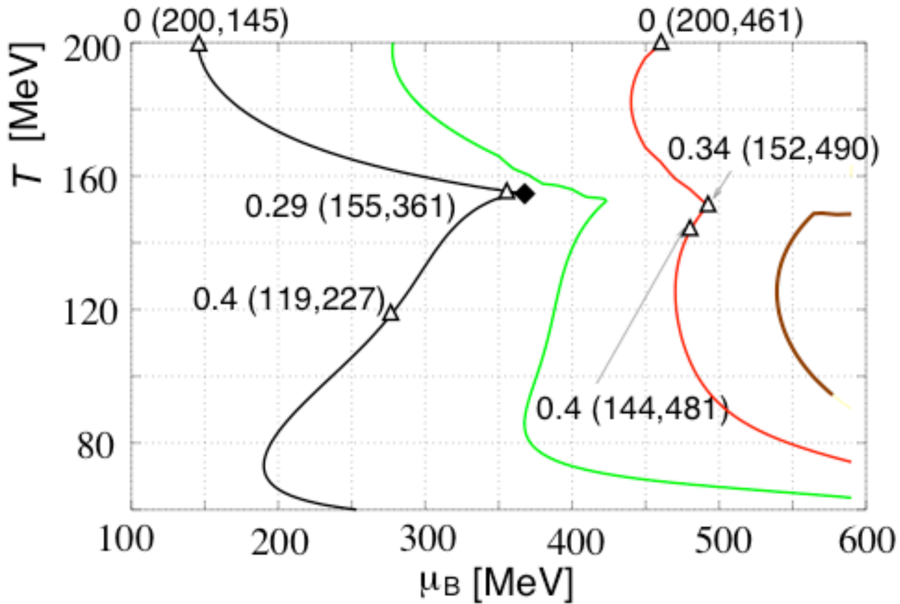


Effect of mixed phase?

Sound Velocity in EoS with CEP

Effect on Time Evolution

Collective flow ⇦ EOS

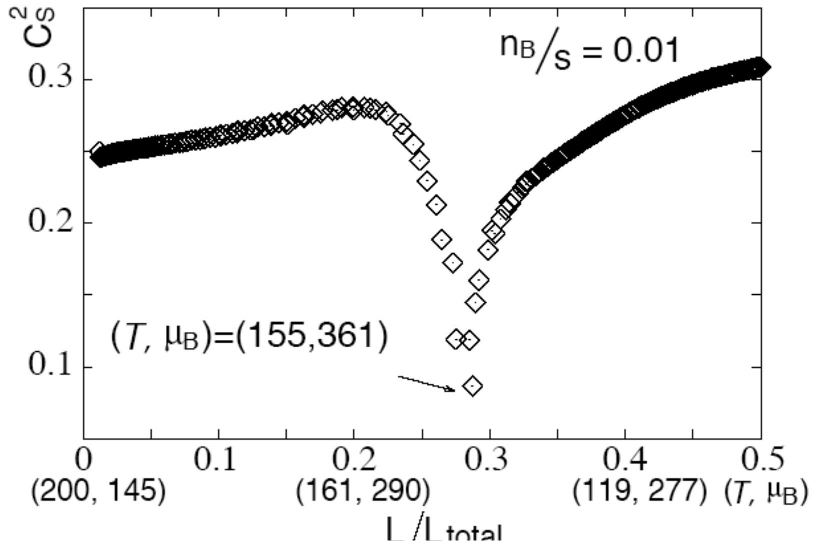


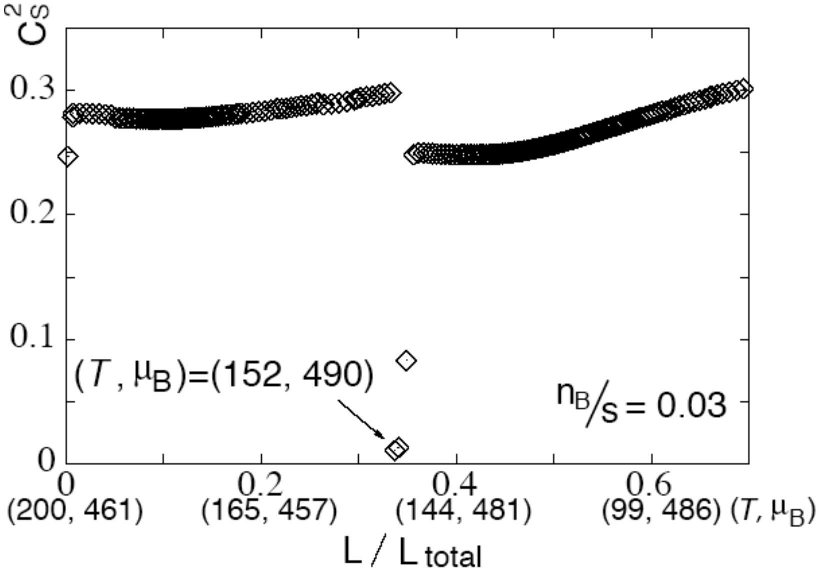
$$C_s^2 = \left.\frac{\partial P}{\partial \varepsilon}\right|_{n_B/s}$$

- The system may not expand uniformly.

↓

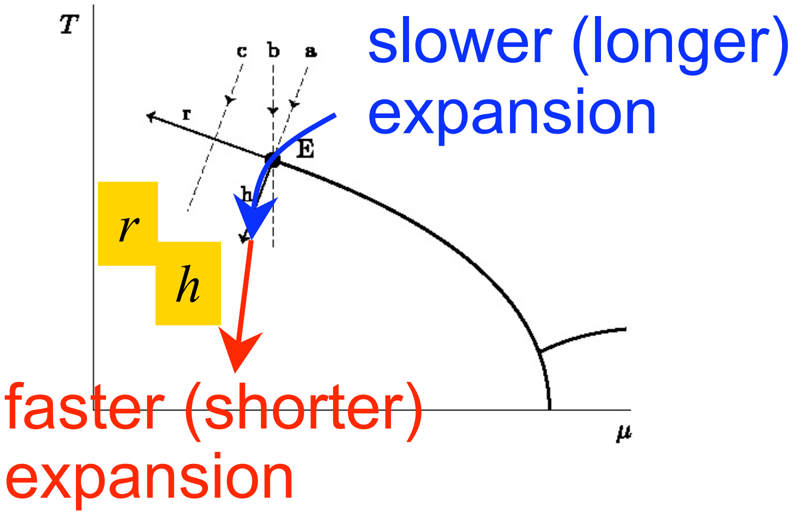
Collective flow & HBT





Slowing out of Equilibrium

- **Berdnikov and Rajagopal's Schematic Argument**

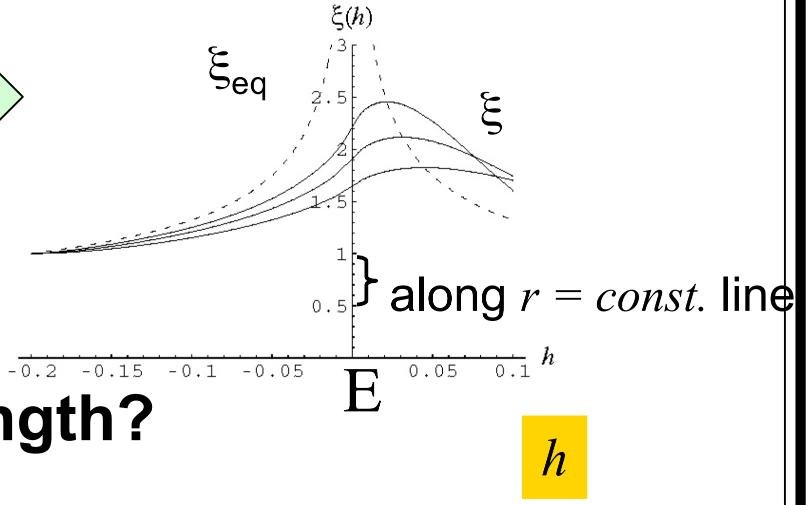


slower (longer) expansion

faster (shorter) expansion

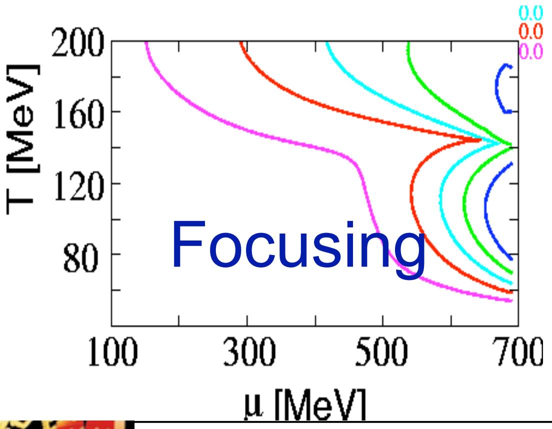
B. Berdnikov and K. Rajagopal,
Phys. Rev. D61 (2000) 105017

Correlation length longer than ξ_{eq}



} along $r = const.$ line

- **Effect of Focusing on correlation length?**



Focusing

Time evolution : Bjorken's scaling solution along n_B/s

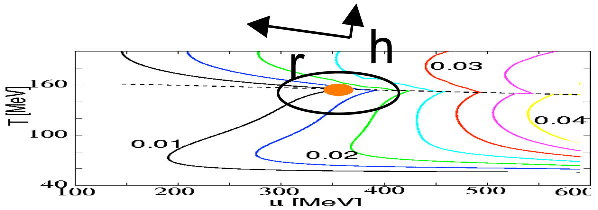
$\tau_0 = 1$ fm, T_0 = 200 MeV

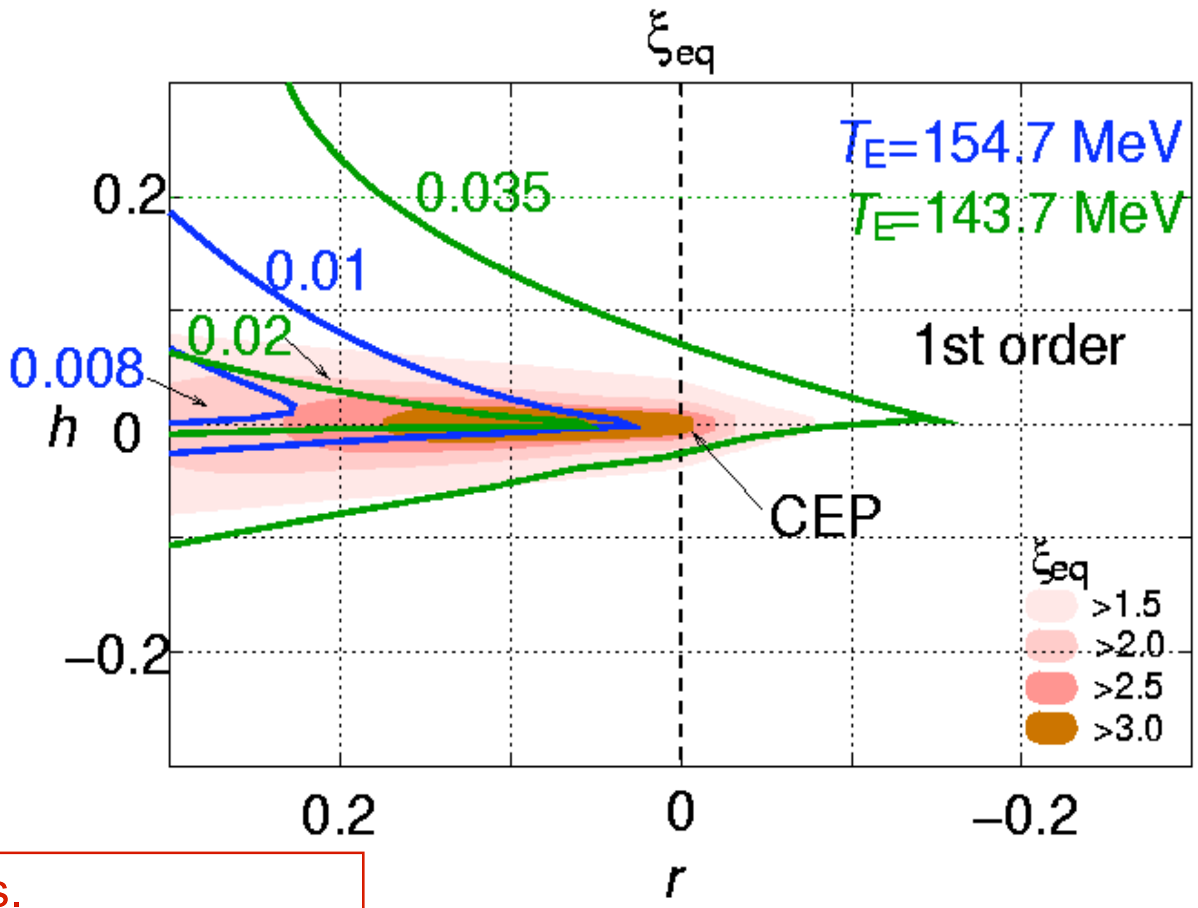
Correlation Length in Equilibrium

- ξ_{eq}

  Widom's scaling low

$$\xi_{\text{eq}}^2(r,M) = f^2 M^{-2\nu/\beta} g\left(\frac{|r|}{|M|^{1/\beta}}\right)$$





- Max. ξ_{eq} depends on n_B/s.
- Trajectories pass through the region where ξ_{eq} is large. (focusing)
- n_B/s ← non-critical component of the EOS

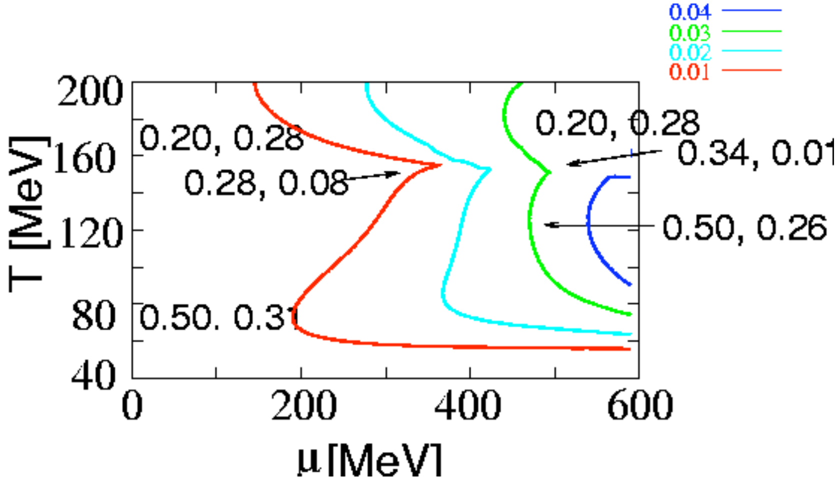
Evolution of Correlation Length

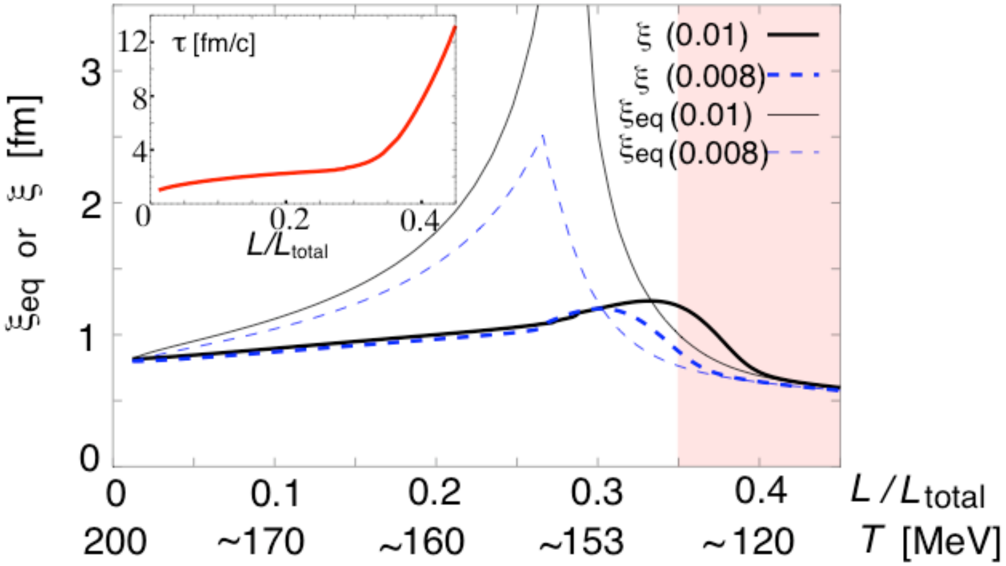
- ξ : time evolution (1-d)

$$\frac{d}{d\tau} m_\sigma(\tau) = -\Gamma\left[m_\sigma(\tau)\right]\left(m_\sigma(\tau) - \frac{1}{\xi_{\mathrm{eq}}(\tau)}\right)$$

$$\Gamma(m_\sigma) = \frac{a}{\xi_0}(m_\sigma \xi_0)^z, \quad m_\sigma(\tau) = 1/\xi(\tau)$$

$z = 3.0$ Model H (Halperin RMP49(77)435)

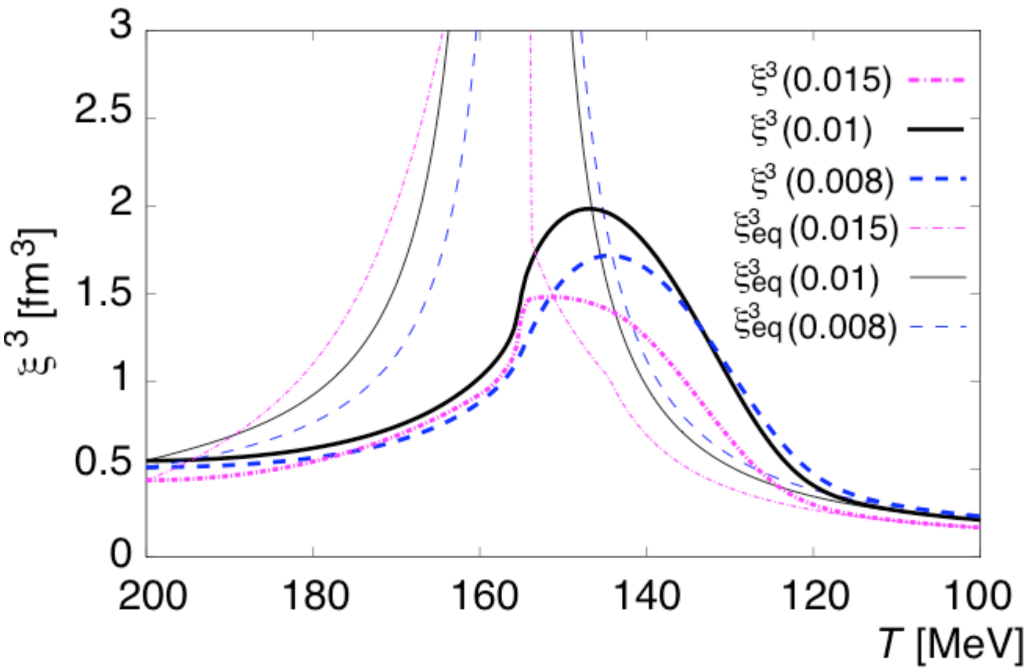




- ξ is larger than ξ_{eq} at T_f. ← Critical slowing down
- Differences among ξs on n_B/s are small.
- In 3-d, the difference between ξ_{eq} and ξ becomes large due to transverse expansion.

Fluctuation

Cube of the equilibrium correlation length



Fluctuation $\propto \xi^3$

- There is the possibility that the fluctuation shows some enhancement.

Fluctuations (I)

- **Fluctuations**

  CERES

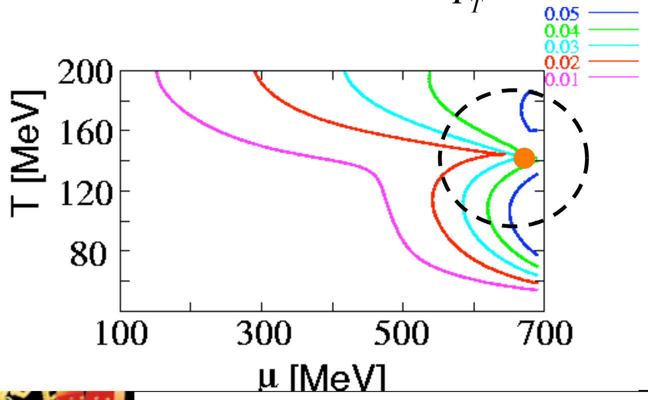
  40,80,158 AGeV Pb+Au collisions

Mean P_T Fluctuation

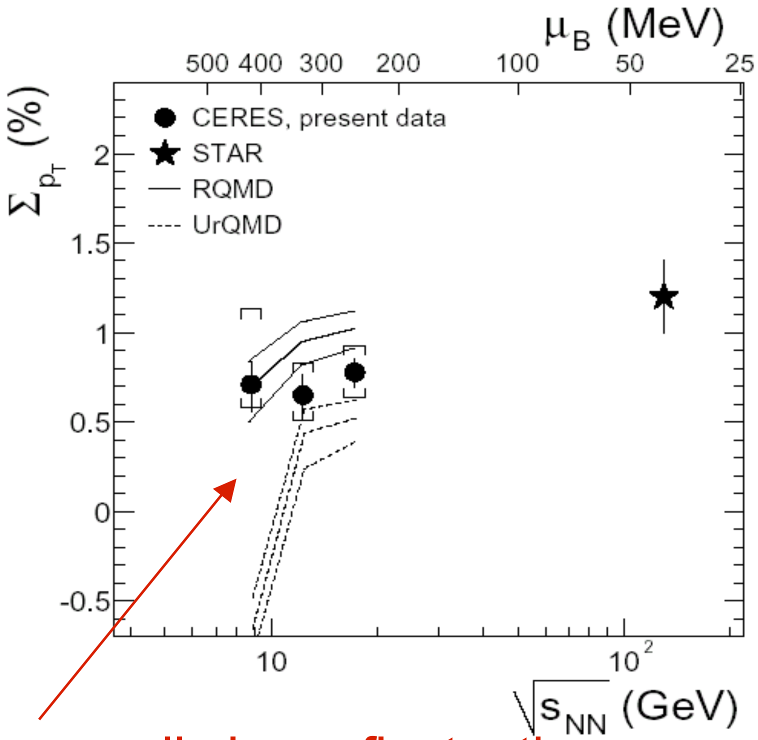
$$\sigma^2_{PT,dyb} \equiv \left\langle \Delta M^2_{PT} \right\rangle - \frac{\Delta P_T^2}{\langle N \rangle}$$

$$\left\langle M_x^2 \right\rangle \equiv \frac{\sum_{j=1}^{n} N_j \left(M_x^j - \langle M_x \rangle \right)^2}{\sum_{j=1}^{n} N_j}$$

$$\Sigma_{\mathrm{PT}} \equiv \mathrm{sgn}(\sigma^2_{PT,dyn}) \frac{\sqrt{\left|\sigma^2_{dyn}\right|}}{P_T}$$



CERES:Nucl.Phys.A727(2003)97



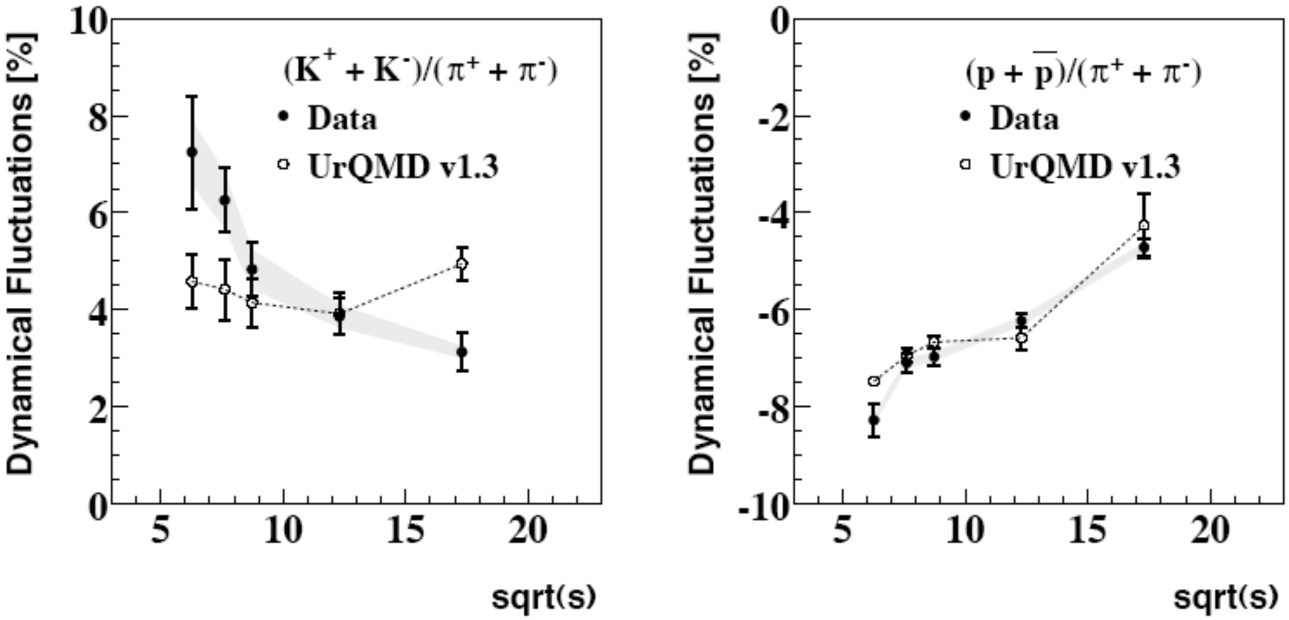
No unusually large fluctuation or non-monotonic behavior

CEP: attractor of isentropic trajectories

Similar correlation length and fluctuation is observed near CEP.

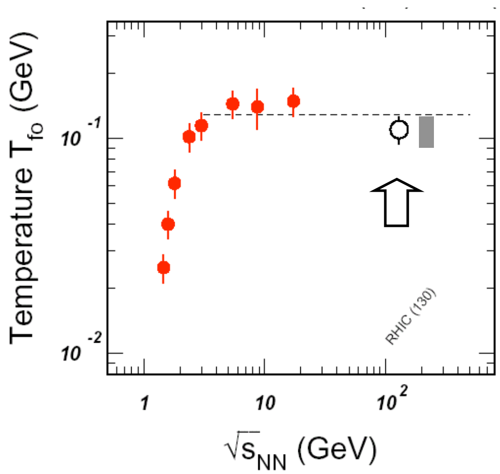
Fluctuations (II)

- NA49



- Suggestion of QCD critical end point?
- Fluctuation ← Conservation

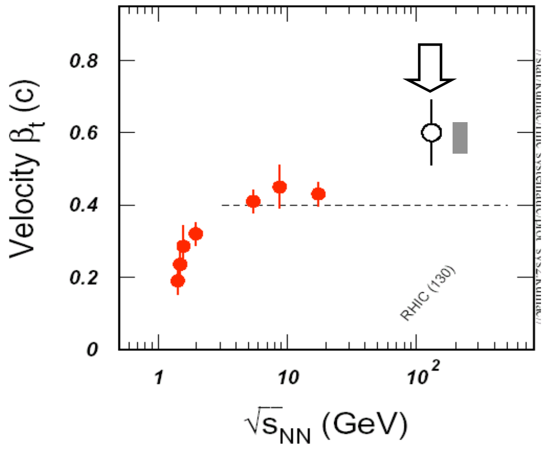
Kinetic Freeze-out Temperature



Low T_f comes from large flow.

?

Xu and Kaneta,
nucl-ex/0104021(QM2001)
BRAHMS nucl-ex/0404011



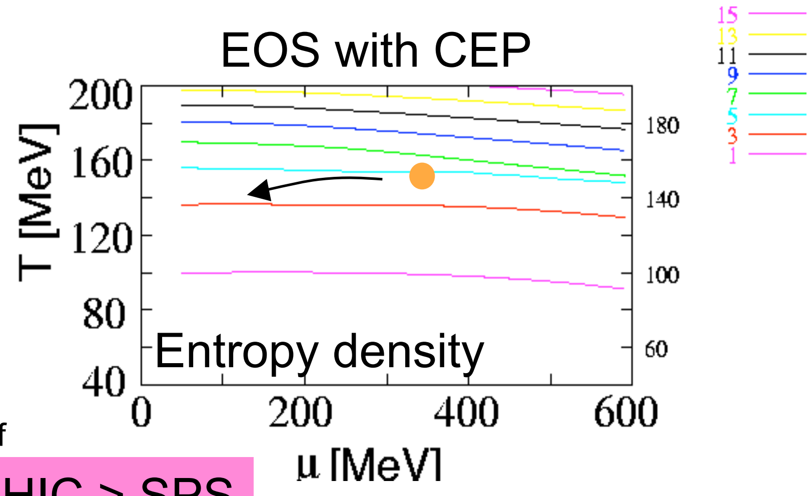
- Kinetic Freeze-out
  mean (elastic) collision rate
  expansion rate
- π–N cross section

  $(p+\overline{p})/(\pi^+ + \pi^-)$

  SPS ~ 0.09
  RHIC ~ 0.09

- Expansion rate
  RHIC > SPS
- Collision rate
  SPS ~ RHIC

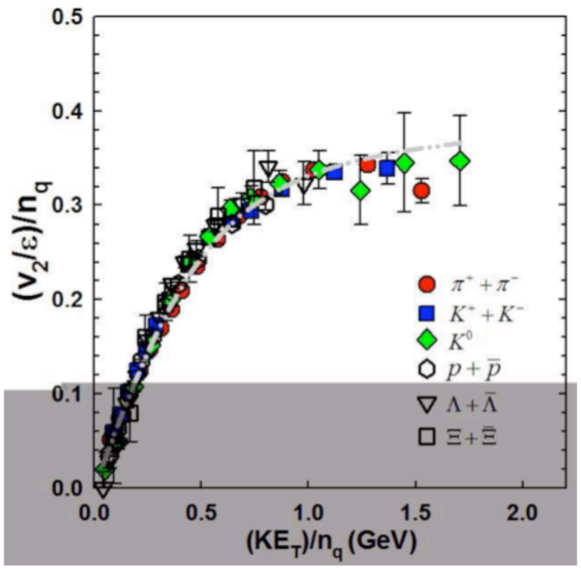
EOS with CEP

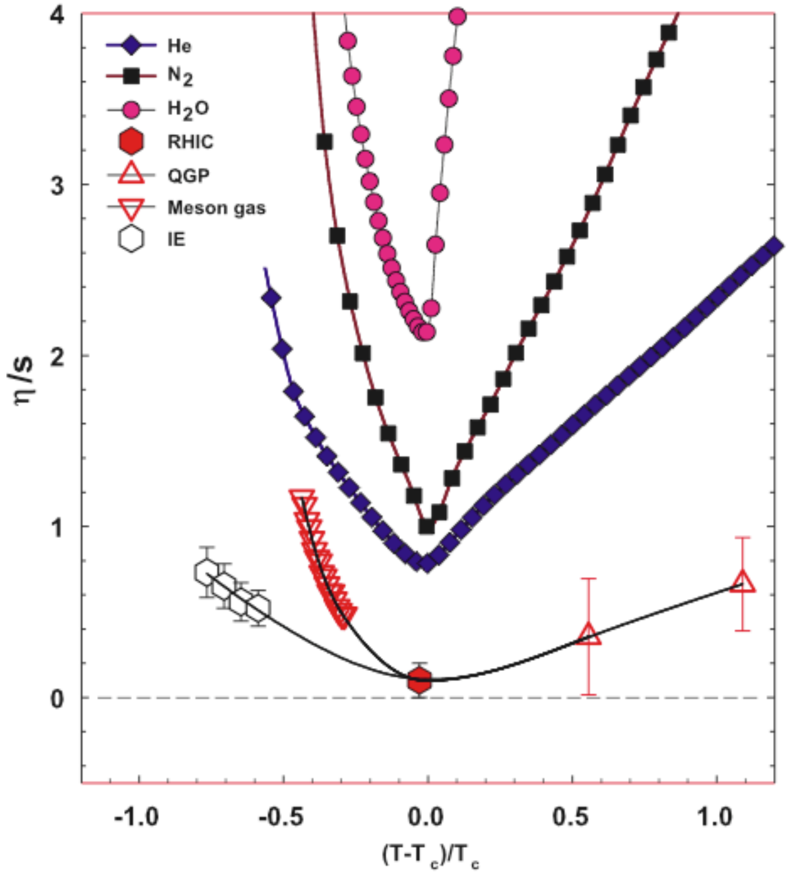


- T_f

RHIC > SPS

CEP at RHIC?



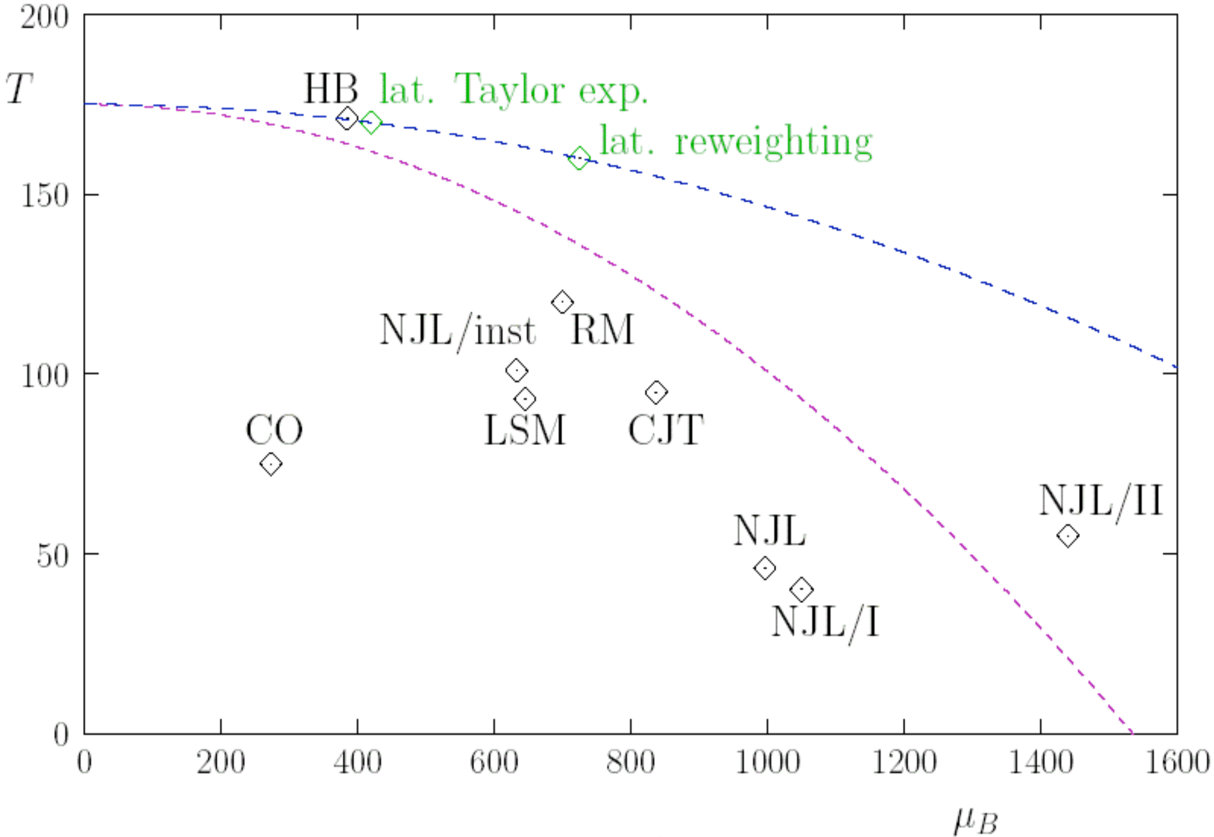


Au+Au Collisions

$$\left.\begin{array}{l} \dfrac{\eta}{s} \sim T\lambda_f c_s \\ T = 165 \pm 3\ \text{MeV} \\ c_s = 0.35 \pm 0.05 \\ \lambda_f = 0.3 \pm 0.03\text{fm} \end{array}\right\} \quad \frac{\eta}{s} \sim 0.09 \pm 0.015$$

Lacey et al, Phys.Rev.Lett.98:092301,2007

Where is CEP on T-μ_B plane?

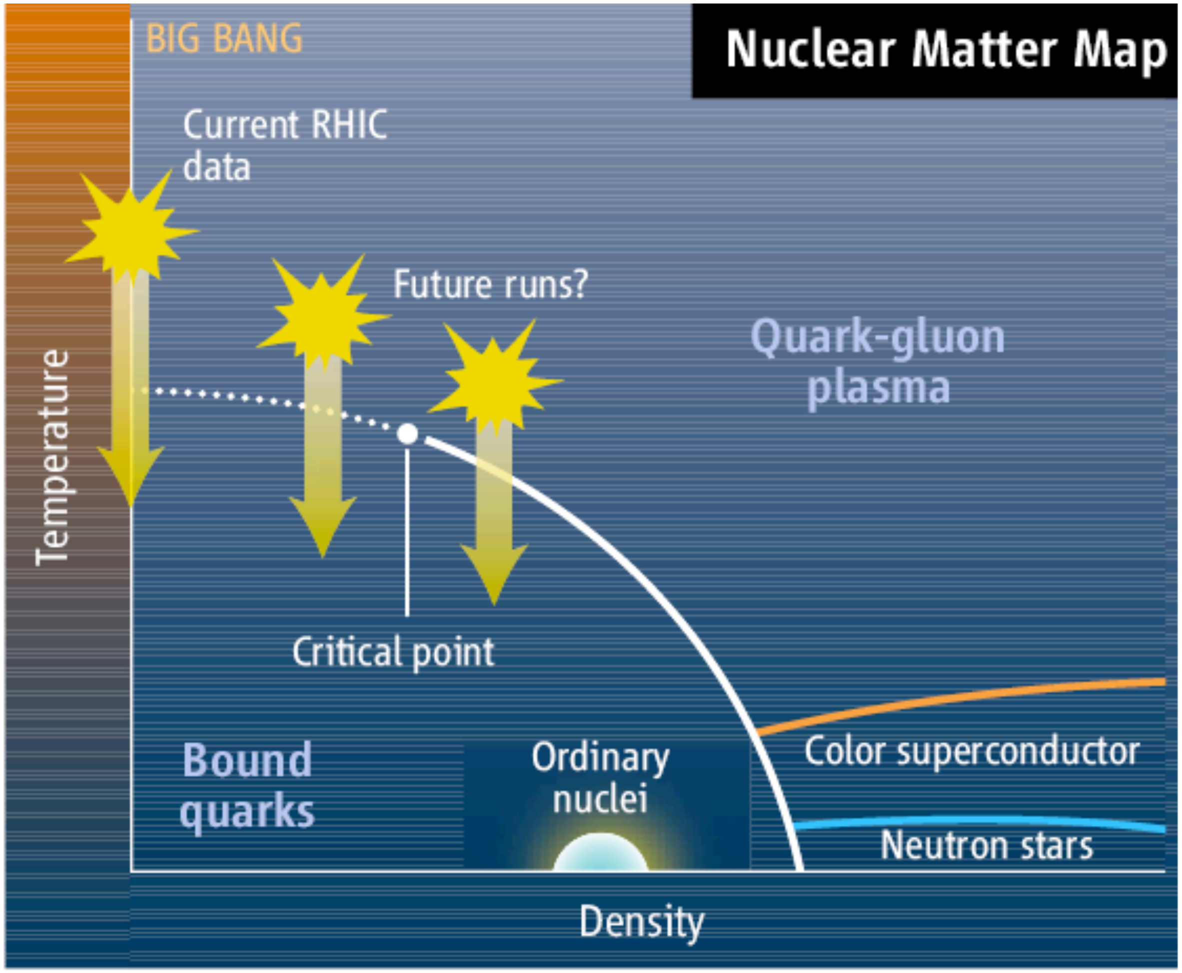


Stephanov:hep-ph/0402115

- NJL/I, NJL/II
  Asakawa & Yazaki
- CO (composite operator)
  Barducci et al.
- NJL/inst (instanton NJL)
  Berges & Rajagopal
- RM (random matrix)
  Halaz et al.
- LSM (linear sigma model)/ NJL
  Scavenius et al.
- CJT(effective potential)
  Hatta & Ikeda
- HB (hadronic bootstrap)
  Antoniou et al.

⇨ experiments

Where is the Critical Point ?



Landmark study. Physicists have seen a smooth transition from bound quarks to quark-gluon plasma (dotted line). They now hope to find the point beyond which the transition becomes violent (white line).

Science,**312**,190(2006)

Energy Scan:
- Necessity of lower energy experiments at RHIC
- SPS

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