

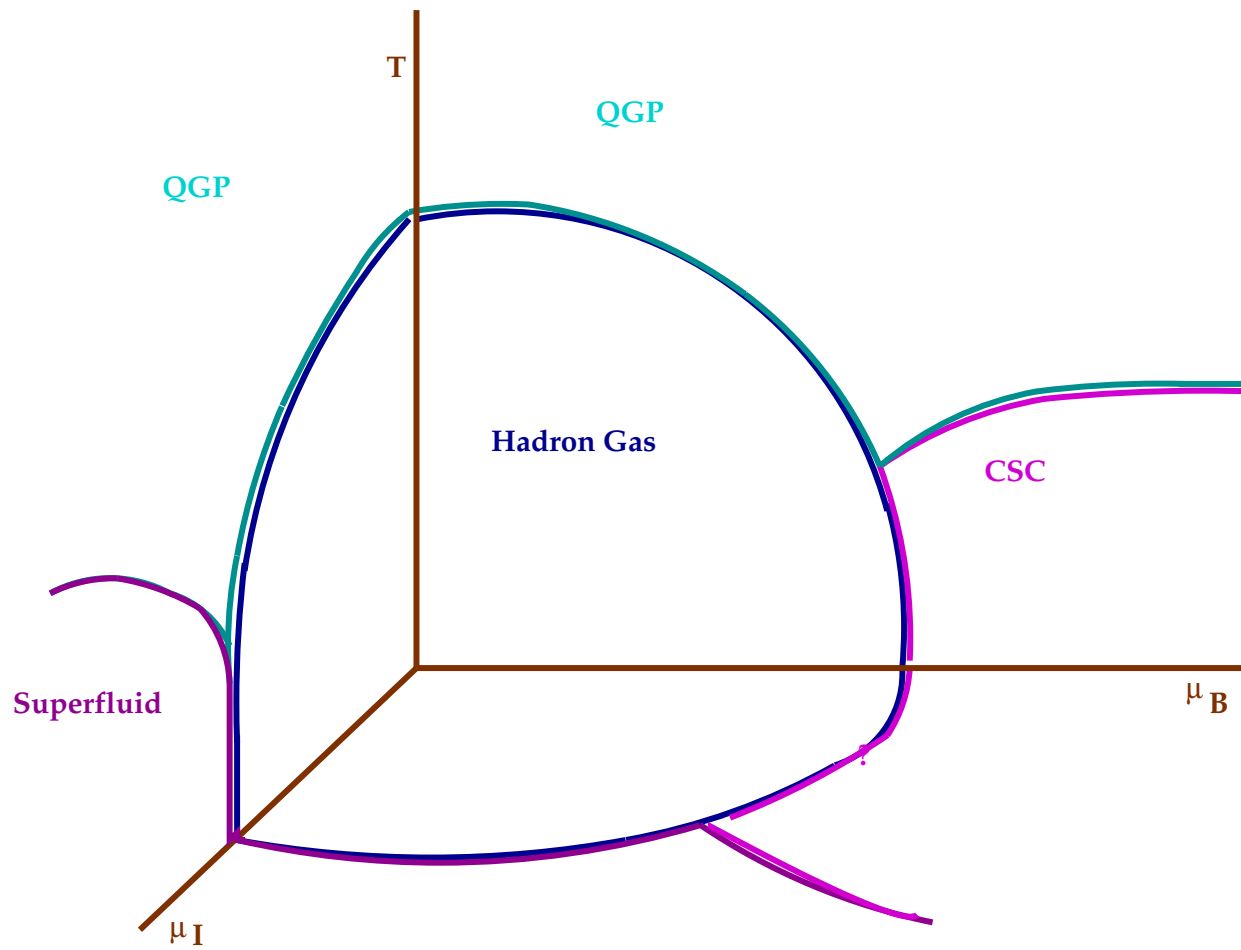
# Quark Matter 2008

## QCD at Non-Zero Density : Lattice Results

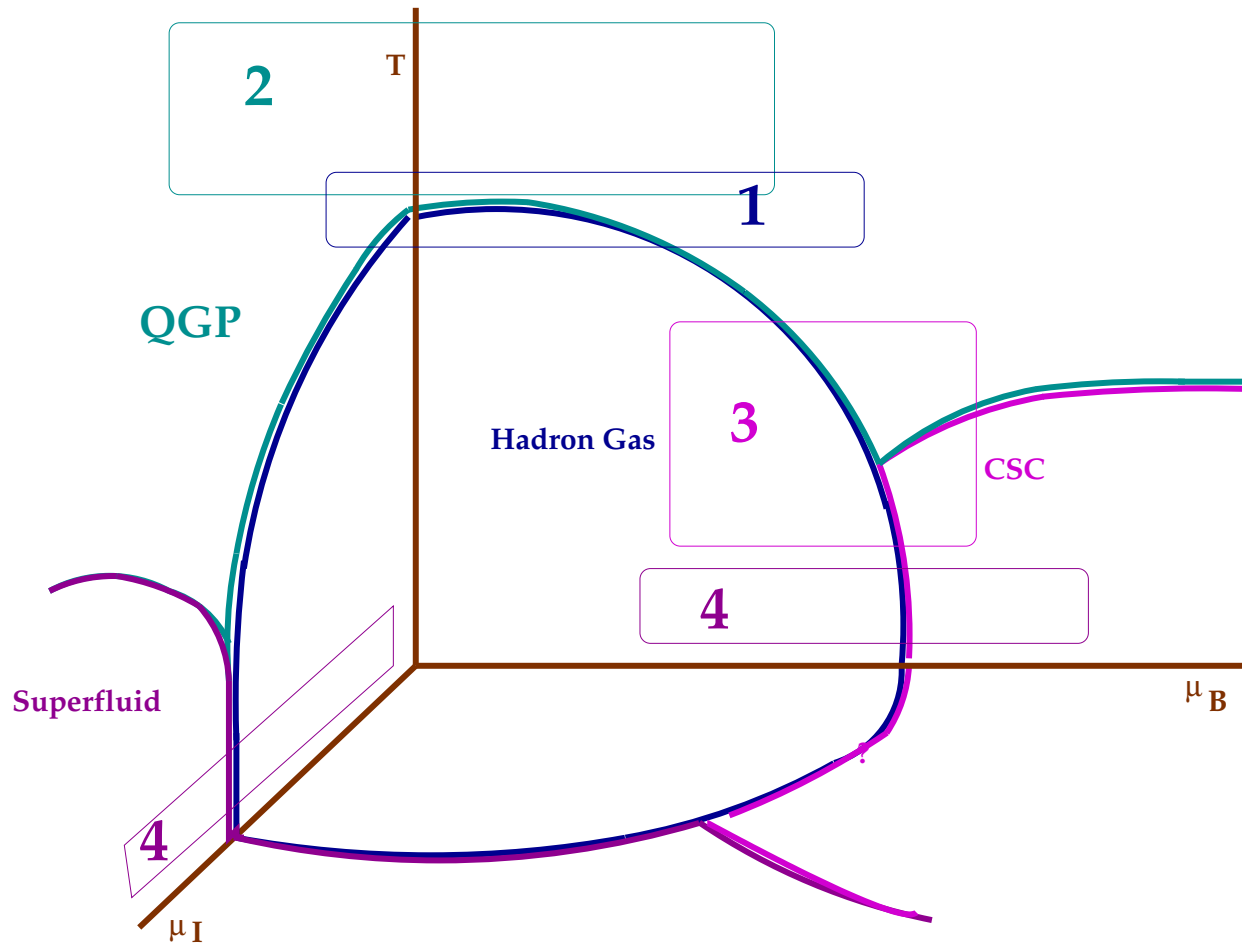
*M.P. Lombardo - LNF INFN*



# LATTICE FIELD THEORY : FIRST PRINCIPLE APPROACH TO THE QCD PHASE DIAGRAM



# PLAN

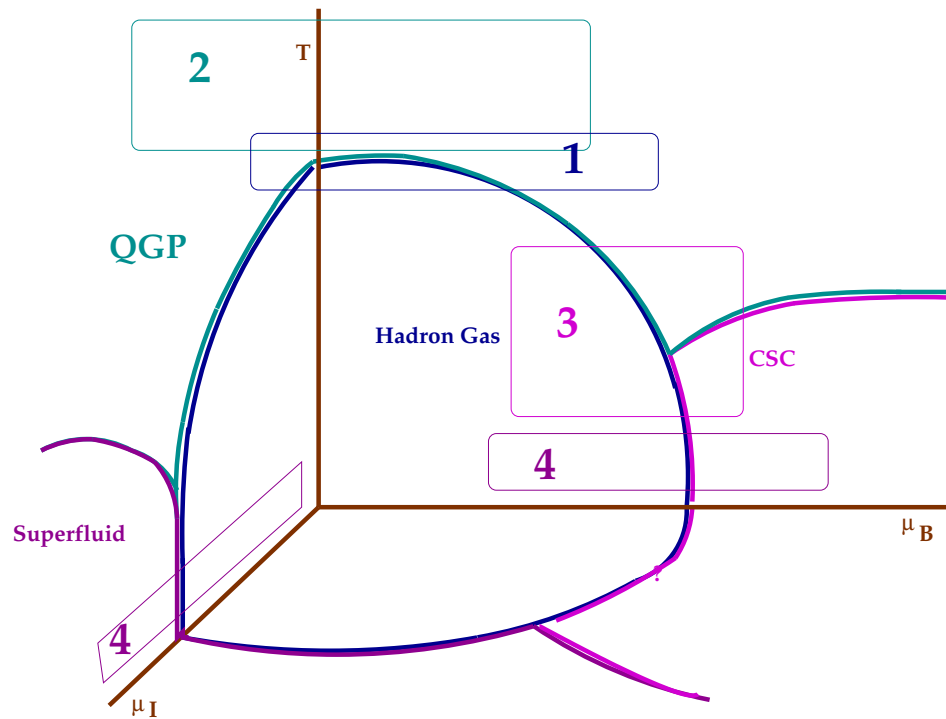


1. Critical line and Critical Point

2. EoS and Critical Behaviour

3. Towards FAIR

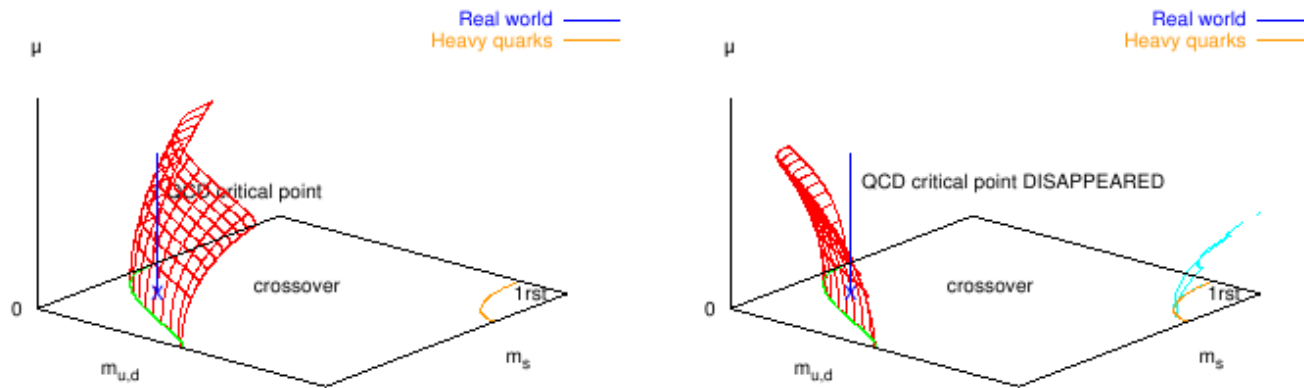
4. Cold and Dense Matter :  
OCD-like models



# 1. THE CRITICAL LINE AND THE CRITICAL POINT

# DOES A CRITICAL POINT AT SMALL $\mu_B$ EXIST IN QCD?

## Forcrand-Philipsen : strategy



Scenario I or Scenario II ? To decide, measure slope  $K$  in

$$\frac{m_c(\mu)}{m_c(0)} = 1 + K \left( \frac{\mu}{T} \right)^2 + \dots$$

$K > 0$  : Scenario I , critical endpoint at small  $\mu_B$

$K < 0$  : Scenario II, NO critical endpoint at small  $\mu_B$ : **avored at QM2006**

## Forcrand-Philipsen Lat07 : improve and confirm previous results

$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(5) \left(\frac{\mu}{T}\right)^2 + \dots$$

## Kogut-Sinclair 2007 : QCD at finite isospin density

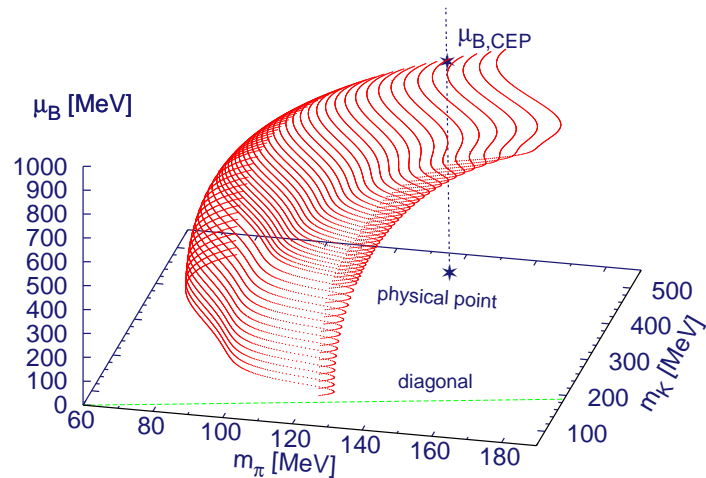
NB :  $\mu_I \simeq \mu_B$  at  $T \simeq T_c$  Toublan, Kogut, Sinclair 2004

$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.(1) \left(\frac{\mu_I}{T}\right)^2 + \dots$$

## Current Results:

- **Confirm unusual scenario for  $N_t = 4$**
- **Suggest NO critical endpoint for  $\mu_B < 600 MeV$**
- **See talk by Philippe de Forcrand**

## $SU(3)_L \times SU(3)_R$ NJL MODEL

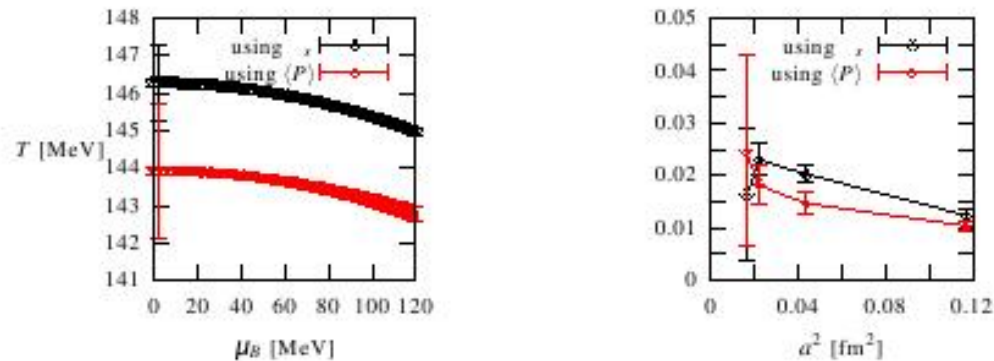


**Kovács and Szép, 2006** compute critical surface of the  $SU(3)_L \times SU(3)_R$  chiral quark model at non-zero baryon density.

**Conventional Scenario realized in NJL**,  $\mu_B^{CEP} \sim 900$  MeV.

**Qualitative difference between QCD and NJL at small  $\mu_B$ :**  
Details of the dynamics important.

# TOWARDS THE CONTINUUM LIMIT CURVATURE OF THE PHASE TRANSITION LINE IN THE $\mu$ - $T$ PLANE



## Results for $N_f=2+1$

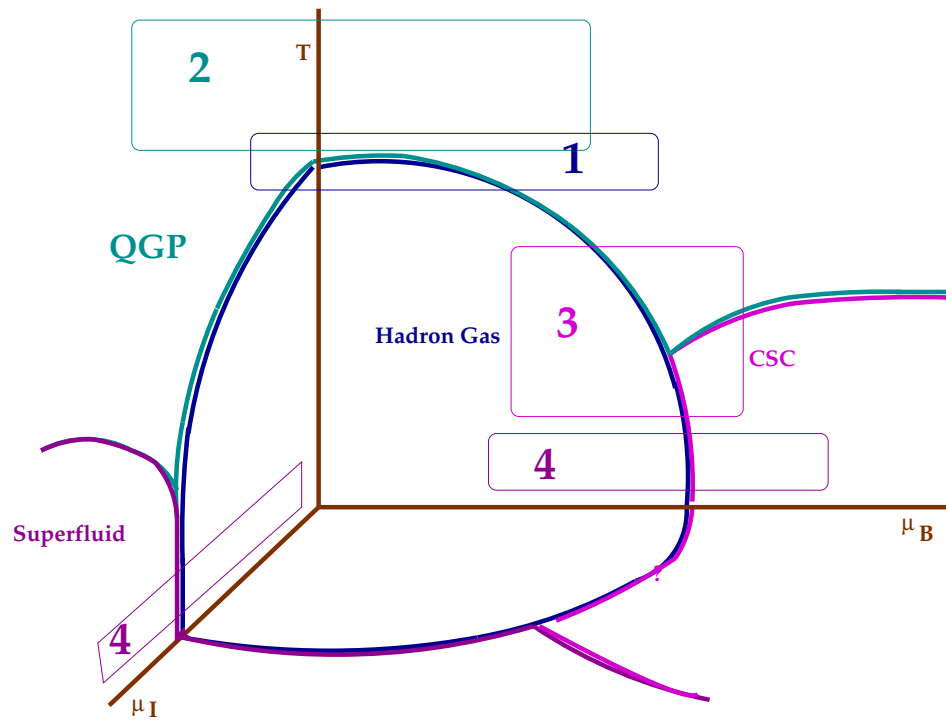
from the Polyakov loop and the strange quark number susceptibility

$$N_t = 4, 6, 8, 10$$

Fodor, Guse, Katz, Kálmán K. Szabó, Lattice2007

See Zoltan Fodor talk.





## 2. EOS AND CRITICAL BEHAVIOUR

## LATTICE OBSERVABLES FOR THERMODYNAMICS

Number Density : accessible at **imaginary chemical potential**.

$$n_{u,d}(T, \mu_u, \mu_d, m_u, m_d) = \frac{\partial p(T, \mu_u, \mu_d)}{\partial \mu_{u,d}}; p(T, \mu_u, \mu_d) = \frac{T}{V} \ln Z(T, \mu_u, \mu_d)$$

Susceptibilities: accessible at **at  $\mu = 0$**

$$\chi_{j_u, j_d}(T) = \left. \frac{\partial^{(j_u + j_d)} p(T, \mu_u, \mu_d)}{\partial \mu_u^{j_u} \partial \mu_d^{j_d}} \right|_{\mu_u = \mu_d = 0}.$$

Test for fluctuations.

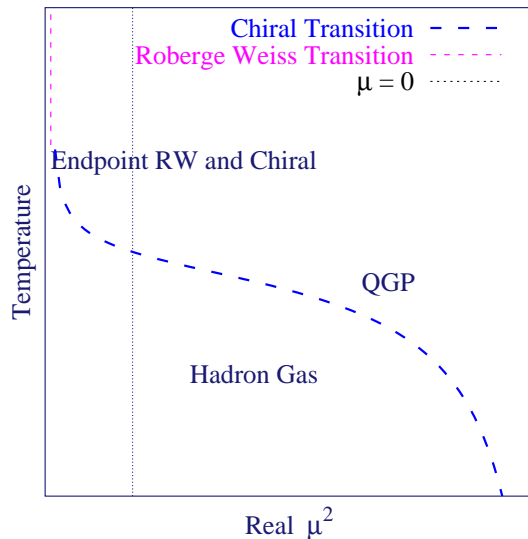
Taylor coefficients of the excess pressure:

$$\Delta p(T, \mu_u, \mu_d) \equiv p(T, \mu_u, \mu_d) - p(T, \mu_u = 0, \mu_d = 0) = \sum_{j_u, j_d} \chi_{j_u, j_d}(T) \frac{\mu_u^{j_u}}{j_u!} \frac{\mu_d^{j_d}}{j_d!},$$

containing information about baryon density effects in the EoS.

# THERMODYNAMICS AND CRITICAL LINES in THE $T - \mu^2$ PLANE

Three regimes for thermodynamics:



- **Low Temperature,**

away from critical lines:

**Hadron Gas QM2006**

$$n(T, \mu) = K(T) \sin(N_c \mu / T)$$

- **In the critical region:**

$$p(T, \mu) = b(T) |t + a(T) (\mu^2 - \mu_c^2)|^{(2-\alpha)}$$

implying

$$n(T, \mu) = A(T) \mu (\mu^{c^2} - \mu^2)^{(2-\alpha)}$$

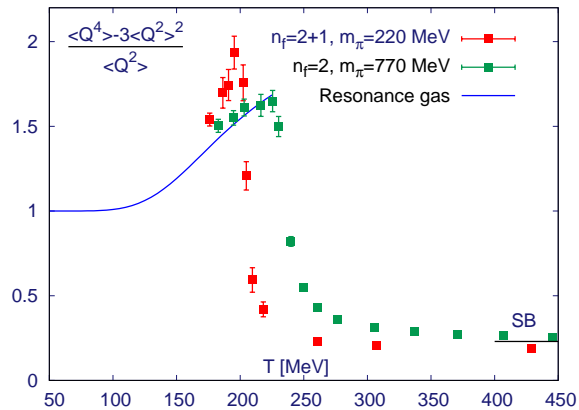
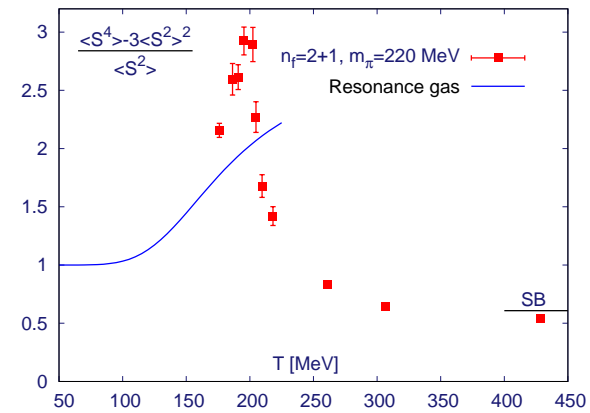
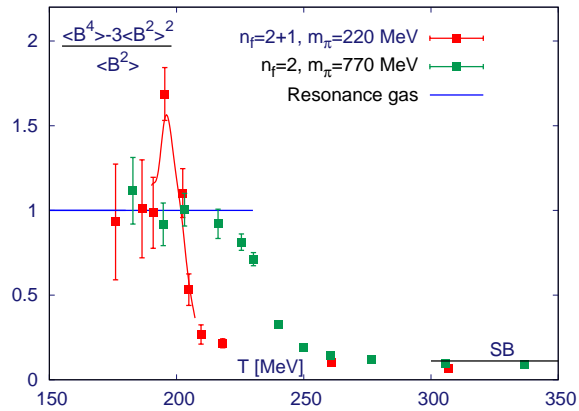
- **High Temperature,**

away from critical line

**Approach to Free Field**

$$n(T, \mu) \rightarrow n_{SB}(T, \mu)$$

# HADRON GAS; CRITICAL BEHAVIOUR; SB from SUSCEPTIBILITIES



**2+1 Flavor**  
**RBC-Bielefeld collaboration 2008;**  
**courtesy C. Schmidt**  
**See Christian Schmidt talk**

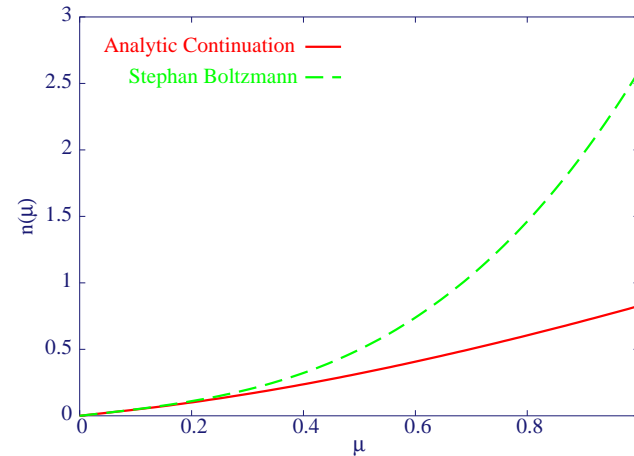
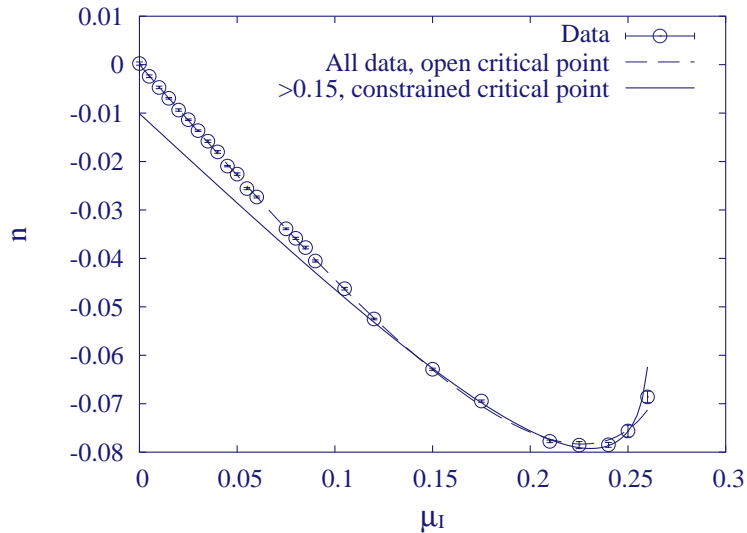
# CRITICAL BEHAVIOR AND THERMODYNAMICS AT THE ENDPOINT OF THE RW TRANSITION

**Critical behavior at imaginary  $\mu$**   
 $n(\mu_I) = A(T)\mu_I(\mu^{c2} - \mu_I^2)^{(2-\alpha)}$

**Continued to real  $\mu$ ..**

$$n(\mu) = A(T)\mu(\mu^{c2} + \mu^2)^{(2-\alpha)}$$

$$n_{SB}(\mu) = A\mu + B\mu^3 \rightarrow \alpha = 1$$



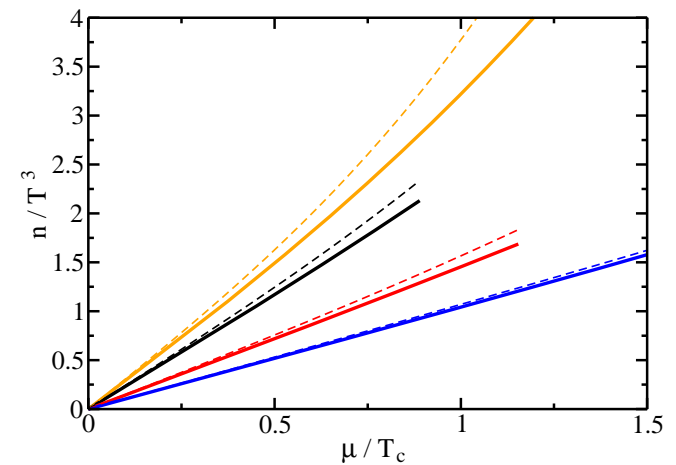
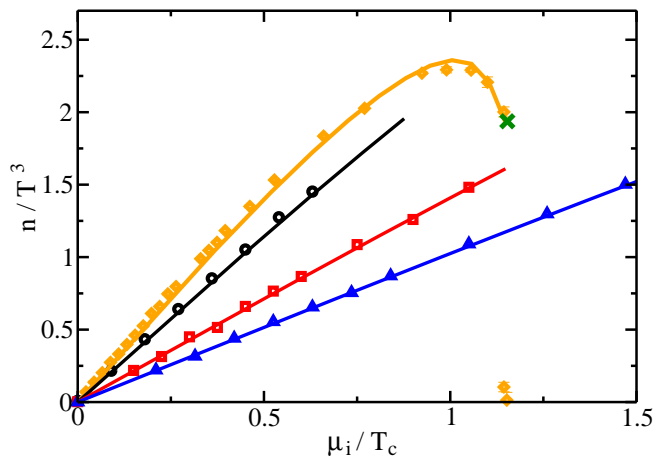
D'Elia, Di Renzo, Lombardo, 2007  
 See Poster

# CRITICAL BEHAVIOR, THERMODYNAMICS, QUASIPARTICLE MODELS

Kämpfer, Bluhm Proposal

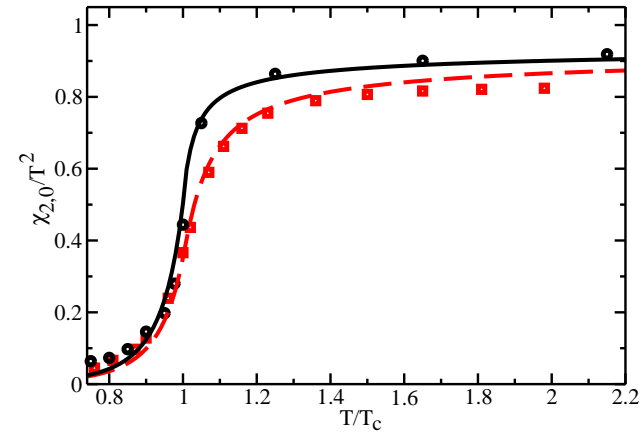
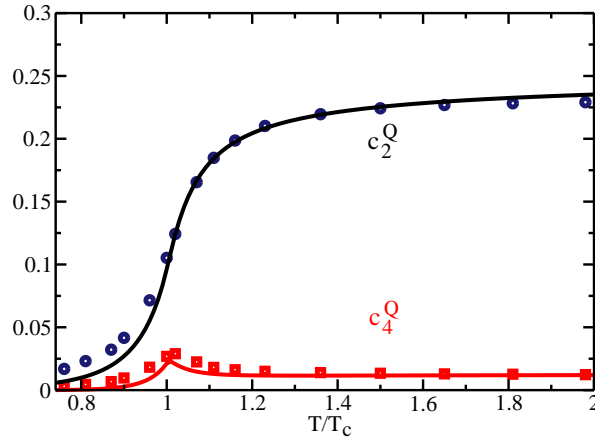
amenable to an easy comparison with lattice data.

## I. Quasiparticlemodel vs Imaginary Chemical Potential Lattice Data, and analytic continuation to Real Chemical Potential



Kämpfer, Bluhm , 2007

## II. Quasiparticlemodel vs Taylor Coefficients



### Crucial ingredients:

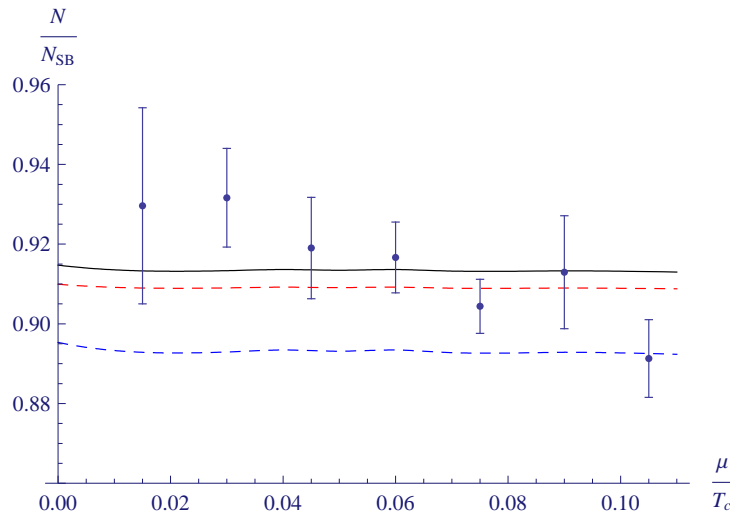
- Explicit dependence of the self-energy parts on  $\mu_i = \mu_{u,d}$  and  $T$
- Implicit dependence via the effective coupling  $G^2(T, \mu_u, \mu_d)$ .

$$\omega_i^2 = k^2 + m_i^2 + \Pi_i, \quad \Pi_i = \frac{1}{3} \left( T^2 + \frac{\mu_i^2}{\pi^2} \right) G^2(T, \mu_u, \mu_d).$$

Kämpfer-Bluhm : see Poster

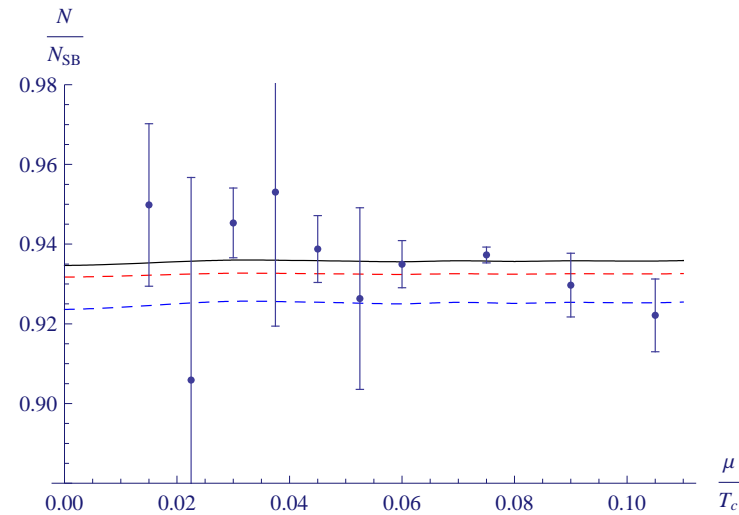
# APPROACH TO SB : ANALYTIC RESULTS VS. LATTICE DATA

Based on A. Vuorinen, 2004



$$T = 1.5T_c$$

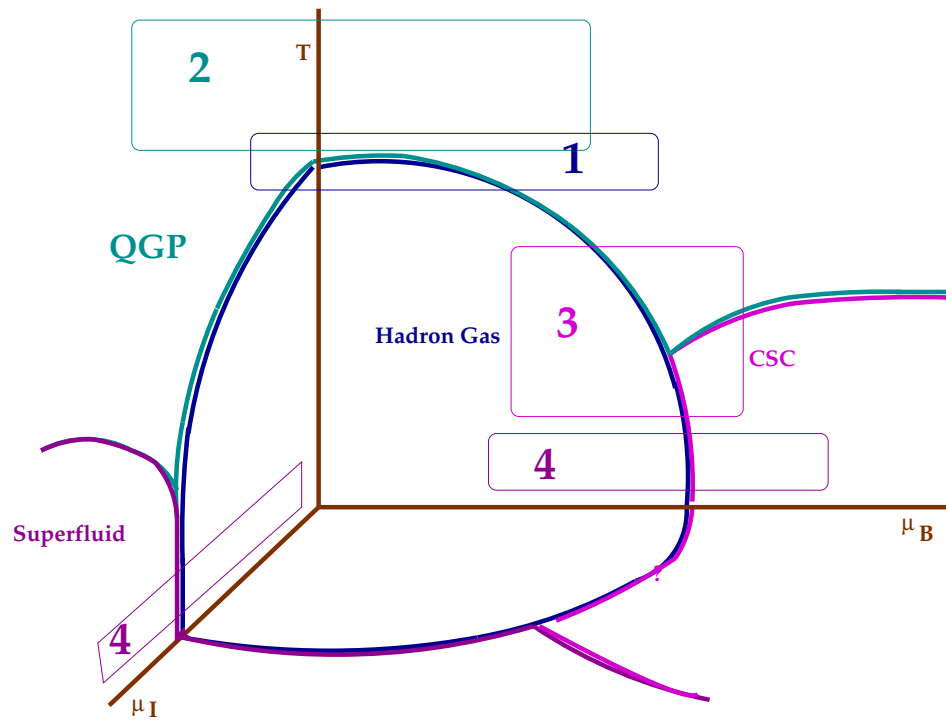
D'Elia, Di Renzo, Lombardo, Vuorinen, in progress



$$T = 3.5T_c$$

Lattice Cutoff Effects : Karsch, Laermann, 2008

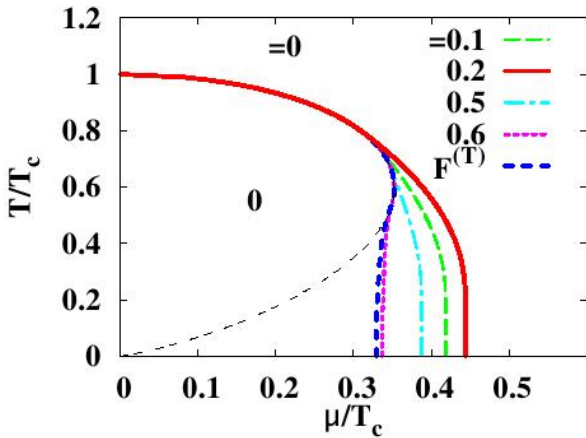




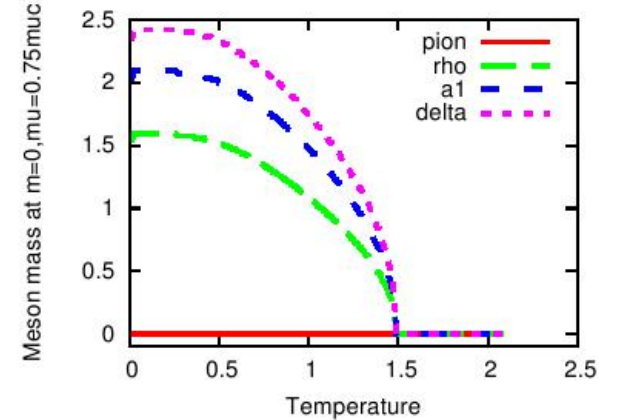
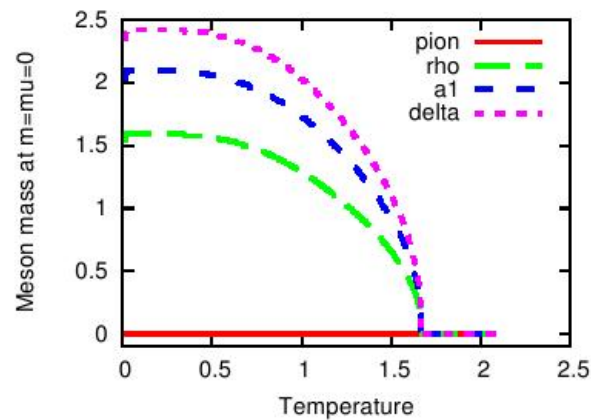
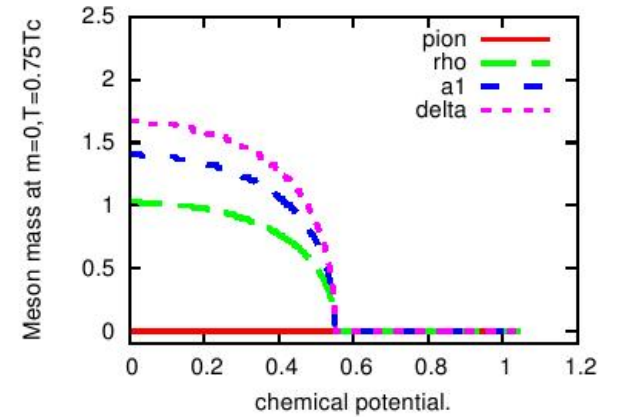
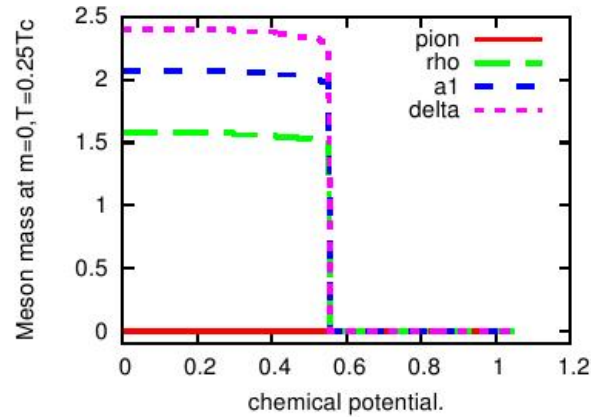
### 3. TOWARDS FAIR

# STRONG COUPLING

## New Results on The Mass Spectrum



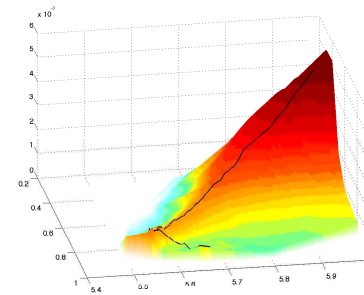
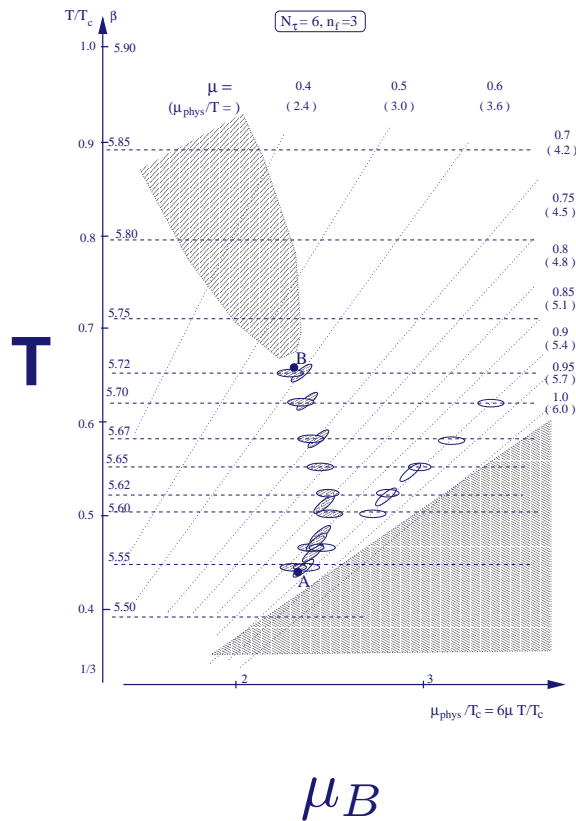
Kawamoto, Miura 2007



# HEAVY QUARK EFFECTIVE MODEL

**Double limit:**  $M \rightarrow \infty, \mu \rightarrow \infty, \zeta \equiv \exp(\mu - \ln M)$  : **Fixed**  
**Evolved ‘quenched approximation’** in the presence of charged matter

## Polyakov Loop



**Results for  $N_f = 3$  :**

- Identified phase transition
- Identified Ridge in the  $T, \mu$  plane
- Studies of diquark in progress

**Di Pietro, Feo, Seiler, Stamatescu 2008**

## CANONICAL FORMALISM

$$\mathcal{Z}_C(T, N) =$$

$$\frac{3}{2\pi} \int_0^{2\pi T} d(\mu_I/T) e^{-N i \mu_I/T} \mathcal{Z}_{GC}(T, i\mu_I)$$

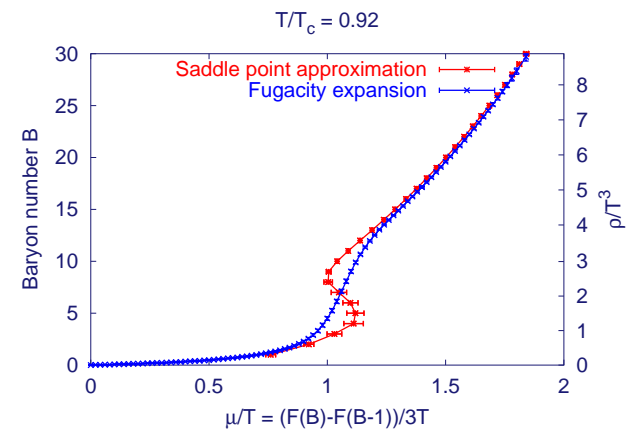
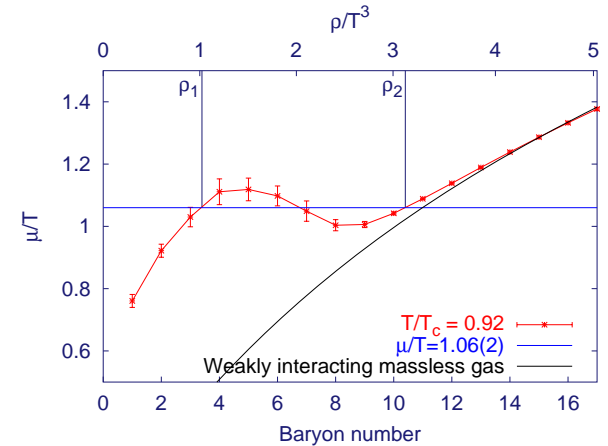
**Forcrand and Kratovchila, 2006,2007**

**Canonical partition a la Hasenfratz-Toussaint:**

$$\begin{aligned} & \frac{Z_C(B, \beta)}{Z_{GC}(\beta_0 = \beta, \mu = i\mu_{I_0})} \\ &= \left\langle \frac{1}{2\pi} \int_{-\pi}^{\pi} d\left(\frac{\mu_I}{T}\right) e^{-i3B \frac{\mu_I}{T}} \frac{\det(U; i\mu_I)}{\det(U; i\mu_{I_0})} \right\rangle_{\beta_0, i\mu_{I_0}} \\ &\equiv \left\langle \frac{\hat{Z}_C(U; B)}{\det(U; i\mu_{I_0})} \right\rangle_{\beta_0, i\mu_{I_0}} \end{aligned}$$

**Results for  $N_f = 4$  :**

- **First order line-coexistence region**
- **Critical  $\mu$  and Critical densities**  $\rightarrow$

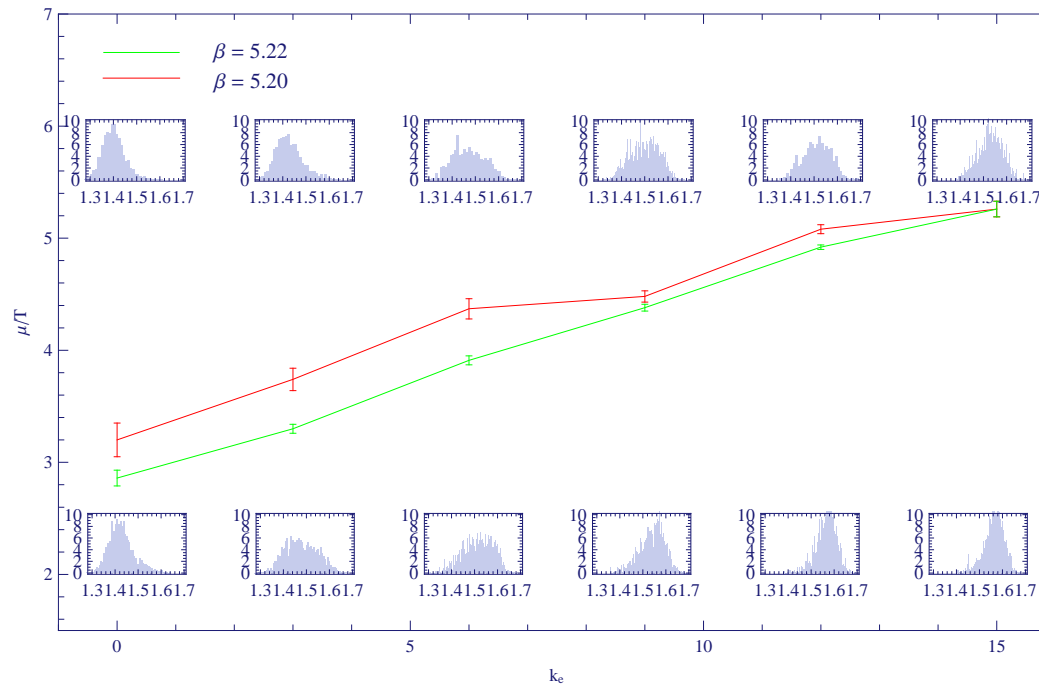


# CANONICAL APPROACH II : HISTOGRAMS

Proposed by Ejiri,2007

## Alternative search for a critical point in $N_f = 2$ QCD, large masses

Alexandru, Li, Liu, 2007 :

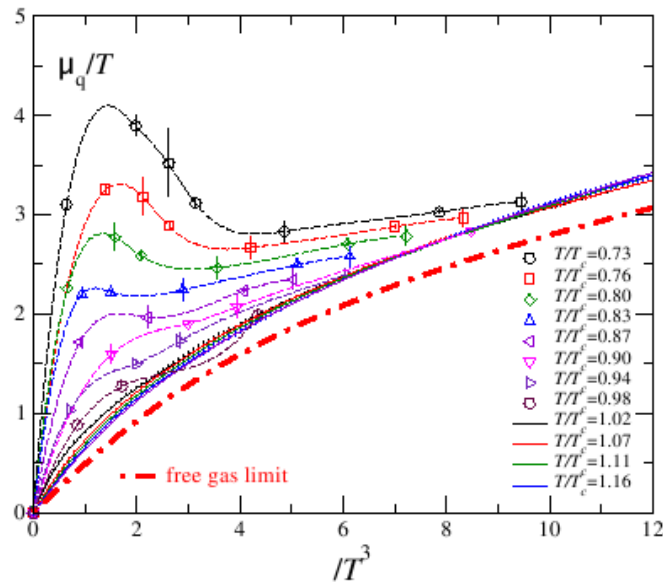


## CANONICAL APPROACH III : Grand Partition Function via a Taylor Expansion.

Ejiri, 2007,2008

$$\frac{Z_{GC}(T, \mu_q)}{Z_{GC}(T, 0)} \equiv \left\langle e \left[ N_f N_t V \sum_{n=1}^{\infty} D_n \left( \frac{\mu_q}{T} \right)^n \right] \right\rangle_{(T, \mu_q=0)},$$

$$\frac{Z_C(T, \bar{\rho}V)}{Z_{GC}(T, 0)} \text{ at large } V \approx \left\langle \exp [V (N_f N_t \sum_{n=1}^{\infty} D_n z_0^n - \bar{\rho}z_0)] e^{-i\alpha/2} \sqrt{\frac{1}{V|D''(z_0)|}} \right\rangle_{(T, \mu=0)}$$



Results for  $N_f = 2$  :

- Qualitative change at  $T/T_c \simeq 0.8$
- Indication of First Order Line

Ejiri 2008

## DENSITY OF STATES

Luo, Azcoiti et al., Ambjorn et al., Anagnostopoulos and J. Nishimura,

$$\langle O \rangle = \int d\phi \langle O f(U) \rangle_{\phi} \rho(\phi) / \int d\phi \langle f(U) \rangle_{\phi} \rho(\phi), \phi \text{ fixed}$$

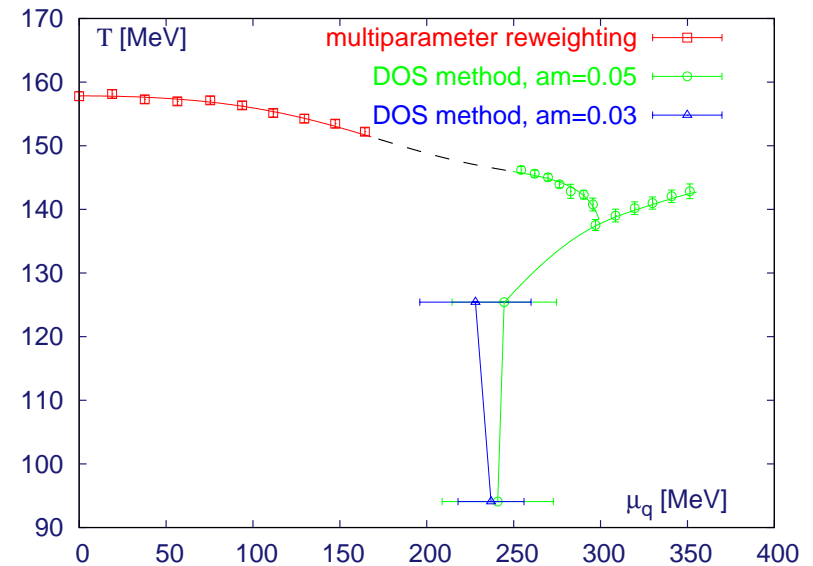
Density of states –  $\rho$  –  
constrained partition function:

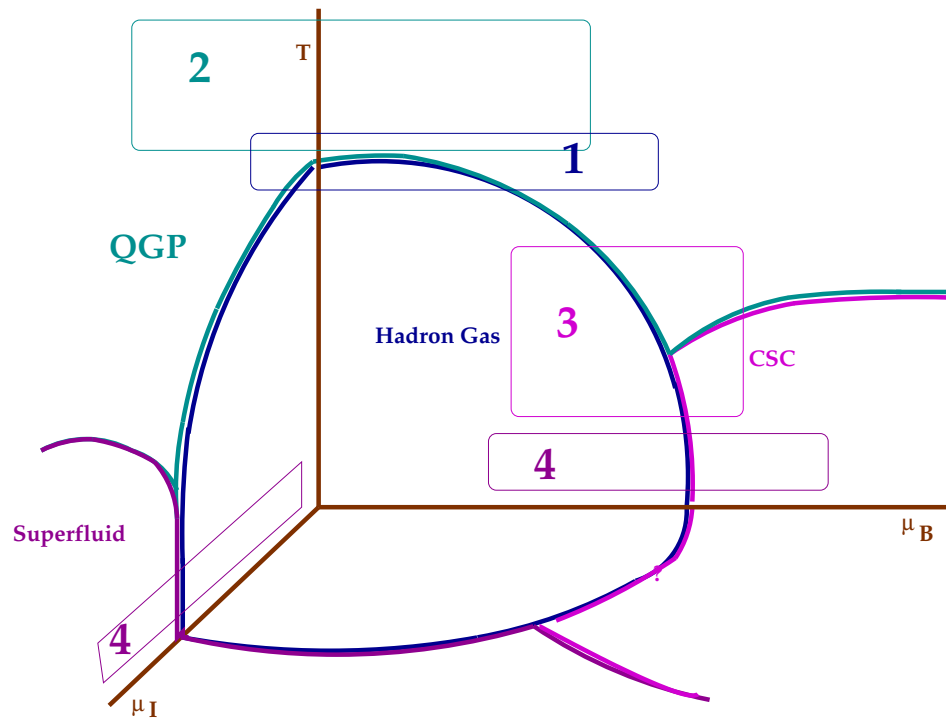
$$\rho(x) = \int \mathcal{D}U g(U) \delta(\phi - x).$$

Results for  $N_f = 4$

- Signal of two phase transition lines
- Indication for a triple point

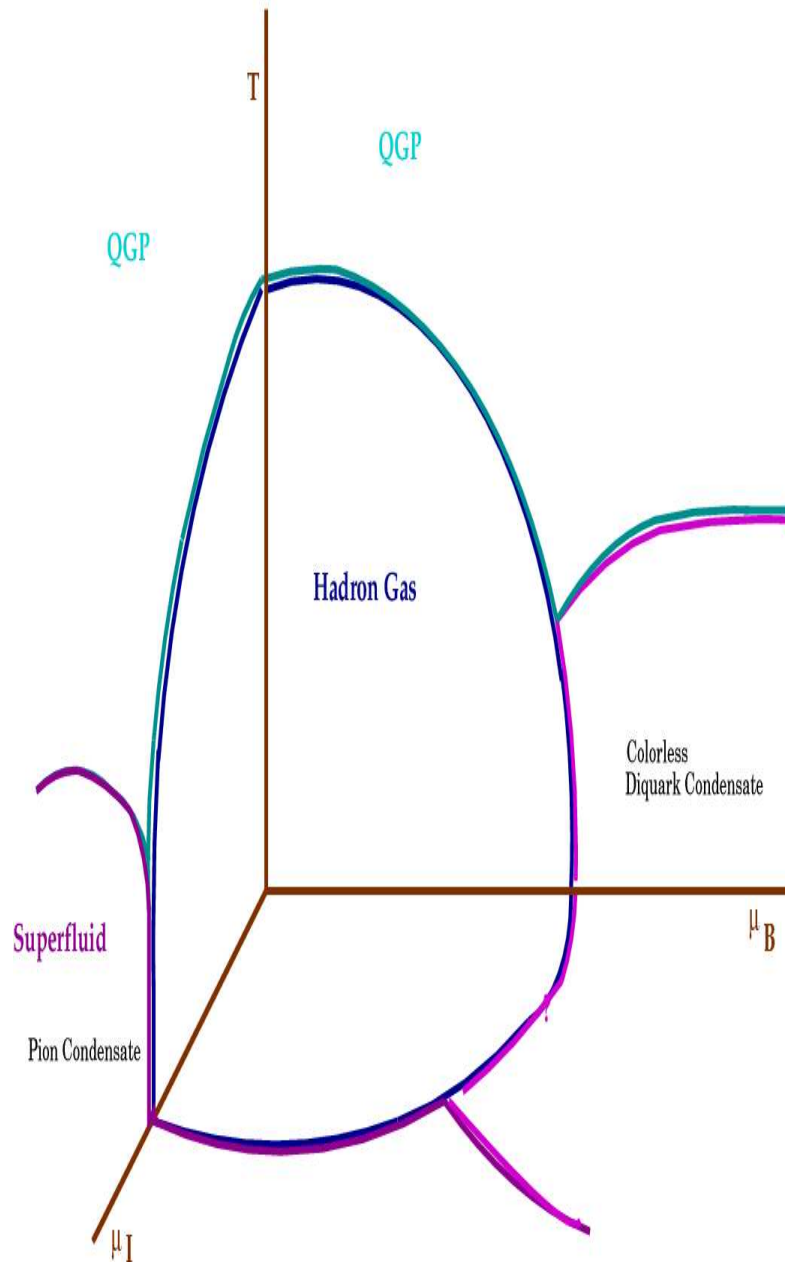
Fodor, Katz, Schmidt 2007





## 4. COLD AND DENSE MATTER





## TWO COLOR QCD

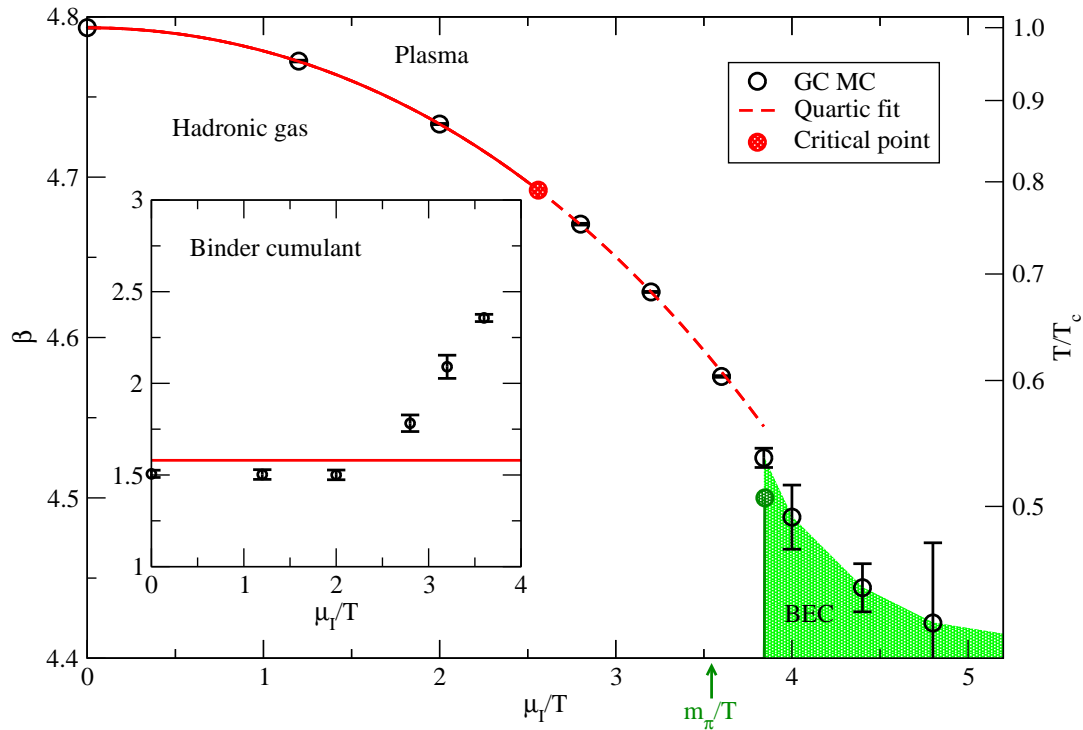
- **Symmetry :**  $qq \longleftrightarrow \bar{q}q$
- **Symmetry :**  $\mu_B \longleftrightarrow \mu_I$
- **At T=0**  $\mu_B^c = \mu_I^c = m_\pi/2$
- **Diquark Condensate for**  $\mu_B > \mu_B^c$
- **Pion Condensate for**  $\mu_B > \mu_I^c$
- **No sign problem**

## QCD at $\mu_I \neq 0$ and TWO COLOR QCD at $\mu_B \neq 0$

### No sign problem, many studies!

- Low Temperature, Fermionic Sector
  - Strong coupling analysis [Dagotto, Karsch, Moreo, Wolff, 1986]
  - Laboratory for diquark condensation [Alford, Rajagopal and Wilczek; Rapp, Schäfer, Shuryak and Velkovsky, 1998]
  - Numerical studies of symmetries and spectrum [Hands, Kogut, Morrison, MpL 1998]
  - Lattice studies of diquark condensation [Hands, Kogut, Morrison, Sinclair; 1998; Aloisio, Azcoiti, di Carlo, Galante, 1999]
  - RMT analysis [Akemann, Splittorff, Toublan, Verbaarschot, 2001]
  - Studies of the Dirac spectrum [Bittner, Markum, Pullirsch, MpL, 2001]
  - $\mu_c(T=0) = m_\pi/2$  : amenable to  $\chi$ PT analysis.  
[Kogut, Stephanov, Toublan, Verbaarschot and Zhitnitsky, 2001; Kogut, Toublan, Sinclair 2002]
  - EoS and Gluon Condensate from  $\chi$ PT [Metlitsky, Toublan, Zhitnisky, 2005, 2006]
  - Model studies of the vector sector [Lenaghan, Sannino, Splittorff 2002]
  - Vector spectroscopy on the lattice [Alles, D'Elia, Pepe, MpL, 2001; Muroya, Nakamura, Nonaka, 2003]
  - Test Bed for Imaginary  $\mu$  [Cea, Cosmai, Giudice, D'Elia, Papa, 2006, 2007]
- High T , Gluon Sector finite  $\mu$  transition from hadron gas to QGP, similar to the T=0 transition
  - Critical Line [Kogut, Sinclair, 2003-2008]
  - Topological Susceptibility [ Alles, D'Elia, Pepe, MpL 2002]
  - Polyakov Loop [Muroya, Nakamura, 2004; Alles, D'Elia, MpL 2006]
  - Order Parameter for Confinement [D'Elia Conradi 2007]
  - Critical Line [Forcrand, Stephanov, Wenger, 2007]
- Low T : Chiral Symmetry , Confinement, BEC Phase
  - Confinement/Deconfinement, Screening [Hands, Kim, Skullerud 2006–2008]
  - Glueballs spectrum, screening [Lombardo, Paciello, Petrarca, Taglienti, 2007]
  - BEC Phase, interface BEC/QGP [Forcrand, Stephanov, Wenger, 2007]

# THE PHASE DIAGRAM FOR A FINITE ISOSPIN DENSITY



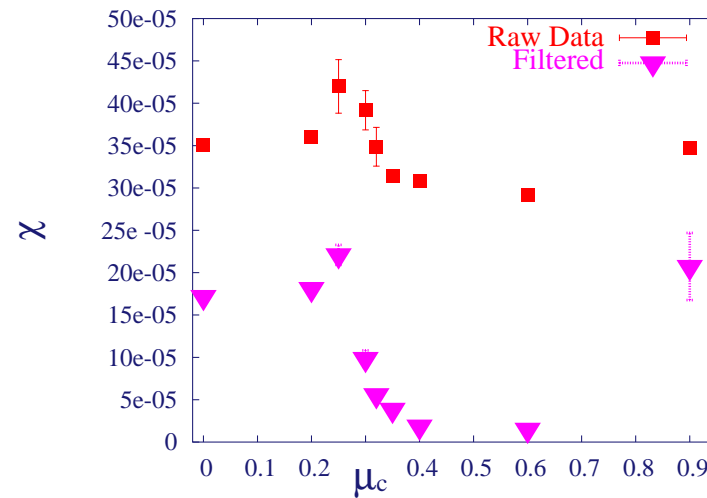
Forcrand, Stephanov, Wenger, 2007

# GLUONIC OBSERVABLES IN THE BEC PHASE of QC<sub>2</sub>D

0<sup>++</sup> Glueball : lighter in the BEC phase

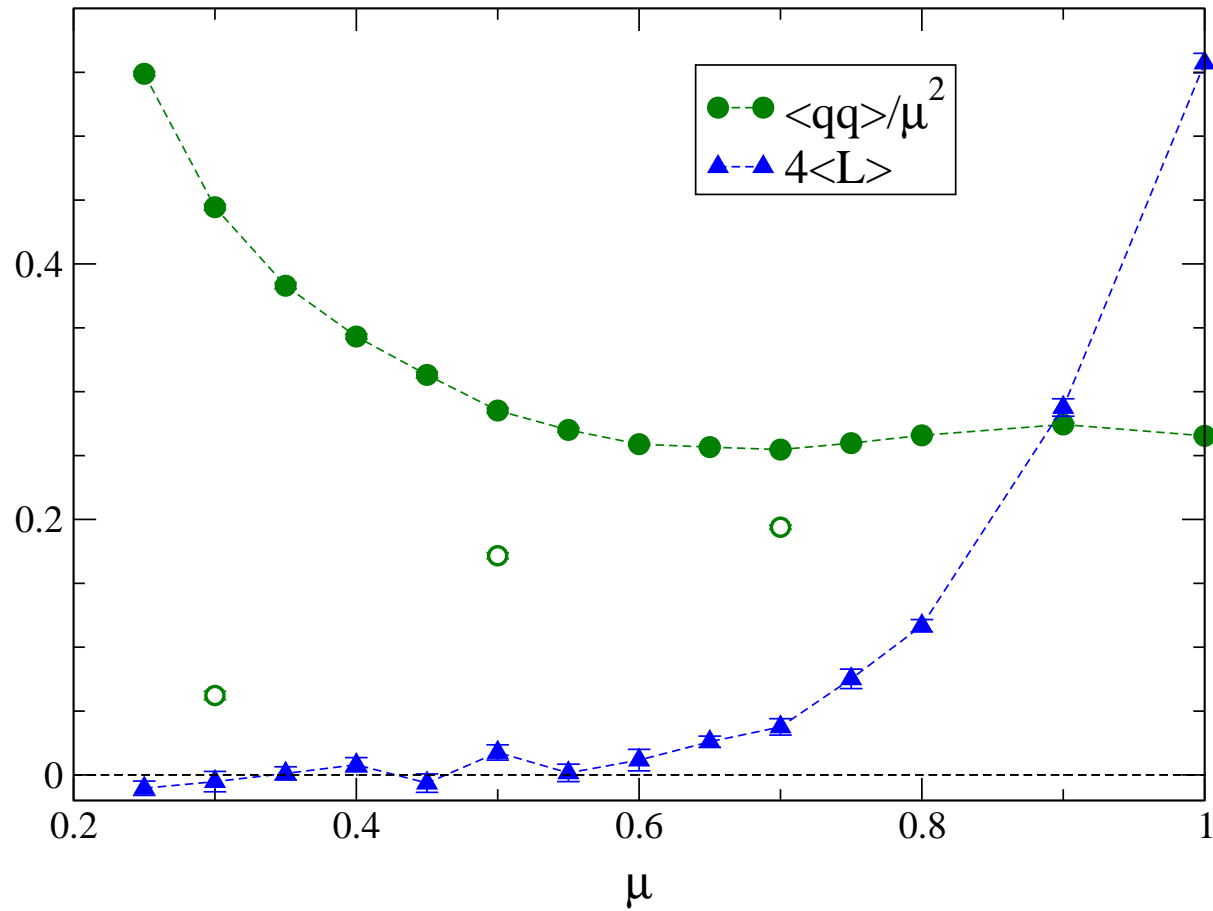
Susceptibility:  $\chi = \langle P^2 \rangle - \langle P \rangle^2$  peaks at  $\mu_c$

Normal Phase	
$m_\pi/m_\rho$	$m_{0^{++}}/m_\rho$
0.40	1.07
0.42	1.26
BEC	
0.64	0.80
0.80	0.23



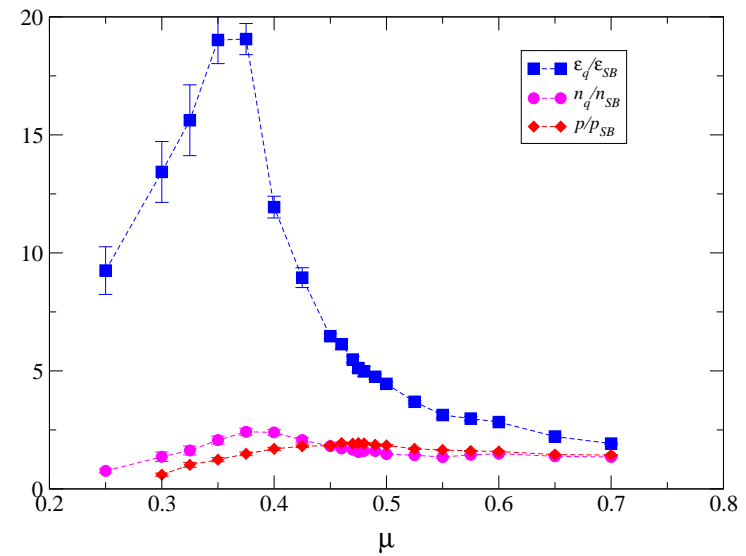
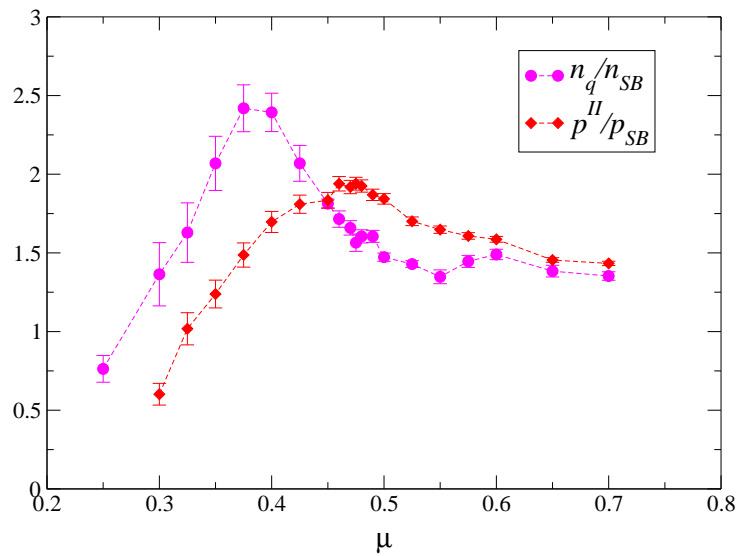
Lombardo, Petrarca, Paciello, Taglienti, 2007

## BEC AND CONFINEMENT: deconfinement deep in the BEC phase

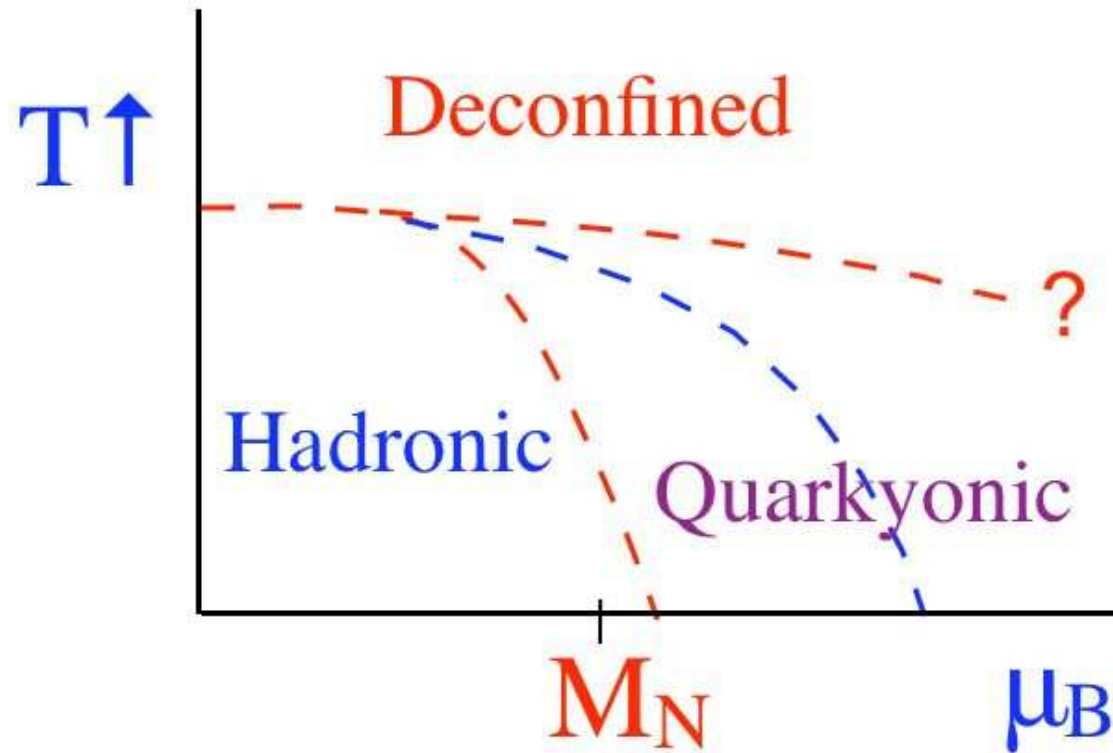


Hands, Kim, Skullerud 2006,2007

## DECONFINEMENT IN THE BEC PHASE – BEC/BCS crossover



Hands, Kim, Skullerud 2008; preliminary, courtesy S. Hands



From Small  $N_c = 2$  to  
**Large  $N_c$  : Proposed Phase Diagram**  
 McLerran, Pisarski, 2007



## QCD AT NON-ZERO DENSITY: LATTICE SUMMARY

**Small  $\mu$  : Mature calculations.**

**Deviation from Hadron Resonance Gas Model in the critical region**

**Mass spectrum depends on  $\mu_B$**

**Qualitative difference critical surface QCD / NJL**

**Role of confinement in the determining the fate of the endpoint?**

**Critical line towards the continuum limit.**

**Attempts at beating the sign problem.**

**Critical line at lower temperature 'bends down'**

**First evidences of a triple point in the  $T, \mu_B$  plane**

**Cold Phases studied at  $\mu_I \neq 0$  /  $QC_2D$  at  $\mu_B \neq 0$**

**Examples of interesting patterns confinement/chiral symmetry.**

**BEC/BCS Crossover observed in two color QCD.**